Erratum to: Semilocal convergence of an eighth-order method in Banach spaces and its computational efficiency

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1 Scalar functions, recurrence relations and semilocal convergence

The method discussed here in the Banach spaces is given by [1]

$$\begin{aligned}y_k &= \frac{1}{2}(r_k + x_k), \\z_k &= \frac{1}{3}(4y_k - x_k), \\u_k &= y_k + [F'(x_k) - 3F'(z_k)]^{-1}F(x_k), \\v_k &= u_k + 2[F'(x_k) - 3F'(z_k)]^{-1}F(u_k), \\x_{k+1} &= v_k + 2[F'(x_k) - 3F'(z_k)]^{-1}F(v_k),\end{aligned}\quad (1.1)$$

where I is the identity operator on X , $\Gamma_k = F'(x_k)^{-1}$, $r_k = x_k - \Gamma_k F(x_k)$. Here we give only the modified scalar function appearing in [1], rest are the same. The correct value of $g(t)$ is

$$g(t) = \frac{71t^7 - 576t^6 + 1832t^5 - 3104t^4 + 3120t^3 - 1920t^2 + 704t - 128}{128(t-1)^7}. \quad (1.2)$$

The online versions of the original articles can be found at
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Table 1 Results of recurrence relations

k	η_k	β_k	a_k	b_k	c_k	$h(a_k)p_3(a_k, b_k, c_k)$
0	0.228817	1.94541	0.222571	0.0509278	0.0116531	2.0496e-5
1	4.68982e-6	3.20034	7.50452e-6	3.51948e-11	1.65058e-16	1.01684e-37
2	4.76880e-43	3.20037	7.63096e-43	3.63905e-85	1.73539e-127	1.14294e-296
3	5.45048-339	3.20037	8.72176e-339	4.75377e-677	2.59103e-1015	2.91206e-2368

Other scalar functions such as $h(t)$, p_1 , p_2 and p_3 are correct. Although the expression for $g(t)$ is altered, the theoretical results are as it is but the some tabulated numerical values are changed and the corrected numerical results are mentioned in the Tables 1 and 2.

2 Numerical results

Consider as in [1], the nonlinear integral equation $F(x) = 0$, where

$$F(x)(s) = x(s) - \frac{4}{3} + \frac{1}{2} \int_0^1 s \cos(x(t)) dt, \quad (2.1)$$

where $s \in [0, 1]$, $x \in \Omega = B(0, 2) \subset X$ and $X = C[0, 1]$ with the max-norm

$$\|x\| = \max_{s \in [0, 1]} |x(s)|.$$

It can be seen that

$$F'(x)y(s) = y(s) - \frac{1}{2} \int_0^1 s \sin(x(t))y(t)dt, \quad y \in \Omega,$$

$$\|F''(x)\| \leq \frac{1}{2} = M, \quad \|F'''(x)\| \leq \frac{1}{2} = N, \quad x \in \Omega,$$

$$\|F'''(x) - F'''(y)\| \leq \frac{1}{2} \|x - y\|, \quad x, y \in \Omega,$$

with $L = 1/2$. By assuming $x_0 = 4/3$,

$$\|F(x_0)\| \leq \frac{1}{2} \cos \frac{4}{3}, \quad \|I - F'(x_0)\| \leq \frac{1}{2} \sin \frac{4}{3}.$$

Table 2 Comparison results for the error bounds

k	Super Halley Method	Method M_5	Method M_6	Method MZG_6	Method (1.1)
1	1.7963e-3	1.9957e-4	7.9728e-3	3.9875e-4	8.2625e-6
2	9.6411e-12	1.4148e-19	4.4764e-10	6.9776e-21	8.3901e-42
3	1.9302e-44	8.7504e-95	2.0144e-48	1.6065e-120	3.0923e-328

The above relations implies that

$$\begin{aligned} \|\Gamma_0\| \leq \beta = 1.94541, \|\Gamma_0 F(x_0)\| \leq \eta = 0.228817, a_0 = 0.222571, \\ b_0 = 0.0509278, c_0 = 0.0116531 \end{aligned}$$

Due to modification in $g(t)$, now we have

$$z(a_0) = a_0 g(a_0) - 1 = -1.39 < 0.$$

and

$$h^2(a_0)p_3(a_0, b_0, c_0) = 6.43 \times 10^{-6} < 1, .$$

The above values fulfill the assumptions of the main theorem and the corrected recurrence relations for the method (1.1) are listed in the Table 1. Due to change in $g(t)$, the ball containing solution and unique solution changed and thus the solution $x^* \in \overline{B(x_0, R\eta)} = \overline{B(4/3, 0.40314\dots)} \subset \Omega$ and it is unique in $B(4/3, 1.2499\dots) \cap \Omega$. The correct error bounds obtained by method (1.1) are mentioned in the Table 2; for other schemes the results are unchanged. There is no change in the Tables 3-5, since these value does not depend on the value of $g(t)$.

References

1. Jaiswal, J.P.: Semilocal convergence of an eighth-order method in Banach spaces and its computational efficiency. Numer. Algor. **71**(4), 933–951 (2016)