Erratum to: Parameter identification and synchronization of uncertain general complex networks via adaptive-impulsive control

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In the original article [1] there are some errors in the proof of Theorem 1. Now, we point out the mistake as follows:

The inequality (9) $[2\alpha\tau_k + \ln(\xi) < 0]$, with $\xi > 1$ and $\tau_k > 0$ in Theorem 1 implies that $\alpha < 0$, where $\alpha = (L + \frac{1}{2}\lambda - d^*)$. Thus, with the Lyapunov function (12) in [1]

$$\begin{aligned} V(t) = & \frac{1}{2} \sum_{i=1}^N e_i^T(t) e_i(t) \\ & + \frac{1}{2} \sum_{i=1}^N (\hat{\Theta}_i - \Theta_i)^T (\hat{\Theta}_i - \Theta_i) \\ & + \frac{1}{2} \sum_{i=1}^N \frac{(d_i - d_i^*)^2}{k_i}, \end{aligned}$$

the enlargement $(L + \frac{1}{2}\lambda - d^*) \sum_{i=1}^N e_i^T(t) e_i(t) \leq 2\alpha V(t)$ will not surely hold, which is appeared in the line 5 of the right column on page 356 [1].

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For correcting the mistakes in the original paper, we slightly revised them and a correct version of Theorem 1 and Corollary 1 are given in this paper.

Theorem 1 Suppose that A1 holds. Let λ be the largest eigenvalue of $(C \otimes A) + (C \otimes A)^T$, if

$$\beta_{ik} = \lambda_{\max}[(I + B_{ik})^T (I + B_{ik})] < 1 \quad (8)$$

and there exist a constant $\xi > 1$ such that

$$2\alpha\tau_k + \ln(\xi) < 0 \quad (9)$$

under the following restriction conditions

$$\begin{aligned} U_i = & -d^* e_i(t) - \mu \|\hat{\Theta} - \Theta\|^2 \frac{e_i}{\|E\|^2}, \\ \dot{\hat{\Theta}}_i = & -g_i^T(t, y_i(t)) e_i(t), \end{aligned} \quad (10)$$

where $d^* > 0$, $\mu > 0$, $\Theta = (\Theta_1^T, \Theta_2^T, \dots, \Theta_N^T)^T$, and $\hat{\Theta}$ is an estimation vector to Θ , $E(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$. Then, the impulsively controlled network (4) and (3) in Ref. [1] is asymptotically synchronous. Moreover,

$$\hat{\Theta}_i \rightarrow \Theta_i, \quad i = 1, 2, \dots, N. \quad (11)$$

Proof Construct the Lyapunov function as follows:

$$\begin{aligned}
V(t) &= \frac{1}{2} \|E(t)\|^2 + \frac{1}{2} \|\hat{\Theta} - \Theta\|^2 \\
&= \frac{1}{2} \sum_{i=1}^N e_i^T(t) e_i(t) \\
&\quad + \frac{1}{2} \sum_{i=1}^N (\hat{\Theta}_i - \Theta_i)^T (\hat{\Theta}_i - \Theta_i). \quad (12)
\end{aligned}$$

Then

$$\dot{V}(t) = \sum_{i=1}^N e_i^T(t) \dot{e}_i(t) + \sum_{i=1}^N \dot{\hat{\Theta}}_i^T (\hat{\Theta}_i - \Theta_i),$$

along with error systems (6) and Assumption 1 in [1], we have

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^N e_i^T(t) [g_i(t, y_i(t)) \cdot (\hat{\Theta}_i - \Theta_i)] \\
&\quad + \sum_{i=1}^N L_i e_i^T(t) e_i(t) \\
&\quad + \sum_{i=1}^N \sum_{j=1}^N e_i^T(t) c_{ij} A e_j(t) + \sum_{i=1}^N e_i^T(t) U_i \\
&\quad + \sum_{i=1}^N \dot{\hat{\Theta}}_i^T (\hat{\Theta}_i - \Theta_i).
\end{aligned}$$

Denote $L = \max_i \{L_i\}$, and notice that $E(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$, substitute Eq. (10) into the above inequality, we further have

$$\begin{aligned}
\dot{V}(t) &\leq L \sum_{i=1}^N e_i^T(t) e_i(t) + E^T(t) (C \otimes A) E(t) \\
&\quad - \sum_{i=1}^N d^* e_i^T(t) e_i(t) \\
&\quad - \mu \sum_{i=1}^N \|\hat{\Theta} - \Theta\|^2 e_i^T \frac{e_i}{\|E\|^2}.
\end{aligned}$$

In fact, $\sum_{i=1}^N e_i^T \frac{e_i}{\|E\|^2} = 1$, thus,

$$\dot{V}(t) \leq L \sum_{i=1}^N e_i^T(t) e_i(t) + E^T(t)$$

$$\begin{aligned}
&\times \frac{(C \otimes A) + (C \otimes A)^T}{2} E(t) \\
&\quad - d^* \sum_{i=1}^N e_i^T(t) e_i(t) - \mu \|\hat{\Theta} - \Theta\|^2.
\end{aligned}$$

Further,

$$\begin{aligned}
\dot{V}(t) &\leq \left(L + \frac{1}{2}\lambda - d^*\right) \sum_{i=1}^N e_i^T(t) e_i(t) - \mu \|\hat{\Theta} - \Theta\|^2 \\
&= \left(L + \frac{1}{2}\lambda - d^*\right) \|E(t)\|^2 - \mu \|\hat{\Theta} - \Theta\|^2 \\
&\leq 2\alpha V(t),
\end{aligned}$$

where $\alpha = \max\{\nu, -\mu\} < 0$ and $\nu = L + \frac{1}{2}\lambda - d^* < 0$ for large enough d^* .

This implies that

$$V(t) \leq V(t_{k-1}^+) e^{2\alpha(t-t_{k-1})}, \quad t \in (t_{k-1}, t_k] \quad (13)$$

then we get the same inequality as Eq. (13) in [1].

For $t = t_k$, from Eq. (7) [1], we have

$$\begin{aligned}
V(t_k^+) &= \frac{1}{2} \sum_{i=1}^N e_i^T(t_k) (I + B_{ik})^T (I + B_{ik}) e_i(t_k) \\
&\quad + \frac{1}{2} \sum_{i=1}^N (\hat{\Theta}_i - \Theta_i)^T (\hat{\Theta}_i - \Theta_i) \\
&\leq V(t_k), \quad (14)
\end{aligned}$$

due to $\beta_{ik} = \lambda_{\max}[(I + B_{ik})^T (I + B_{ik})] < 1$.

From Eqs. (13) and (14), for $t \in (t_k, t_{k+1}]$, there is

$$V(t) \leq V(t_0^+) e^{2\alpha(t-t_0)}. \quad (15)$$

In virtue of the inequality (9) given in Theorem 1, we know that

$$e^{2\alpha\tau_k} < \frac{1}{\xi}, \quad k = 1, 2, \dots$$

Thus, the inequality (15) can be further rewritten as

$$\begin{aligned}
V(t) &\leq V(t_0^+) (e^{2\alpha\tau_1}) \dots (e^{2\alpha\tau_k}) e^{2\alpha(t-t_k)} \\
&< V(t_0^+) \frac{1}{\xi^k} e^{2\alpha\tau_{k+1}}, \quad (16)
\end{aligned}$$

therefore $V(t) \rightarrow 0$ as $k \rightarrow \infty$ because $\xi > 1$, which implies that all the errors $e_i(t) \rightarrow 0$ and $\hat{\Theta}_i \rightarrow \Theta_i$ ($i = 1, 2, \dots, N$). So the synchronization between the

Fig. 1 Synchronization errors $e_i(t)$ ($1 \leq i \leq 5$)

impulsively controlled complex network (4) and network (3) in [1] is realized and the unknown system parameters are identified simultaneously. This completes the proof. $\square$

Remark 1 After the synchronization occurs, that is, all the $e_{ij} \rightarrow 0$ as $t \rightarrow \infty$, from the error systems (6) in [1], one can get $g_i(t, y_i(t))(\hat{\Theta}_i - \Theta_i) = 0$, therefore, the conclusion (11) holds under the condition that all the column vectors of $g_i(t, y_i(t))$ are linear independent.

Corollary 1 If a complex network consists of N identical nodes, which can be described by

$$\dot{x}_i(t) = f(t, x_i(t)) + g(t, x_i(t)) \cdot \Theta + \sum_{j=1}^N c_{ij} A x_j(t), \quad (17)$$

then unknown system parameters Θ can be identified by using the estimated values $\hat{\Theta}$ with the following impulsively controlled response network

$$\begin{cases} \dot{y}_i(t) = f(t, y_i(t)) + g(t, y_i(t)) \cdot \hat{\Theta} \\ \quad + \sum_{j=1}^N c_{ij} A y_j(t) + U_i, & t \neq t_k \\ \Delta y_i(t^+) = B_{ik} e_i(t), & t = t_k, k = 1, 2, \dots \\ y_i(t_0^+) = y_{i0}, \\ U_i = -d^* e_i(t) - \mu \|\hat{\Theta} - \Theta\|^2 \frac{e_i}{\|E\|^2}, \end{cases} \quad (18)$$

Fig. 2 Identification of system parameters

if

$$2\alpha\tau_k + \ln(\xi) < 0 \quad (19)$$

and

$$\dot{\hat{\Theta}} = - \sum_{i=1}^N g^T(t, y_i(t)) e_i(t), \quad (20)$$

where λ is the largest eigenvalue of $(C \otimes A) + (C \otimes A)^T$, the constant $\xi > 1$ and α is defined as in Theorem 1.

In the numerical simulation, let $d^* = 100$, $\mu = 1.5$, $B_{ik} = \text{diag}\{-0.5, -0.5, -0.5\}$, $\tau_k = 0.1$. Figures 1 and 2, respectively show the synchronization errors of $e_{i1}(t)$, $e_{i2}(t)$, $e_{i3}(t)$ and the identified parameters under the updating laws (20).

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References

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