Erratum to: Delayed-state-feedback exponential stabilization for uncertain Markovian jump systems with mode-dependent time-varying state delays

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In Ref. [2] [Nonlinear Dyn. 69:1023–1039 (2012)], since the Lyapunov functionals $V_2(t, x_t, i)$ and $V_3(t, x_t, i)$ are inappropriate, the proofs of Theorems 3.1–3.2 and Theorems 4.1–4.2 need some important improvements. Besides, the results of Examples 5.1–5.3 are necessarily modified.

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1 Introduction

In [2] [Nonlinear Dyn. 69:1023–1039 (2012)], the authors mainly studied the robustly exponential stabilization for the uncertain Markovian jump systems with mode-dependent time-varying state delays. Firstly, by constructing a modified Lyapunov functional and using the free-weighting matrices technique, some delay-dependent robustly exponential stability criteria on such systems can be obtained in terms of linear matrix inequalities (LMIs). Then, based on the obtained stability criteria, a state-feedback controller is designed, which can guarantee the robustly exponential stability of the uncertain closed-loop systems. In [2] [Nonlinear Dyn. 69:1023–1039 (2012)], although the exponential stability of such systems can be obtained by constructing the following Lyapunov functional:

$$V(t, x_t, i) = V_1(t, x_t, i) + V_2(t, x_t, i) + V_3(t, x_t, i),$$

where

$$\begin{aligned} V_1(t, x_t, i) &= e^{\kappa t} x^T(t) P_i x(t), \\ V_2(t, x_t, i) &= \int_{t-h_i(t)}^t e^{\kappa(s+h)} x^T(s) Q_{1i} x(s) ds \\ &\quad + \int_{t-h}^t e^{\kappa(s+h)} x^T(s) Q_{1i} x(s) ds, \end{aligned}$$

$$\begin{aligned}
V_3(t, x_t, i) &= \int_{-h}^0 \int_{t+\theta}^t e^{\kappa(s-\theta)} x^T(s) (Q_2 + Q_3) x(s) ds d\theta \\
&\quad + \int_{-h}^0 \int_{t+\theta}^t e^{\kappa(s-\theta)} \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta,
\end{aligned}$$

the state-feedback controller is very difficult to design since there are two important problems to be considered. Firstly, the inequalities in (4.1) of [2] [Nonlinear Dyn. 69:1023–1039 (2012)] are bilinear matrix inequalities, in which the bilinear constraints are not convex and are very hard to be solved. Thus, there is a confusion in Example 5.3 [Nonlinear Dyn. 69:1023–1039 (2012)]; secondly, in order to design the state-feedback controller, two variables

$$R_i = X_i^T Q_1 X_i, \quad (1.1)$$

and

$$T_i = X_i^T Q_2 X_i, \quad (1.1)$$

defined in Theorem 4.1 of [2] [Nonlinear Dyn. 69:1023–1039 (2012)] are not rational, where X_i , R_i and T_i are mode-dependent design parameters. These two equalities are impossible for each $i \in S$ since matrices Q_1 and Q_2 are mode-independent. In detail, finding a constant Q_1 or Q_2 for all obtained parameters X_i , R_i or T_i such that $Q_1 = X_i^{-T} R_i X_i^{-1}$, for each $i \in S$, is generally impossible.

To solve these two problems, we revisit to construct an appropriate Lyapunov functional, and thus the proofs of Theorems 3.1–3.2 and Theorems 4.1–4.2 need some important improvements. Besides, the results of Examples 5.1–5.3 are necessarily modified.

3 Main results

In the following, the new proof of Theorem 3.1 is presented, in which (2.1), (2.3)–(2.4), (2.6), (2.8) represent the equations in the publication work [Nonlinear Dyn. 69:1023–1039 (2012)].

Theorem 3.1 *For a given scalar $\kappa > 0$, and the time-varying delays $h_i(t)$ satisfies $0 \leq h_i(t) \leq h_i \leq h$ and $\dot{h}_i(t) \leq \mu_i$ ($i \in S$). If there exist some positive definite symmetric matrices P , Q_1 , Q_2 , Q_3 , Q_4 and some appropriately dimensional matrices: $N_i = [N_{1i}^T, N_{2i}^T, N_{3i}^T, N_{4i}^T, N_{5i}^T]^T$, $M_i = [M_{1i}^T, M_{2i}^T, M_{3i}^T,$*

$M_{4i}^T, M_{5i}^T]^T$ ($i \in S$), such that the following linear matrix inequalities (LMIs) hold:

$$\begin{aligned}
\Pi_i &= \begin{bmatrix} \Omega_i & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22} \end{bmatrix} < 0, \\
\Psi_i &= \begin{bmatrix} \Omega_i & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22} \end{bmatrix} < 0, \\
\Phi_i &= \begin{bmatrix} \Omega_i & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22} \end{bmatrix} < 0, \\
\Upsilon_i &= \begin{bmatrix} \Omega_i & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22} \end{bmatrix} < 0,
\end{aligned} \quad (3.1)$$

where

$$\begin{aligned}
\Omega_i &= \begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} \\ * & \Omega_{i22} & \Omega_{i23} \\ * & * & \Omega_{i33} \end{bmatrix}, \\
\Pi_{i12} = \Psi_{i12} &= \begin{bmatrix} hN_{4i} & -hN_{1i}^T & A_i^T Q_4 \\ -hN_{4i} + hM_{4i} & -hN_{2i}^T & A_{id}^T Q_4 \\ -hM_{4i} & -hD_{3i}^T & 0_{n \times n} \end{bmatrix}, \\
\Pi_{i22} &= \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - hQ_3 & -h^2 N_{4i}^T & 0_{n \times n} \\ * & -hQ_4 & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 \end{bmatrix}, \\
\Psi_{i22} &= \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - hQ_3 & -h^2 N_{5i}^T & 0_{n \times n} \\ * & -hQ_4 & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 \end{bmatrix}, \\
\Phi_{i12} = \Upsilon_{i12} &= \begin{bmatrix} hN_{5i} & -hM_{1i}^T & A_i^T Q_4 \\ -hN_{5i} + hM_{5i} & -hM_{2i}^T & A_{id}^T Q_4 \\ -hM_{5i} & -hM_{3i}^T & 0_{n \times n} \end{bmatrix}, \\
\Phi_{i22} &= \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - hQ_3 & -h^2 M_{4i}^T & 0_{n \times n} \\ * & -hQ_4 & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 \end{bmatrix}, \\
\Upsilon_{i22} &= \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - hQ_3 & -h^2 M_{5i}^T & 0_{n \times n} \\ * & -hQ_4 & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\Omega_{i11} &= \kappa P_i + P_i A_i + A_i^T P_i + \sum_{j=1}^N \gamma_{ij} P_j \\
&\quad + e^{\kappa h} Q_1 + e^{\kappa h} Q_2 + \frac{e^{\kappa h} - 1}{\kappa} Q_3 + N_{1i}^T + N_{1i}, \\
\Omega_{i12} &= P_i A_{id} + N_{2i} - N_{1i}^T + M_{1i}^T, \\
\Omega_{i13} &= N_{3i} - M_{1i}^T,
\end{aligned}$$

$$\begin{aligned}\Omega_{i22} &= [-(1 - \mu_i)e^{\kappa h} Q_1] \vee [-(1 - \mu_i) Q_1] - N_{2i}^T \\ &\quad - N_{2i} + M_{2i}^T + M_{2i},\end{aligned}$$

$$\Omega_{i23} = -N_{3i} - M_{2i}^T + M_{3i},$$

$$\Omega_{i33} = -Q_2 - M_{3i}^T - M_{3i},$$

and $*$ means symmetric terms, the nominal Markovian jump systems (2.6) when $u(t) = 0$ are exponentially stable.

Proof Define a Lyapunov–Krasovskii functional candidate for systems (2.6) as

$$V(t, x_t, i) = V_1(t, x_t, i) + V_2(t, x_t, i) + V_3(t, x_t, i), \quad (3.2)$$

where

$$x_t = x(t + \theta), \quad -2h \leq \theta \leq 0,$$

and

$$V_1(t, x_t, i) = e^{\kappa t} x^T(t) P_i x(t),$$

$$\begin{aligned}V_2(t, x_t, i) &= \int_{t-h_i(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds \\ &\quad + \int_{t-h}^t e^{\kappa(s+h)} x^T(s) Q_2 x(s) ds,\end{aligned}$$

$$\begin{aligned}V_3(t, x_t, i) &= \int_{-h}^0 \int_{t+\theta}^t e^{\kappa(s-\theta)} x^T(s) Q_3 x(s) ds d\theta \\ &\quad + \int_{-h}^0 \int_{t+\theta}^t e^{\kappa(s-\theta)} \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta.\end{aligned}$$

Let $\mathfrak{L}$ be the weak infinitesimal generator of the random process $\{(x_t, r(t)), t \geq 0\}$. For each $i \in S$ and $t > h$, we have

$$\begin{aligned}\mathfrak{L}V(t, x_t, i) &= \mathfrak{L}V_1(t, x_t, i) + \mathfrak{L}V_2(t, x_t, i) \\ &\quad + \mathfrak{L}V_3(t, x_t, i),\end{aligned} \quad (3.3)$$

where

$$\begin{aligned}\mathfrak{L}V_1(t, x_t, i) &= e^{\kappa t} x^T(t) \left[\kappa P + P_i A_i + A_i^T P_i + \sum_{j=1}^N \gamma_{ij} P_j \right] x(t) \\ &\quad + 2e^{\kappa t} x^T(t) P_i A_{id} x(t - h_i(t)),\end{aligned} \quad (3.4)$$

$$\begin{aligned}\mathfrak{L}V_2(t, x_t, i) &= e^{\kappa t} [x^T(t) [e^{\kappa h} Q_1 + e^{\kappa h} Q_2] x(t) \\ &\quad - (1 - \dot{h}_i(t)) x^T(t - h_i(t)) \\ &\quad \times e^{-\kappa(h_i(t)-h)} Q_1 x(t - h_i(t)) \\ &\quad - x^T(t - h) Q_2 x(t - h)] \\ &\quad + \sum_{j=1}^N \gamma_{ij} \int_{t-h_j(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds,\end{aligned} \quad (3.5)$$

$$\begin{aligned}\mathfrak{L}V_3(t, x_t, i) &= e^{\kappa t} \left[x^T(t) \frac{e^{\kappa h} - 1}{\kappa} Q_3 x(t) \right. \\ &\quad \left. + \dot{x}^T(t) \frac{e^{\kappa h} - 1}{\kappa} Q_4 \dot{x}(t) \right] \\ &\quad - e^{\kappa t} \int_{t-h_i(t)}^t e^{\kappa h} x^T(s) Q_3 x(s) ds \\ &\quad - e^{\kappa t} \int_{t-h}^{t-h_i(t)} e^{\kappa h} x^T(s) Q_3 x(s) ds \\ &\quad - e^{\kappa t} \int_{t-h_i(t)}^t e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s) ds \\ &\quad - e^{\kappa t} \int_{t-h}^{t-h_i(t)} e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s) ds.\end{aligned} \quad (3.6)$$

By calculation, as $\mu_i \leq 1$ ($i \in S$),

$$\begin{aligned}&-(1 - \dot{h}_i(t)) e^{\kappa h - \kappa h_i(t)} x^T(t - h_i(t)) Q_1 x(t - h_i(t)) \\ &\leq -(1 - \mu_i) e^{\kappa h - \kappa h_i(t)} x^T(t - h_i(t)) Q_1 x(t - h_i(t)) \\ &\leq -(1 - \mu_i) x^T(t - h_i(t)) Q_1 x(t - h_i(t)),\end{aligned} \quad (3.7)$$

and as $\mu_i > 1$ ($i \in S$),

$$\begin{aligned}&-(1 - \dot{h}_i(t)) e^{\kappa h - \kappa h_i(t)} x^T(t - h_i(t)) Q_1 x(t - h_i(t)) \\ &\leq -(1 - \mu_i) e^{\kappa h - \kappa h_i(t)} x^T(t - h_i(t)) Q_1 x(t - h_i(t)) \\ &\leq -(1 - \mu_i) e^{\kappa h} x^T(t - h_i(t)) Q_1 x(t - h_i(t)).\end{aligned} \quad (3.8)$$

Thus, from (3.8) and (3.9) it follows that

$$\begin{aligned}&-(1 - \dot{h}_i(t)) e^{\kappa h - \kappa h_i(t)} x^T(t - h_i(t)) Q_1 x(t - h_i(t)) \\ &\leq x^T(t - h_i(t)) [-(1 - \mu_i) e^{\kappa h} Q_1] \\ &\quad \vee [-(1 - \mu_i) Q_1] x(t - h_i(t)).\end{aligned} \quad (3.9)$$

Noting that $\gamma_{ij} \geq 0$, for $j \neq i$ and $\gamma_{ii} \leq 0$, it follows that

$$\begin{aligned} & \sum_{j=1}^N \gamma_{ij} \int_{t-h_j(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds \\ & \leq \sum_{j \neq i} \gamma_{ij} \int_{t-h_j(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds \\ & \quad - |\gamma_{ii}| \int_{t-h_i(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds \\ & \leq \sum_{j \neq i} \gamma_{ij} \int_{t-h}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds \\ & = |\gamma_{ii}| \int_{t-h}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds. \end{aligned} \quad (3.10)$$

On the other hand, from the Newton–Leibniz formula [9, 10, 15, 16, 18, 21, 23], it is clear that

$$\begin{aligned} a_{1i}(t) &:= 2e^{\kappa t} \zeta^T(t) N_i \left[x(t) - x(t-h_i(t)) \right. \\ & \quad \left. - \int_{t-h_i(t)}^t \dot{x}(s) ds \right] = 0, \end{aligned} \quad (3.11)$$

$$\begin{aligned} a_{2i}(t) &:= 2e^{\kappa t} \zeta^T(t) M_i \left[x(t-h_i(t)) - x(t-h) \right. \\ & \quad \left. - \int_{t-h}^{t-h_i(t)} \dot{x}(s) ds \right] = 0, \end{aligned} \quad (3.12)$$

where

$$\begin{aligned} \zeta(t) &= \left[x^T(t), x^T(t-h_i(t)), x^T(t-h), \right. \\ & \quad \left. \left[\int_{t-h_i(t)}^t x(s) ds \right]^T, \left[\int_{t-h}^{t-h_i(t)} x(s) ds \right]^T \right]^T. \end{aligned}$$

Substituting (3.4)–(3.12) into (3.3) yields

$$\begin{aligned} & \mathfrak{L}V(t, x_t, i) \\ & \leq e^{\kappa t} \left\{ x^T(t) \left[\kappa P_i + P_i A_i + A_i^T P_i + \sum_{j=1}^N \gamma_{ij} P_j \right. \right. \\ & \quad \left. \left. + e^{\kappa h} Q_1 + e^{\kappa h} Q_2 + \frac{e^{\kappa h} - 1}{\kappa} Q_3 \right] x(t) \right. \\ & \quad \left. + 2x^T(t) P_i A_{id} x(t-h_i(t)) \right. \\ & \quad \left. + x^T(t-h_i(t)) [-(1-\mu_i)e^{\kappa h} Q_1] \right. \end{aligned}$$

$$\begin{aligned} & \left. + [-(1-\mu_i)Q_1] x(t-h_i(t)) \right. \\ & \quad \left. - x^T(t-h) Q_2 x(t-h) \right. \\ & \quad \left. + |\gamma_{ii}| \int_{t-h}^t e^{\kappa h} x^T(s) Q_1 x(s) ds \right. \\ & \quad \left. - e^{\kappa t} \int_{t-h_i(t)}^t e^{\kappa h} x^T(s) Q_3 x(s) ds \right. \\ & \quad \left. - e^{\kappa t} \int_{t-h}^{t-h_i(t)} e^{\kappa h} x^T(s) Q_3 x(s) ds \right. \\ & \quad \left. + \dot{x}^T(t) \frac{e^{\kappa h} - 1}{\kappa} Q_4 \dot{x}(t) \right. \\ & \quad \left. - e^{\kappa t} \int_{t-h_i(t)}^t e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s) ds \right. \\ & \quad \left. - e^{\kappa t} \int_{t-h}^{t-h_i(t)} e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s) ds \right\} \\ & + a_{1i}(t) + a_{2i}(t) \\ & \leq \frac{1}{h^2} \int_{t-h_i(t)}^t \int_{t-h_i(t)}^t e^{\kappa t} \eta^T(t, s, \alpha, i) \\ & \quad \times \Pi'_i \eta(t, s, \alpha, i) ds d\alpha \\ & \quad + \frac{1}{h^2} \int_{t-h_i(t)}^{t-h_i(t)} \int_{t-h_i(t)}^t e^{\kappa t} \eta^T(t, s, \alpha, i) \\ & \quad \times \Psi'_i \eta(t, s, \alpha, i) ds d\alpha \\ & \quad + \frac{1}{h^2} \int_{t-h_i(t)}^t \int_{t-h}^{t-h_i(t)} e^{\kappa t} \eta^T(t, s, \alpha, i) \\ & \quad \times \Phi'_i \eta(t, s, \alpha, i) ds d\alpha \\ & \quad + \frac{1}{h^2} \int_{t-h}^{t-h_i(t)} \int_{t-h}^{t-h_i(t)} e^{\kappa t} \eta^T(t, s, \alpha, i) \\ & \quad \times \Upsilon'_i \eta(t, s, \alpha, i) ds d\alpha, \end{aligned} \quad (3.13)$$

where $\xi(t, s, \alpha, i) = [x^T(t), x^T(t-h_i(t)), x^T(t-h), \dot{x}^T(s)]^T$, and

$$\begin{aligned} \Pi'_i &= \begin{bmatrix} \Omega_i & \Pi'_{i12} \\ \Pi'^T_{i12} & \Pi'_{i22} \end{bmatrix} + \frac{e^{\kappa h} - 1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i, \\ \Psi'_i &= \begin{bmatrix} \Omega_i & \Psi'_{i12} \\ \Psi'^T_{i12} & \Psi'_{i22} \end{bmatrix} + \frac{e^{\kappa h} - 1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i, \\ \Phi'_i &= \begin{bmatrix} \Omega_i & \Phi'_{i12} \\ \Phi'^T_{i12} & \Phi'_{i22} \end{bmatrix} + \frac{e^{\kappa h} - 1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i, \\ \Upsilon'_i &= \begin{bmatrix} \Omega_i & \Upsilon'_{i12} \\ \Upsilon'^T_{i12} & \Upsilon'_{i22} \end{bmatrix} + \frac{e^{\kappa h} - 1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i, \end{aligned}$$

$$\begin{aligned}\Pi'_{i12} &= \Psi'_{i12} = \begin{bmatrix} hN_{4i} & -hN_{1i}^T \\ -hN_{4i} + hM_{4i} & -hN_{2i}^T \\ -hM_{4i} & -hN_{3i}^T \end{bmatrix}, \\ \Phi'_{i12} &= \Upsilon'_{i12} = \begin{bmatrix} hN_{5i} & -hM_{1i}^T \\ -hN_{5i} + hM_{5i} & -hM_{2i}^T \\ -hM_{5i} & -hM_{3i}^T \end{bmatrix}, \\ \Pi'_{i22} &= \begin{bmatrix} he^{\kappa h}|\gamma_{ii}|Q_1 - hQ_3 & -h^2N_{4i} \\ * & -hQ_4 \end{bmatrix}, \\ \Psi'_{i22} &= \begin{bmatrix} he^{\kappa h}|\gamma_{ii}|Q_1 - hQ_3 & -h^2N_{5i}^T \\ * & -hQ_4 \end{bmatrix}, \\ \Psi'_{i22} &= \begin{bmatrix} he^{\kappa h}|\gamma_{ii}|Q_1 - hQ_3 & -h^2M_{4i} \\ * & -hQ_4 \end{bmatrix}, \\ \Upsilon'_{i22} &= \begin{bmatrix} he^{\kappa h}|\gamma_{ii}|Q_1 - hQ_3 & -h^2M_{5i}^T \\ * & -hQ_4 \end{bmatrix}. \\ \tilde{A}_i &= [Q_4^T A_i, Q_4^T A_{id}, 0].\end{aligned}$$

It follows from (3.1), by applying Lemma 2.3 in [2] [Nonlinear Dyn. 69:1023–1039 (2012)], that for each $i \in S$

$$\Pi'_i < 0, \quad \Psi'_i < 0, \quad \Phi'_i < 0, \quad \Upsilon'_i < 0.$$

And, by virtue of (3.1), when $t > h$, it from (3.13) follows that

$$\mathcal{L}V(t, x_t, i) \leq 0. \quad (3.14)$$

Let $\alpha_1 = \max_{i \in S} \{|A_i|\}$ and $\alpha_2 = \max_{i \in S} \{|A_{id}|\}$. Thus, it follows from $\dot{x}(t) = A_i x(t) + A_{id} x(t - h_i(t))$ that for any $t \geq 0$,

$$\begin{aligned}|x(t)| &= \left| x(0) + \int_0^t [A_i x(s) + A_{id} x(s - h_i(s))] ds \right| \\ &\leq |x(0)| + \int_0^t \alpha_2 |x(s - h_i(s))| ds \\ &\quad + \int_0^t \alpha_1 |x(s)| ds.\end{aligned}$$

When $0 \leq t \leq h$, the above inequality implies that

$$|x(t)| \leq [\alpha_2 h + 1] \sup_{-h \leq \theta \leq 0} |\varphi(\theta)| + \int_0^t \alpha_1 |x(s)| ds.$$

By applying the Gronwall inequality to this inequality, we obtain, when $0 \leq t \leq h$,

$$|x(t)| \leq [\alpha_2 h + 1] e^{\alpha_1 h} \sup_{\theta \in [-h, 0]} |\varphi(\theta)|. \quad (3.15)$$

Hence, we have, when $0 \leq t \leq h$,

$$|x(t)|^2 \leq \nu \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2, \quad (3.16)$$

where $\nu = [\alpha_2 h + 1]^2 e^{2\alpha_1 h}$. Now, by employing the Dynkin's formula, (3.14) and (3.16), it follows that, when $t > h$,

$$\begin{aligned}EV(t, x_t, r(t)) &= EV(h, x_h, r(h)) + E \int_h^t \mathcal{L}V(x_s, r(s), s) ds \\ &\leq EV(h, x_h, r(h)) \\ &= e^{\kappa h} E x^T(h) P_i x(h) \\ &\quad + E \int_{h-h_i(h)}^h e^{\kappa(s+h)} x^T(s) Q_1 x(s) ds \\ &\quad + E \int_0^h e^{\kappa(s+h)} x^T(s) Q_2 x(s) ds \\ &\quad + E \int_{-h}^0 \int_{h+s}^h e^{\kappa h} x^T(s) Q_3 x(s) ds d\theta \\ &\quad + E \int_{-h}^0 \int_{h+s}^h e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta \\ &\leq e^{\kappa h} \max_{i \in S} \{|P_i|\} \nu E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 \\ &\quad + e^{\kappa h} |Q_1| \frac{e^{\kappa h} - 1}{\kappa} E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 \\ &\quad + |Q_2| e^{\kappa h} \frac{e^{\kappa h} - 1}{\kappa} E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 \\ &\quad + |Q_3| e^{\kappa h} \frac{e^{\kappa h} - \kappa h - 1}{\kappa^2} E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 \\ &\quad + 2|Q_4| (\alpha_1^2 \nu + \alpha_2^2) e^{\kappa h} \frac{e^{\kappa h} - \kappa h - 1}{\kappa^2} \\ &\quad \times E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2.\end{aligned}$$

On the other hand,

$$EV(t, x_t, i) \geq \frac{1}{\max_{i \in S} \{|P_i^{-1}|\}} e^{\kappa t} E |x(t)|^2.$$

Hence, when $t > h$,

$$E|x(t)|^2 \triangleq \sigma(h)E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 e^{-\kappa t}, \quad (3.17)$$

where

$$\begin{aligned} \sigma(h) = & \max_{i \in S} \{ |P_i^{-1}| \} \left\{ e^{\kappa h} \max_{i \in S} \{ |P_i| \} v \right. \\ & + e^{\kappa h} |Q_1| \frac{e^{\kappa h} - 1}{\kappa} + |Q_2| e^{\kappa h} \frac{e^{\kappa h} - 1}{\kappa} \\ & + |Q_3| e^{\kappa h} \frac{e^{\kappa h} - \kappa h - 1}{\kappa^2} \\ & \left. + 2|Q_4| (\alpha_1^2 v + \alpha_2^2) e^{\kappa h} \frac{e^{\kappa h} - \kappa h - 1}{\kappa^2} \right\}. \end{aligned}$$

When $0 \leq t \leq h$, we obtain

$$\begin{aligned} & \frac{1}{\max_{i \in S} \{ |P_i^{-1}| \}} e^{\kappa t} E|x(t)|^2 \\ & \leq e^{\kappa t} E x^T(t) P_i x(t) \\ & \leq e^{\kappa h} \max_{i \in S} \{ |P_i| \} v E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2. \end{aligned}$$

That is,

$$\begin{aligned} E|x(t)|^2 & \leq \max_{i \in S} \{ |P_i^{-1}| \} e^{\kappa h} \max_{i \in S} \{ |P_i| \} v \\ & \quad \times E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 e^{-\kappa t}. \end{aligned} \quad (3.18)$$

It follows from (3.17) and (3.18) that when $t > 0$,

$$E|x(t)|^2 \leq \sigma'(h)E \sup_{\theta \in [-h, 0]} |\varphi(\theta)|^2 e^{-\kappa t},$$

where $\sigma'(h) = \max\{\sigma(h), \max_{i \in S} \{ |P_i^{-1}| \} e^{\kappa h} \times \max_{i \in S} \{ |P_i| \} v\}$. Thus, the nominal systems (2.6) are exponentially stable with the decay rate κ . The proof of this theorem is completed. $\square$

Now, we generalize Theorem 3.1 to the corresponding uncertain case, and give the following sufficient conditions on the robustly exponential stability for the uncertain Markovian jump systems (2.1) when $u(t) = 0$.

Theorem 3.2 For a given scalar $\kappa > 0$, and the time-varying delays $h_i(t)$ satisfies $0 \leq h_i(t) \leq h_i \leq h$ and $\dot{h}_i(t) \leq \mu_i$ ($i \in S$). If there exist some positive definite symmetric matrices P , Q_1 , Q_2 , Q_3 , Q_4 and some appropriately dimensional matrices $N_i = [N_{1i}^T, N_{2i}^T, N_{3i}^T, N_{4i}^T, N_{5i}^T]^T$, $M_i = [M_{1i}^T, M_{2i}^T, M_{3i}^T, M_{4i}^T, M_{5i}^T]^T$ ($i \in S$), and some positive scalars ε_{1i} , ε_{2i} , ε_{3i} , ε_{4i} ($i \in S$) such that the following linear matrix inequalities (LMIs) hold:

$$\begin{aligned} \Pi'_i &= \begin{bmatrix} \Omega_i^1 & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22} \end{bmatrix} < 0, \\ \Psi'_i &= \begin{bmatrix} \Omega_i^2 & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22} \end{bmatrix} < 0, \\ \Phi'_i &= \begin{bmatrix} \Omega_i^3 & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22} \end{bmatrix} < 0, \\ \Upsilon'_i &= \begin{bmatrix} \Omega_i^4 & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22} \end{bmatrix} < 0, \end{aligned} \quad (3.19)$$

where

$$\begin{aligned} \Omega_i^k &= \begin{bmatrix} \Omega_{i11}^k & \Omega_{i12}^k & \Omega_{i13}^k \\ * & \Omega_{i22}^k & \Omega_{i23}^k \\ * & * & \Omega_{i33}^k \end{bmatrix}, \quad k = 1, 2, 3, 4, \\ \Pi_{i12} &= \Psi_{i12} = \begin{bmatrix} hN_{4i} & -hN_{1i}^T & A_i^T Q_4 & P_i^T E_i \\ -hN_{4i} + hM_{4i} & -hN_{2i}^T & A_{id}^T Q_4 & 0_{n \times n} \\ -hM_{4i} & -hN_{3i}^T & 0_{n \times n} & 0_{n \times n} \end{bmatrix}, \\ \Pi_{i22} &= \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - hQ_3 & -h^2 N_{4i}^T & 0_{n \times n} & 0_{n \times n} \\ * & -hQ_4 & 0_{n \times n} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 & Q_4^T E_i \\ * & * & * & -\varepsilon_{1i} I \end{bmatrix}, \end{aligned}$$

$$\Psi_{i22} = \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - h Q_3 & -h^2 N_{5i}^T & 0_{n \times n} & 0_{n \times n} \\ * & -h Q_4 & 0_{n \times n} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 & Q_4^T E_i \\ * & * & * & -\varepsilon_{2i} I \end{bmatrix},$$

$$\Phi_{i12} = \Upsilon_{i12} = \begin{bmatrix} h N_{5i} & -h M_{1i}^T & A_i^T Q_4 & P_i^T E_i \\ -h N_{5i} + h M_{5i} & -h M_{2i}^T & A_{id}^T Q_4 & 0_{n \times n} \\ -h M_{5i} & -h M_{3i}^T & 0_{n \times n} & 0_{n \times n} \end{bmatrix},$$

$$\Phi_{i22} = \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - h Q_3 & -h^2 M_{4i}^T & 0_{n \times n} & 0_{n \times n} \\ * & -h Q_4 & 0_{n \times n} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 & Q_4^T E_i \\ * & * & * & -\varepsilon_{3i} I \end{bmatrix},$$

$$\Upsilon_{i22} = \begin{bmatrix} he^{\kappa h} |\gamma_{ii}| Q_1 - h Q_3 & -h^2 M_{5i}^T & 0_{n \times n} & 0_{n \times n} \\ * & -h Q_4 & 0_{n \times n} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} Q_4 & Q_4^T E_i \\ * & * & * & -\varepsilon_{4i} I \end{bmatrix},$$

$$\Omega_{i11}^k = \kappa P_i + P_i A_i + A_i^T P_i + \sum_{j=1}^N \gamma_{ij} P_j + e^{\kappa h} Q_1 + e^{\kappa h} Q_2 + \frac{e^{\kappa h} - 1}{\kappa} Q_3 + N_{1i}^T + N_{1i} + \varepsilon_{ki} H_{1i}^T H_{1i},$$

$$\Omega_{i12}^k = P_i A_{id} + N_{2i} - N_{1i}^T + M_{1i}^T + \varepsilon_{ki} H_{1i}^T H_{2i}, \quad k = 1, 2, 3, 4, i \in S.$$

Ω_{i13} , Ω_{i22} , Ω_{i23} , Ω_{i33} are given in Theorem 3.1 and * means the symmetric terms, the uncertain Markovian jump systems (2.1) when $u(t) = 0$ are robustly exponentially stable.

Proof Replacing A_i , A_{id} in (3.1) with $A_i + \Delta A_i(t)$, $A_{id} + \Delta A_{id}(t)$, $\Delta A_i(t)$, $\Delta A_{id}(t)$ are described in (2.3) and (2.4), we obtain

$$\Pi'_i = \Pi_i + \Gamma_i^T F_i(t) \Delta_i + \Delta_i^T F_i(t) \Gamma_i, \quad i \in S,$$

where

$$\Gamma_i = [E_i^T P_i \quad 0 \quad 0 \quad 0 \quad 0 \quad E_i^T Q_4],$$

$$\Delta_i = [H_{1i} \quad H_{2i} \quad 0 \quad 0 \quad 0 \quad 0].$$

From Lemma 2.4 in [2] [Nonlinear Dyn. 69:1023–1039 (2012)], we know $\Pi'_i < 0$ ($i \in S$) and this is equivalent to

$$\Pi''_i = \Pi_i + \varepsilon_{1i}^{-1} \Gamma_i^T \Gamma_i + \varepsilon_{1i} \Delta_i^T \Delta_i,$$

where $\varepsilon_{1i} > 0$ ($i \in S$). From $\Pi'_i < 0$ ($i \in S$), by using Lemma 2.3 in [2] [Nonlinear Dyn. 69:1023–1039 (2012)], we know that $\Pi''_i < 0$ ($i \in S$). Similarly, the

other three cases can be proved. Hence, the proof is completed. $\square$

4 A state-feedback controller design

In this section, we consider the robust stabilization for the uncertain Markovian jump systems. When a control in law (2.6) is applied to systems (2.1), the closed-loop systems (2.8) become

$$\dot{x}(t) = (\hat{A}_i + E_i F_i(t) \hat{H}_{1i}) x(t) + (\hat{A}_{id} + E_i F_i(t) \hat{H}_{2i}) x(t - h_i(t)), \quad (4.1)$$

where $\hat{A}_i = A_i + B_i K_i$, $\hat{A}_{id} = A_{id} + B_i K_{id}$, $\hat{H}_{1i} = H_{1i} + H_{3i} K_i$ and $\hat{H}_{2i} = H_{2i} + H_{3i} K_{id}$ ($i \in S$).

Theorem 4.1 For a given scalar $\kappa > 0$, and the time-varying delays $h_i(t)$ satisfies $0 \leq h_i(t) \leq h_i \leq h$ and $\dot{h}_i(t) \leq \mu_i$ ($i \in S$) and tuning matrices: $\tilde{N}_i = [\tilde{N}_{1i}^T, \tilde{N}_{2i}^T, \tilde{N}_{3i}^T, \tilde{N}_{4i}^T, \tilde{N}_{5i}^T]^T$, $\tilde{M}_i = [\tilde{M}_{1i}^T, \tilde{M}_{2i}^T, \tilde{M}_{3i}^T, \tilde{M}_{4i}^T, \tilde{M}_{5i}^T]^T$ ($i \in S$). If there exist some positive definite symmetric matrices $\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Q}_4, X_i, \tilde{L}_i, \tilde{L}_{id} \in R^{n \times n}$ ($i \in S$) such that the following linear matrix inequalities (LMIs) hold:

$$\begin{aligned} \Pi_i &= \begin{bmatrix} \Omega_i & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22} \end{bmatrix} < 0, & \Psi_i &= \begin{bmatrix} \Omega_i & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22} \end{bmatrix} < 0, \\ \Phi_i &= \begin{bmatrix} \Omega_i & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22} \end{bmatrix} < 0, & \Upsilon_i &= \begin{bmatrix} \Omega_i & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22} \end{bmatrix} < 0, \end{aligned} \quad (4.2)$$

where

$$\begin{aligned} \Omega_i &= \begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} \\ * & \Omega_{i22} & \Omega_{i23} \\ * & * & \Omega_{i33} \end{bmatrix}, \\ \Pi_{i12} = \Psi_{i12} &= \begin{bmatrix} hX_i^T \tilde{N}_{4i} & -h\tilde{N}_{1i}^T \tilde{Q}_4 & X_i^T A_i^T + \tilde{L}_i^T B_i^T & \Xi_i & 0_{n \times n} \\ -h\tilde{Q}_1^T \tilde{N}_{4i} + h\tilde{Q}_1^T \tilde{M}_{4i} & -h\tilde{N}_{2i}^T \tilde{Q}_4 & \tilde{Q}_1^T A_{id}^T + \tilde{L}_{id}^T B_i^T & 0_{n \times n(N+2)} & 0_{n \times n} \\ -h\tilde{Q}_2^T \tilde{M}_{4i} & -h\tilde{N}_{3i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \end{bmatrix}, \\ \Pi_{i22} &= \begin{bmatrix} -h\tilde{Q}_3 & -h^2 \tilde{N}_{4i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & \tilde{Q}_3 \\ * & -h\tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} \tilde{Q}_4 & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & * & -\Gamma_i & 0_{n \times n} \\ * & * & * & * & -\frac{e^{-\kappa h}}{h|\gamma_{ii}|} \tilde{Q}_1 \end{bmatrix}, \\ \Psi_{i22} &= \begin{bmatrix} -h\tilde{Q}_3 & -h^2 \tilde{N}_{5i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & \tilde{Q}_3 \\ * & -h\tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} \tilde{Q}_4 & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & * & -\Gamma_i & 0_{n \times n} \\ * & * & * & * & -\frac{e^{-\kappa h}}{h|\gamma_{ii}|} \tilde{Q}_1 \end{bmatrix}, \\ \Phi_{i12} = \Upsilon_{i12} &= \begin{bmatrix} hX_i^T \tilde{N}_{5i} & -h\tilde{N}_{1i}^T \tilde{Q}_4 & X_i^T A_i^T + \tilde{L}_i^T B_i^T & \Xi_i & 0_{n \times n} \\ -h\tilde{Q}_1^T \tilde{N}_{5i} + h\tilde{Q}_1^T \tilde{M}_{5i} & -h\tilde{N}_{2i}^T \tilde{Q}_4 & \tilde{Q}_1^T A_{id}^T + \tilde{L}_{id}^T B_i^T & 0_{n \times n(N+2)} & 0_{n \times n} \\ -h\tilde{Q}_2^T \tilde{M}_{5i} & -h\tilde{N}_{3i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \end{bmatrix}, \\ \Phi_{i22} &= \begin{bmatrix} -h\tilde{Q}_3 & -h^2 \tilde{M}_{4i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & \tilde{Q}_3 \\ * & -h\tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} \tilde{Q}_4 & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & * & -\Gamma_i & 0_{n \times n} \\ * & * & * & * & -\frac{e^{-\kappa h}}{h|\gamma_{ii}|} \tilde{Q}_1 \end{bmatrix}, \\ \Upsilon_{i22} &= \begin{bmatrix} -h\tilde{Q}_3 & -h^2 \tilde{M}_{5i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & \tilde{Q}_3 \\ * & -h\tilde{Q}_4 & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & -\frac{\kappa}{e^{\kappa h} - 1} \tilde{Q}_4 & 0_{n \times n(N+2)} & 0_{n \times n} \\ * & * & * & -\Gamma_i & 0_{n \times n} \\ * & * & * & * & -\frac{e^{-\kappa h}}{h|\gamma_{ii}|} \tilde{Q}_1 \end{bmatrix}, \\ \Omega_{i11} &= \kappa X_i + A_i X_i + X_i^T A_i^T + B_i \tilde{L}_i + \tilde{L}_i^T B_i^T + \gamma_{ii} X_i + \tilde{N}_{1i}^T X_i + X_i^T \tilde{N}_{1i}, \\ \Omega_{i12} &= A_{id} X_i + B_i \tilde{L}_{id} + X_i^T \tilde{N}_{2i} - \tilde{N}_{1i}^T \tilde{Q}_1 + \tilde{M}_{1i}^T \tilde{Q}_1, \\ \Omega_{i13} &= X_i^T \tilde{N}_{3i} - X_i^T \tilde{M}_{2i}^T, \end{aligned}$$

$$\begin{aligned}\Omega_{i22} &= [-(1 - \mu_i)e^{\kappa h} \tilde{Q}_1] \vee [-(1 - \mu_i) \tilde{Q}_1] - \tilde{N}_{2i}^T \tilde{Q}_1 - \tilde{Q}_1^T \tilde{N}_{2i} + \tilde{M}_{2i}^T \tilde{Q}_1 + \tilde{Q}_1^T \tilde{M}_{2i}, \\ \Omega_{i23} &= -\tilde{Q}_1^T \tilde{N}_{3i} - M_{2i}^T \tilde{Q}_2 + \tilde{Q}_1^T \tilde{M}_{3i}, \\ \Omega_{i33} &= \tilde{Q}_2 - \tilde{M}_{3i}^T \tilde{Q}_2 - \tilde{Q}_2^T \tilde{M}_{3i}, \\ \Xi_i &= [\sqrt{\gamma_{i1}} X_i^T, \dots, \sqrt{\gamma_{i(i-1)}} X_i^T, \sqrt{\gamma_{i(i+1)}} X_i^T, \dots, \sqrt{\gamma_{iN}} X_i^T, X_i^T, X_i^T, X_i^T], \\ \Gamma_i &= \text{diag} \left\{ X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_N, \frac{\kappa}{e^{\kappa h} - 1} \tilde{Q}_3, e^{-\kappa h} \tilde{Q}_1, e^{-\kappa h} \tilde{Q}_2, \right\}.\end{aligned}$$

Here $*$ means the symmetric terms, then the nominal closed-loop systems (4.1) are exponentially stable. Moreover, the gain matrices K_i and K_{id} can be chosen as

$$K_i = \tilde{L}_i X_i^{-1} \quad \text{and} \quad K_{id} = \tilde{L}_{id} \tilde{Q}_1^{-1},$$

for each $i \in S$.

Proof Setting

$$\begin{aligned}P_i &= X_i^{-1}, & Q_j &= \tilde{Q}_j^{-1}, & j &= 1, 2, 3, 4, \\ N_{1i} &= \tilde{N}_{1i} X_i^{-1}, & M_{1i} &= \tilde{M}_{1i} X_i^{-1}, \\ N_{2i} &= \tilde{N}_{2i} \tilde{Q}_1^{-1}, & M_{2i} &= \tilde{M}_{2i} \tilde{Q}_1^{-1}, \\ N_{3i} &= \tilde{N}_{3i} \tilde{Q}_2^{-1}, & M_{3i} &= \tilde{M}_{3i} \tilde{Q}_2^{-1}, \\ N_{4i} &= \tilde{N}_{4i} \tilde{Q}_3^{-1}, & M_{4i} &= \tilde{M}_{4i} \tilde{Q}_3^{-1}, \\ N_{5i} &= \tilde{N}_{5i} \tilde{Q}_3^{-1}, & M_{5i} &= \tilde{M}_{5i} \tilde{Q}_3^{-1}, & i &\in S.\end{aligned}$$

Then, pre-multiplying and post-multiplying (4.2) with $\text{diag}\{X_i, \tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Q}_4, \tilde{Q}_4\}$ ($i \in S$) can result in (3.1) by using Lemma 2.3 in [2] [Nonlinear Dyn. 69:1023–1039 (2012)]. Thus, by virtue of Theorem 3.1, the closed-loop systems (4.1) are exponential stability. The proof of this Theorem is completed. $\square$

Next, we also generalize Theorem 4.1 to the uncertain case, and the design theorem is given as follows:

Theorem 4.2 For a given scalar $\kappa > 0$, and the time-varying delays $h_i(t)$ satisfies $0 \leq h_i(t) \leq h_i \leq h$ and $\dot{h}_i(t) \leq \mu_i$ ($i \in S$) and tuning matrices: $\tilde{N}_i = [\tilde{N}_{1i}^T, \tilde{N}_{2i}^T, \tilde{N}_{3i}^T, \tilde{N}_{4i}^T, \tilde{N}_{5i}^T]^T$, $\tilde{M}_i = [\tilde{M}_{1i}^T, \tilde{M}_{2i}^T, \tilde{M}_{3i}^T, \tilde{M}_{4i}^T, \tilde{M}_{5i}^T]^T$ ($i \in S$). If there exist some positive definite symmetric matrices $\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Q}_4, X_i, \tilde{L}_i, \tilde{L}_{id} \in R^{n \times n}$ ($i \in S$), and there exist some positive scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}$ ($i \in S$) such that the following linear matrix inequalities (LMIs) hold:

$$\Pi_i = \begin{bmatrix} \Omega_i & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22} \end{bmatrix} < 0, \quad \Psi_i = \begin{bmatrix} \Omega_i & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22} \end{bmatrix} < 0, \quad \Phi_i = \begin{bmatrix} \Omega_i & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22} \end{bmatrix} < 0,$$

$$\Upsilon_i = \begin{bmatrix} \Omega_i & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22} \end{bmatrix} < 0,$$

where

$$\Omega_i = \begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} \\ * & \Omega_{i22} & \Omega_{i23} \\ * & * & \Omega_{i33} \end{bmatrix},$$

$$\Pi_{i12} = \begin{bmatrix} h X_i^T \tilde{N}_{4i} & -h \tilde{N}_{1i}^T \tilde{Q}_4 & X_i^T A_i^T + \tilde{L}_i^T B_i^T & \varepsilon_{1i} E_i & X_i^T H_{1i}^T + \tilde{L}_i^T H_{3i}^T & \Xi_i & 0_{n \times n} \\ -h \tilde{Q}_1^T \tilde{N}_{4i} + h \tilde{Q}_1^T \tilde{M}_{4i} & -h \tilde{N}_{2i}^T \tilde{Q}_4 & \tilde{Q}_1^T A_{id}^T + \tilde{L}_{id}^T B_i^T & 0_{n \times n} & \tilde{Q}_1^T H_{2i}^T + \tilde{L}_{id}^T H_{3i}^T & 0_{n \times n(N+2)} & 0_{n \times n} \\ -h \tilde{Q}_2^T \tilde{M}_{4i} & -h \tilde{N}_{3i}^T \tilde{Q}_4 & 0_{n \times n} & 0_{n \times n} & 0_{n \times n} & 0_{n \times n(N+2)} & 0_{n \times n} \end{bmatrix},$$

Table 1 The upper bounds of h with different values of μ in Example 5.1

	μ		
	0.9	1.0	1.5
Yue and Han in [22]	0.9279	–	–
Shen et al. [13]	0.7529	0.7529	0.7255
Xu et al. [19]	0.9359	0.8886	0.8886
Ma et al. [12]	1.0428	1.0428	1.0428
Our result	1.4099	1.4099	1.4099

Table 2 The upper bounds of h with different values of μ_1 and μ_2 in Example 5.1

	Authors		
	Xu et al. in [20]	Ma et al. [11]	Theorem 3.1
$\mu_1 = 0.0, \mu_2 = 0.0$	0.9800	1.5370	1.4253
$\mu_1 = 0.0, \mu_2 = 0.5$	0.9800	1.3550	1.4131
$\mu_1 = 0.5, \mu_2 = 0.5$	0.4899	1.1240	1.4099
$\mu_1 = 1.0, \mu_2 = 1.5$	–	0.8870	1.4099

the other items: $\Omega_{i11}, \Omega_{i12}, \Omega_{i13}, \Omega_{i22}, \Omega_{i23}, \Omega_{i33}, \Xi_i, \Gamma_i$ ($i \in S$) are given in Theorem 4.1 and $*$ means the symmetric terms, then the uncertain closed-loop systems (4.1) are robustly exponential stable. Moreover, the gain matrices K_i and K_{id} can be chosen as

$$K_i = \tilde{L}_i X_i^{-1} \quad \text{and} \quad K_{id} = \tilde{L}_{id} \tilde{Q}_i^{-1},$$

for each $i \in S$.

5 Some illustrative numerical examples

In this section, some illustrative examples are provided to show the feasibility of our obtained results.

Example 5.1 Consider the following nominal Markovian jump systems, considered in [11–14, 19, 20, 22]:

$$\dot{x}(t) = A_i x(t) + A_{id} x(t - h_i(t)), \quad i \in S = \{1, 2\}, \quad (5.1)$$

with system matrices as follows:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_{1d} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \quad A_{2d} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

where $0 \leq h_i(t) \leq h_i \leq h$, $\dot{h}_i(t) \leq \mu_i$ ($i \in S$). The parameter matrix $\Pi = \{\gamma_{ij}\}$ is given by $\Pi = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$. The convergence rate is $\kappa = 0.1$.

Case 1. Table 1 lists the comparison results with respect to [12, 13, 19, 22] when $\mu_1 = \mu_2 = \mu$. It is not hard to see from Table 1 that the upper bound h of the time-varying delay $h(t)$ is better than the ones obtained in [12, 13, 19, 22]. When $h = 0.8$ and $\mu = 1.5$, the upper bound of the decay rate is 0.8065 in [14]. However, by using Theorem 3.1, it can be derived that the suboptimal upper bound of decay rate is 1.6175, which is larger than one given in [14].

Case 2. Table 2 presents the comparison results with respect to [11, 20] with different values μ_1 and μ_2 . From Table 2, it can be seen that Theorem 3.1 is much better than those obtained in [11, 20].

Example 5.2 Consider the nominal Markovian jump system with two modes and the following parameters, which is considered in [1, 3–11, 17, 19, 24]:

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix},$$

$$A_{1d} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix},$$

Table 3 The allowable upper bounds of h for different γ_{11} in Example 5.2

	γ_{11}				
	-0.1	-0.3	-0.5	-0.7	-0.9
Xu et al. [19]	0.4021	0.4010	0.4001	0.3993	0.3987
Wu et al. [17]	0.4252	0.4250	0.4248	0.4246	0.4244
Our result	0.6127	0.6130	0.6133	0.6136	0.6139

Table 4 The allowable upper bounds of h for different γ_{11} in Example 5.2

	γ_{11}			
	-0.1	-0.5	-0.8	-1.0
Fujisaki [7]	0.2224	0.2200	–	0.2174
Boukas et al. [1]	0.2224	0.2200	0.2184	0.2174
Chen et al. [6]	0.5012	0.4941	0.4915	0.4903
Cao et al. [4]	0.5012	0.4941	–	0.4903
Cao et al. [5]	0.5012	0.4941	–	0.4903
Xu et al. [20]	0.6797	0.5794	0.5562	0.5465
Fei et al. [8] ($m = 1$)	0.6797	0.5794	0.5562	0.5465
Zhao et al. [24]	0.6797	0.5794	–	0.5465
Chen et al. [3] ($k = 0.1$)	0.6814	0.5794	0.5563	0.5475
Chen et al. [3] ($k = 0.3$)	0.6979	0.5898	0.5660	0.5568
Chen et al. [3] ($k = 0.5$)	0.7053	0.5991	0.5739	0.5568
Our results	0.9599	0.9182	0.8877	0.8699

$$A_{2d} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}.$$

Case 1. Table 3 presents the maximal allowable delay h for different values γ_{11} compared with those in [17, 19] when $\gamma_{22} = -0.8$ and $\mu_1 = \mu_2 = 0.9$. From Table 3, we see that our result is also better than the ones given in [17, 19].

Case 2. Table 4 gives the maximal allowable delay h for different values γ_{11} compared with those discussed in [1, 3–8, 20, 24] when $\gamma_{22} = -0.8$ and $\mu_1 = \mu_2 = 0.0$. It is easily seen that our result is less conservative than those derived in [1, 3–8, 20, 24].

Example 5.3 Consider the uncertain Markovian jump systems in the form of (2.1) with two modes, that is, $S = \{1, 2\}$. The mode switching is governed by the infinitesimal generator $\Pi = \begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix}$. The systems matrices are shown as follows:

$$A_1 = \begin{bmatrix} -1 & 0.5 \\ -0.2 & -1 \end{bmatrix}, \quad A_{1d} = \begin{bmatrix} -0.25 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.05 \end{bmatrix}, \quad H_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.8 & 0.8 \\ 0.15 & -2 \end{bmatrix}, \quad A_{2d} = \begin{bmatrix} 0.12 & -0.1 \\ 0.1 & -0.11 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.21 \\ 0.1 \end{bmatrix},$$

$$H_{12} = \begin{bmatrix} 0.2 & 0.0 \\ 0.0 & 0.1 \end{bmatrix}, \quad H_{22} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.05 \end{bmatrix}.$$

Case 1. This uncertain close-loop system with $h_1(t) = h_2(t) = h$ is considered in [14]. When $h = 0.8$, by virtue of Theorem 4.2, we also obtain $\kappa = 0.8$. Thus, it can be seen that our result in this paper is the same as one in Shu et al. [14], and the result in [14] can be generalized.

Case 2. The time-varying delays $h_1(t)$, $h_2(t)$ satisfy $\mu_1 = \mu_2 = 0.1$ and $h = 0.5$. The purpose is to design a robust state-feedback controller $u(t)$ satisfying

Fig. 1 The operation modes of Example 5.3

(2.7), which can exponentially stabilize these systems. In this example, we assume the decay rate $\kappa = 0.2$. We choose

$$\begin{aligned}\tilde{N}_{11} &= \tilde{N}_{21} = \tilde{N}_{31} = \tilde{M}_{11} = \tilde{M}_{21} = \tilde{M}_{31} \\ &= \begin{bmatrix} -0.05 & 0 \\ -0.01 & 0 \end{bmatrix}, \\ \tilde{N}_{12} &= \tilde{N}_{22} = \tilde{N}_{32} = \tilde{M}_{12} = \tilde{M}_{22} = \tilde{M}_{32} \\ &= \begin{bmatrix} -0.01 & 0 \\ 0 & -0.05 \end{bmatrix}, \\ \tilde{N}_{41} &= \tilde{N}_{42} = \tilde{N}_{51} = \tilde{N}_{52} = \tilde{M}_{41} = \tilde{M}_{42} = \tilde{M}_{51} = \tilde{M}_{52} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.\end{aligned}$$

By resorting to the Matlab LMI Control Toolbox to solve the LMIs in (4.3) in Theorem 4.1 and (4.4) in Theorem 4.2, the gain matrices of a robustly exponentially stabilizing controller can be obtained as

$$K_1 = [737.7994 \quad 126.4296],$$

$$K_2 = [-44.3834 \quad -2.5750],$$

$$K_{1d} = [-0.85819 \quad -1.1983],$$

$$K_{2d} = [-0.2273 \quad 0.0491].$$

The simulation is displayed for $h_1(t) = h_2(t) = \frac{1}{0.17+1}$ under the initial condition $\varphi(t) = [2.0, 0.5]$ for $t \in [-1, 0]$ with $r(0) = 1, t \in [-1, 0]$. The simulation

Fig. 2 The operation modes of Example 5.3 Case 2

Fig. 3 The control input curve of Example 5.3 (Case 2)

results are given in Figs. 1, 2, 3. Figures 1–3 shows the operation modes, the response and the control input curve, respectively. From these simulation results, it is easily seen that the uncertain closed-loop systems (4.1) are robustly exponentially stable.

6 Conclusion

In this paper, our main aim is to make up the shortage in [2] [Nonlinear Dyn. 69:1023–1039 (2012)]. Since a memory state feedback controller needed to be designed in the section four, the Lyapunov functional constructed in [2] [Nonlinear Dyn. 69:1023–1039 (2012)] is very difficult to realize it. Thus, we re-

visit to consider this problem. Meanwhile, the results of Examples 5.1–5.3 in [2] [Nonlinear Dyn. 69:1023–1039 (2012)] are necessarily to be modified.

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