Erratum to: Surjective isometries on spaces of vector valued continuous and Lipschitz functions

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In the original article, the authors claimed incorrectly that the set of extreme points of the unit ball of a subspace A of $C(X, F)$ (X a compact Hausdorff space and F a Banach space) containing the constant functions is equal to $\{v^* \circ \delta_x : A \rightarrow \mathbb{C}, v^* \in \text{ext}(F_1^*), x \in X\}$, $\text{ext}(F_1^*)$ denoting the set of extreme points of the unit ball of the dual space F^* . We obtain the same conclusion but under some constraints on the range space and on A . More precisely, we assume that the range space is a reflexive Banach space with strictly convex dual and A separates X in the sense of Definition 3.1 in the original article. Strict convexity of the dual implies that F is smooth and then for every unit vector $u \in F$ there exists a unique functional $u^* \in F_1^*$ such that $u^*(u) = 1$. Reflexivity implies that every functional is norm attaining. We prove the following result.

Corollary (cf. Corollary 2.3 in the original article) *Suppose F is a reflexive Banach space with strictly convex dual space, X is a compact Hausdorff topological space, and A is a subspace of $C(X, F)$ that contains the constant functions (i.e. $A \supseteq \text{Const}(X, F)$) and separates X . Then*

$$\text{ext}(A_1^*) = \{v^* \circ \delta_x : A \rightarrow \mathbb{C}, v^* \in \text{ext}(F_1^*), x \in X\}.$$

Professor Jamison passed away in November 2014.

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First, we recall the definition of separation and a result on extreme points of the dual of a subspace of $C(X, F)$ due to Kalkurni and Limaye.

Definition (cf. Definition 3.1 in the original article) A subspace A of $C(X, F)$ is said to separate X if and only if for every pair of distinct points x_0 and x_1 in X , there exists a continuous scalar valued function $\lambda : X \rightarrow [0, 1]$ with $\lambda(x_0) = 0$ and $\lambda(x_1) = 1$ such that all functions of the form $\lambda \cdot v$ ($v \in F$) are in A .

We adapt the Kalkurni and Limaye theorem to our notation.

Theorem (cf. Theorem 2 (b) in [2]) Let A be a nonzero subspace of $C(X, F)$ and $\phi \in A_1^*$. Assume that whenever

$\psi \in \text{ext}(C(X, F)_1^*)$ and $\psi(f) = 1, \forall f \in A$ with $\phi(f) = 1 = \|f\|$, implies $\psi|_A = \phi$ (henceforth referred as condition $(\star)$), then $\phi \in \text{ext}(A_1^*)$.

Proof of Corollary. Proposition 2.2 in the original article implies that

$$\text{ext}(A_1^*) \subseteq \{v^* \circ \delta_x : A \rightarrow \mathbb{C} \text{ s.t. } v^* \in \text{ext}(F_1^*), x \in X\}.$$

The reverse inclusion follows from Kulkarni–Limaye theorem. We consider $\phi = u^* \circ \delta_x \in A_1^*$, with $u^* \in \text{ext}(F_1^*)$ and $x \in X$. Let ψ be an extreme point of $C(X, F)_1^*$ (i.e. $\psi = w^* \circ \delta_y$, with $w^* \in \text{ext}(F_1^*)$ and $y \in X$). We assume that ψ satisfies the hypothesis in $(\star)$, therefore $w^*(f(y)) = 1$, for every $f \in A$ such that $\phi(f) = 1 = \|f\|$, and we prove that $\psi|_A = \phi$. Since F is reflexive every functional is norm attaining (see Proposition 1.11.11 in [3], p. 101), hence there exists a unit vector $u \in F$ such that $u^*(u) = 1$. The hypothesis in $(\star)$ applied to the constant function equal to u implies that $w^*(u) = 1$. The strict convexity of F^* , and then the smoothness of F , imply that $w^* = u^*$ (see Corollary 5.4.3 in [3], p. 481). Hence $\psi = u^* \circ \delta_y$. Further, if $x \neq y$, there exists a Uryshon's function $\lambda : X \rightarrow [0, 1]$ such that $\lambda(x) = 1$ and $\lambda(y) = 0$. For $f = \lambda \cdot u$, we have $\psi(f) = 1 = \phi(f)$ which leads to an absurdity. Then $x = y$ and $\psi|_A = \phi$. An application of Kulkarni–Limaye theorem completes the proof. $\square$

Remark The strict convexity of the dual of F implies that each functional of norm 1 is an extreme point of the unit ball (see Proposition 5.1.10 in [3], p. 431). Then the conclusion of Corollary can be rephrased as

$$\text{ext}(A_1^*) = \{v^* \circ \delta_x : A \rightarrow \mathbb{C}, x \in X, v^* \in F_1^* \text{ with } \|v^*\| = 1\}.$$

The range spaces in the statements of Propositions 3.2 and 3.3 in the original article are assumed to be reflexive with strictly convex dual spaces. The proofs given are correct. These results were applied in Sect. 4 (and 5) to surjective isometries on vector valued spaces of Lipschitz functions, $\text{Lip}_p^\alpha(X, F)$ (and $\text{lip}_p^\alpha(X, F)$, respectively) with $p > 1$ and $0 < \alpha \leq 1$. The norm $\|f\|_p = \sqrt[p]{\|f\|_\infty^p + L_\alpha(f)^p}$, with $p < \infty$. We refer the reader to [1, 4] for $p = \infty$.

As written in the original article, the surjective isometry T of $\text{Lip}_p^\alpha(X, F)$ induces an isometry T_1 on A , which is the subspace of $C(X \times \beta(\tilde{X}), F \oplus_p F)$ consisting

of all functions f of the form $f(x, \xi) = (g(x), g^\beta(\xi))$, with $g \in Lip_p^\alpha(X, F)$. The dual operator T_1^* defines a bijection between the extreme points of the unit ball of A^* . Extreme points of A_1^* are of the form $(u^*, v^*) \circ \delta_{x, \xi}$ with $\|(u^*, v^*)\|_q = 1$, $x \in X$ and $\xi \in \beta(\tilde{X})$. The author was unable to show that functionals of the form $(u^*, 0) \circ \delta_{x, \xi}$, with $\|u^*\| = 1$, $x \in X$ and $\xi \in \beta(\tilde{X})$, are extreme points of A_1^* . For this reason we reformulate Theorems 4.1 and 5.1. The proofs are as in the aforementioned paper with a correction that the p -norm of $f(z) = \frac{1}{\sqrt[p]{2}} \max\{0, 1 - d(z, \tau(x_0))^\alpha\}u$ is less or equal to 1 and thus $1 \geq 2L_\alpha(T(f))^p$.

We introduce the set

$$S = \{(u^*, v^*) \circ \delta_{x, \xi} \in A_1^* : \|(u^*, v^*)\|_q = 1, x \in X \text{ and } \xi \in \beta(\tilde{X})\}.$$

Definition A linear isometry T on $Lip_p^\alpha(X, F)$ is said to preserve S provided that $T_1^*(S) \subseteq S$. If, in addition, T is surjective then $T_1^*(S) = S$.

Theorem (cf. Theorem 4.1 in the original article) *Let F be a finite dimensional Banach space with strictly convex dual space, X a compact metric space, $p > 1$ and $0 < \alpha \leq 1$. If T is a surjective isometry on $Lip_p^\alpha(X, F)$ that preserves the constant functions and preserves S , then there exist a surjective isometry $U : F \rightarrow F$ and an isometry $\tau : X \rightarrow X$, such that*

$$T(f)(x) = U(f(\tau(x))), \text{ for all } f \in Lip_p^\alpha(X, F).$$

and

Theorem (cf. Theorem 5.1 in the original article) *Let F be a finite dimensional Banach space with strictly convex dual space, X a compact metric space, $p > 1$ and $0 < \alpha \leq 1$. If T is a surjective isometry on $lip_p^\alpha(X, F)$ that preserves the constant functions and preserves S , then there exist a surjective isometry $U : F \rightarrow F$ and an isometry $\tau : X \rightarrow X$ such that*

$$T(f)(x) = U(f(\tau(x))), \text{ for all } f \in lip_p^\alpha(X, F), \text{ and } x \in X.$$

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