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ERRATUM

# Erratum to: Parameter identification and adaptive impulsive synchronization of uncertain complex-variable chaotic systems

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## Introduction

In the original article [1], there are some errors in the proof of Theorem 1. Now, we point out the mistakes as follows:

The inequality (11) $\ln\alpha+(h-2d^*)(t_k-t_{k-1})\le 0$, where $\alpha>1$ and $t_k-t_{k-1}>0$ in Theorem 1 implies that $h-2d^*<0$. Therefore, with the Lyapunov type function (12)

$$V(t)=e^T(t)\overline{e(t)}+\tilde{\Theta}^T\bar{\tilde{\Theta}}+\sum_{i=1}^{n}\frac{(d_i-d_i^*)^2}{k_i},$$

the inequality enlargement $D^+V(t)\le(h-2d^*)e^T(t)\overline{e(t)}\le(h-2d^*)V(t)$ is incorrect, which appeared in the line 27 of the left column on page 960 in Ref. [1].

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For correcting the mistakes in the original paper, we slightly revise them and a correct version of Theorem 1 is given. The corresponding proof is also given in this paper.

## Correct version of Theorem 1 [1]

**Assumption 1** Suppose that there exists a constant diagonal matrix $H=\mathrm{diag}(h_1,h_2,\ldots,h_n)$ such that the complexvalued vector function $F(x,\Theta)$ satisfies

$$(y-x)^T\overline{(F(y,\Theta)-F(x,\Theta))}+(F(y,\Theta)-F(x,\Theta))^T\overline{(y-x)}\le(y-x)^TH\overline{(y-x)}.$$

**Theorem 1** *Suppose Assumption 1 holds. The parameters updating law is designed as*

$$\begin{cases}\dot{\hat{\Theta}}=-\overline{g(y(t))}^Te(t), & t\ne t_k,\\ \Delta\tilde{\Theta}=A_k\tilde{\Theta}, & t=t_k, k=1,2,\ldots,\end{cases}\tag{10}$$

*where* $\tilde{\Theta}=\hat{\Theta}-\Theta$, $A_k\in R^{n\times n}$ *is impulsive gain matrix. Let* $\alpha_k=\lambda_{\max}\left((I+A_k)^T(I+A_k)\right)<1$, $\beta_k=\lambda_{\max}\left((I+B_k)^T(I+B_k)\right)<1$, $\mu_k=\max(\alpha_k,\beta_k)$ *and* $d^*$ *is the minimum value of the initial feedback strength* $d_{i0}$ ($d_{i0}\le d_i, 1\le i\le n$)*. If there exist a constant* $\alpha>1$ *such that*

$$\ln\alpha\mu_k+(h-2d^*)(t_k-t_{k-1})\le 0,\quad k=1,2,\ldots.\tag{11}$$

*Then the response chaotic complex system (3) can synchronize the drive chaotic complex system (2) asymptotically with the impulsive controller (5), the adaptive controller (9) and the parameter update law (10) in Ref. [1]. Moreover, $\hat{\Theta} \to \Theta$ , that is, all the unknown parameters are identified exactly.*

*Proof* Select the following Lyapunov type function defined as

$$V(t) = e^T(t)\overline{e(t)} + \tilde{\Theta}^T\bar{\tilde{\Theta}} \tag{12}$$

For $t \neq t_k$, the derivative of (12) along the trajectories of (7) and Assumption 1, we have

$$\begin{aligned} D^+V(t) &= \dot{e}^T(t)\overline{e(t)} + e^T(t)\overline{\dot{e}(t)} + \dot{\tilde{\Theta}}^T\bar{\tilde{\Theta}} + \tilde{\Theta}^T\dot{\bar{\tilde{\Theta}}} \\ &= [F(y(t),\Theta) - F(x(t),\Theta)]^T\overline{e(t)} \\ &\quad + e^T(t)\overline{[F(y(t),\Theta) - F(x(t),\Theta)]} \\ &\quad + [g(y(t))\tilde{\Theta}]^T\overline{e(t)} + e^T(t)\overline{[g(y(t))\tilde{\Theta}]} \\ &\quad + [U_1]^T\overline{e(t)} + e^T(t)\overline{U_1} \\ &\quad + \dot{\tilde{\Theta}}^T\bar{\tilde{\Theta}} + \tilde{\Theta}^T\dot{\bar{\tilde{\Theta}}} \\ &\le e^T(t)H\overline{e(t)} + [g(y(t))\tilde{\Theta}]^T\overline{e(t)} \\ &\quad + e^T(t)\overline{[g(y(t))\tilde{\Theta}]} \\ &\quad + \dot{\tilde{\Theta}}^T\bar{\tilde{\Theta}} + \tilde{\Theta}^T\dot{\bar{\tilde{\Theta}}} - 2\sum_{i=1}^{n} d_i e_i^T(t)\overline{e_i(t)} \end{aligned}$$

Denot $h = \max(h_1, h_2, \ldots, h_n)$ and $d^* = \min(d_{10}, d_{20}, \ldots, d_{n0})$, substitute Eqs. (9–10) into the above inequality, we further have

$$\begin{aligned} D^+V(t) &\le (h - 2d^*)e^T(t)\overline{e(t)} \\ &\le (h - 2d^*)V(t) \end{aligned}$$

with $h - 2d^* > 0$.

This implies that

$$\begin{aligned} &V(t) \le V(t_{k-1}^+)\exp((h - 2d^*)(t - t_{k-1})), \\ &\quad t \in (t_{k-1}, t_k], \quad k = 1, 2, \ldots. \end{aligned} \tag{13}$$

On the other hand, when $t = t_k$ , from Eqs. (8) and (10) in Ref. [1], we have

$$\begin{aligned} V(t_k^+) &\le e^T(t_k)(I + B_k)^T(I + B_k)\overline{e(t_k)} \\ &\quad + \widetilde{\Theta(t_k)}^T(I + A_k)^T(I + A_k)\overline{\widetilde{\Theta(t_k)}} \\ &\le \beta_k e^T(t_k)\overline{e(t_k)} + \alpha_k\widetilde{\Theta(t_k)}^T\overline{\widetilde{\Theta(t_k)}} \\ &\le \mu_k V(t_k) \end{aligned} \tag{14}$$

When $k = 1$ in inequality (13), then for any $t \in (t_0, t_1]$

$V(t) \le V(t_0^+)\exp\big((h - 2d^*)(t - t_0)\big)$ .

This leads to

$V(t_1) \le V(t_0^+)\exp\big((h - 2d^*)(t_1 - t_0)\big)$ .

Also from (14) we have

$$\begin{aligned} V(t_1^+) &\le \mu_1 V(t_1) \\ &\le \mu_1 V(t_0^+)\exp\big((h - 2d^*)(t_1 - t_0)\big) \end{aligned}$$

In the same way for $t \in (t_1, t_2]$, we have

$$\begin{aligned} V(t) &\le V(t_1^+)\exp\big((h - 2d^*)(t - t_1)\big) \\ &\le \mu_1 V(t_0^+)\exp\big((h - 2d^*)(t - t_0)\big). \end{aligned}$$

In general for any $t \in (t_k, t_{k+1}]$, one finds that

$$V(t) \le \quad \mu_1\mu_2\ldots\mu_k V(t_0^+)\exp\big((h - 2d^*)(t - t_0)\big) \tag{15}$$

From the condition given in the Theorem 1, we have

$\mu_k \exp\big((h - 2d^*)(t_{k+1} - t_k)\big) \le \frac{1}{\alpha}, \quad k = 1, 2, \cdots.$

Notice that

$$\begin{aligned} t - t_0 &= (t - t_k) + (t_k - t_{k-1}) + (t_{k-1} - t_{k-2}) \\ &\quad + \cdots + (t_2 - t_1) + (t_1 - t_0). \end{aligned}$$

Thus, the inequality(15) can be further rewriten as

$$\begin{aligned} V(t) &\le \mu_1\mu_2\ldots\mu_k V(t_0^+)\exp\big((h - 2d^*)(t - t_0)\big) \\ &= V(t_0^+)\big[\mu_1\exp\big((h - 2d^*)(t_1 - t_0)\big)\big] \times \cdots \\ &\quad \times \big[\mu_k\exp\big((h - 2d^*)(t_k - t_{k-1})\big)\big] \\ &\quad \exp\big((h - 2d^*)(t - t_k)\big) \\ &\le V(t_0^+)\frac{1}{\alpha^k}\exp\big((h - 2d^*)(t - t_k)\big) \end{aligned}$$

Therefore, $V(t) \to 0$ when $k \to \infty$ because of $\alpha > 1$, which implies that all the errors $e(t) \to 0$. So the complete synchronization between the controlled chaotic complex system (3) and (2) in [1] is realized.The proof is completed. □

*Remark 1* From proof of Theorem 1, we know that the synchronization error will converge to zero. Fom parameters update laws (10), we find that $\dot{\hat{\Theta}} = 0$ when $e(t) = 0$, which implies that $\hat{\Theta}$ approach to some constants and this does not ensure that $\hat{\Theta} \to \Theta$. To identify the unknown parameters, we suppose that the nonlinear vector function $g^T(y(t))$ should be linearly independent on the synchronization manifold $y = x$. According to the results of Ref. [2], then the unknown parameters $\Theta$ can be identified by $\hat{\Theta}$ as $k \to \infty$.

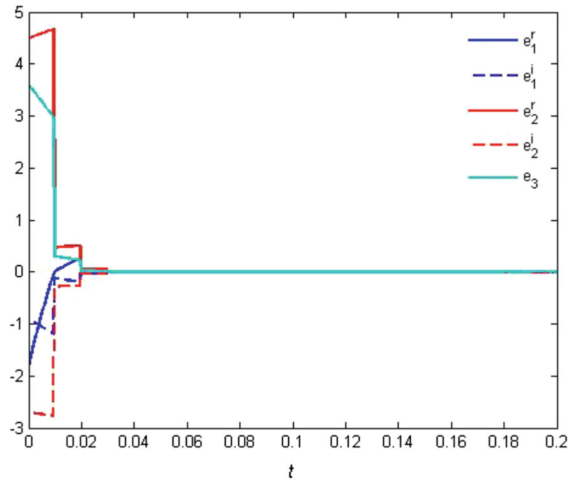


**Fig. 1** Errors of real and imaginary parts of $e_i(t)$.

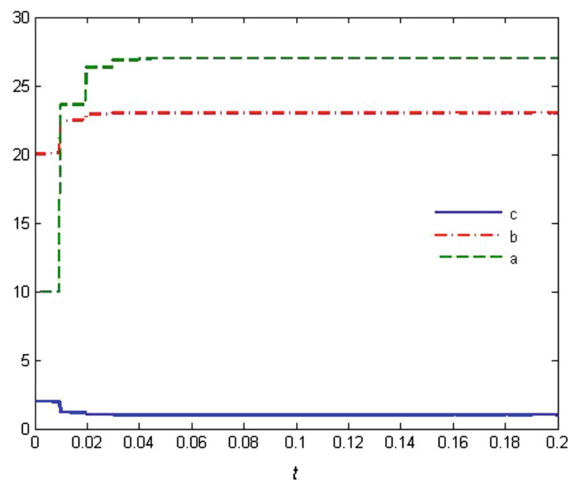


**Fig. 2** Evolution of the three parameters' estimation with time $t$.

In the numerical simulation, let $d_{i0} = 20, \alpha = 2$, $A_k = \mathrm{diag}(-0.8, -0.8, -0.8)$ and $B_k = \mathrm{diag}(-0.5, -0.5, -0.5)$, then, $\mu_k = 0.25$ and $h = 79$ in [1]. From Eq. (11), we can obtain $0 < t_k - t_{k-1} \leq -\frac{\ln \alpha \mu_k}{h-2d^*} = 0.0178$, thus, let the impulsive interval $t_k - t_{k-1} = 0.01$. Figures 1 and 2, respectively, show the synchronization errors $e_i(t)$ and the identified parameters under the updating laws (10).

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## References

1. Zheng, S.: Parameter identification and adaptive impulsive synchronization of uncertain complex-variable chaotic systems. Nonlinear Dyn **74**, 957–967 (2013)
2. Yu, W., Chen, G., Cao, J., Lu, J., Parlitz, U.: Parameter identification of dynamical systems from time series. Phys. Rev. E **75**, 067201 (2007)