

## Erratum to: Analysis on the nonlinear response of cracked rotor in hover flight

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In the original publication, equations (5), (9) and (10) appearing on pages 185 and 186 should read as follows.

The equations of motion of the system can be written as

$$\begin{aligned}
 m\ddot{x} + 2mc_d\omega_t\dot{x} + k_zx & - f[(k_z - k_{11}\cos^2\omega t - k_{22}\sin^2\omega t)x \\
 & + \sin\omega t \cos\omega t(k_{22} - k_{11})y \\
 & + (-k_{13}\cos^2\omega t - k_{24}\sin^2\omega t)\theta_y \\
 & + \sin\omega t \cos\omega t(k_{24} - k_{13})\theta_x] \\
 = me\omega^2 \sin(\omega t + \phi_e + \beta + \vartheta) & + mr_1\omega_1^2 \cos\vartheta - mg \sin\vartheta,
 \end{aligned}$$

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$$\begin{aligned}
 m\ddot{y} + 2mc_d\omega_t\dot{y} + k_zy - f[\sin\omega t \cos\omega t(k_{22} - k_{11})x & + (k_z - k_{11}\sin^2\omega t - k_{22}\cos^2\omega t)y \\
 & + \sin\omega t \cos\omega t(k_{24} - k_{13})\theta_y \\
 & + (-k_{13}\sin^2\omega t - k_{24}\cos^2\omega t)\theta_x] \\
 = me\omega^2 \cos(\omega t + \phi + \beta + \vartheta) - mr_1\omega_1^2 \sin\vartheta & - mg \cos\vartheta,
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 J_d\ddot{\theta}_y + 2J_dc_1\omega_s\dot{\theta}_y - J_p\omega\dot{\theta}_x + k_\theta\theta_y & - f[(-k_{13}\cos^2\omega t - k_{24}\sin^2\omega t)x \\
 & + \sin\omega t \cos\omega t(k_{24} - k_{13})y \\
 & + (k_\theta - k_{33}\cos^2\omega t - k_{44}\sin^2\omega t)\theta_y \\
 & + \sin\omega t \cos\omega t(k_{44} - k_{33})\theta_x] \\
 = J_p\omega\omega_1 \sin\vartheta,
 \end{aligned}$$

$$\begin{aligned}
 J_d\ddot{\theta}_x + 2J_dc_1\omega_s\dot{\theta}_x + J_p\omega\dot{\theta}_y + k_\theta\theta_x & - f[\sin\omega t \cos\omega t(k_{24} - k_{13})x \\
 & + (-k_{13}\sin^2\omega t - k_{24}\cos^2\omega t)y \\
 & + \sin\omega t \cos\omega t(k_{44} - k_{33})\theta_x \\
 & + (k_\theta - k_{33}\sin^2\omega t - k_{44}\cos^2\omega t)\theta_y] \\
 = J_p\omega\omega_1 \cos\vartheta
 \end{aligned}$$

where  $m$  is the mass of disc,  $k_z$  is the bending stiffness of the uncracked rotor, and  $k_\theta$  is the angular stiffness.  $\phi_e$  is the original phase angle.  $c_d$  and  $c_1$  are transverse

and swing vibration damps, respectively.  $f$  is the crack model function.  $\omega_t$  and  $\omega_s$  are transverse and swing vibration critical speeds, respectively.  $k_{ij}$  is the cross stiffness when the crack is open (the force in direction  $i$  caused the unit bending in direction  $j$ , 1, 2, 3 and 4 represent the direction of  $x$ ,  $y$ ,  $\theta_y$  and  $\theta_x$ , respectively) [1].

$$\begin{aligned}
 & X'' + \frac{2c_d}{\Omega^2} X' + \frac{1}{\Omega^2} X - \frac{f}{\Omega^2} \\
 & \times \left[ (1 - \check{k}_{11} \cos^2 \tau - \check{k}_{22} \sin^2 \tau) X \right. \\
 & + \sin \tau \cos \tau (\check{k}_{22} - \check{k}_{11}) Y \\
 & + (-\check{k}_{13} \cos^2 \tau - \check{k}_{24} \sin^2 \tau) \frac{\theta_y}{\delta} \\
 & \left. + \sin \tau \cos \tau (\check{k}_{24} - \check{k}_{13}) \frac{\theta_x}{\delta} \right] \\
 & = U \sin(\tau + \phi_e + \beta + \vartheta) + R_1 \frac{\Omega_1^2}{\Omega^2} \cos \vartheta \\
 & - \frac{1}{\Omega^2} \sin \vartheta, \\
 & Y'' + \frac{2c_d}{\Omega^2} Y' + \frac{1}{\Omega^2} Y - \frac{f}{\Omega^2} \left[ \sin \tau \cos \tau (\check{k}_{22} - \check{k}_{11}) X \right. \\
 & + (1 - \check{k}_{11} \sin^2 \tau - \check{k}_{22} \cos^2 \tau) Y \\
 & + \sin \tau \cos \tau (\check{k}_{24} - \hat{k}_{13}) \frac{\theta_y}{\delta} \\
 & \left. + (-\check{k}_{13} \sin^2 \tau - \check{k}_{24} \cos^2 \tau) \frac{\theta_x}{\delta} \right] \\
 & = U \sin(\tau + \phi_e + \beta + \vartheta) - R_1 \frac{\Omega_1^2}{\Omega^2} \sin \vartheta \\
 & - \frac{1}{\Omega^2} \cos \vartheta, \\
 & \theta_y'' + \frac{2c_1 \Omega_s}{\Omega} \theta_y' - 2\theta_x' \\
 & - \frac{\Omega_s^2 f}{\Omega^2} [(-\check{k}_{13} \cos^2 \tau - \check{k}_{24} \sin^2 \tau) X \delta
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 & + \sin \tau \cos \tau (\check{k}_{24} - \check{k}_{13}) Y \delta \\
 & + (1 - \check{k}_{33} \cos^2 \tau - \check{k}_{44} \sin^2 \tau) \theta_y \\
 & + \sin \tau \cos \tau (\check{k}_{44} - \check{k}_{33}) \theta_x] + \frac{\Omega_s^2}{\Omega^2} \theta_y \\
 & = 2 \frac{\Omega_1}{\Omega} \sin \vartheta, \\
 & \theta_x'' + \frac{2c_1 \Omega_s}{\Omega} \theta_x' + 2\theta_y' \\
 & - \frac{\Omega_s^2 f}{\Omega^2} [\sin \tau \cos \tau (\check{k}_{24} - \check{k}_{13}) X \delta \\
 & + (-\check{k}_{13} \sin^2 \tau - \check{k}_{24} \cos^2 \tau) Y \delta \\
 & + \sin \tau \cos \tau (\check{k}_{44} - \check{k}_{33}) \theta_y \\
 & + (1 - \check{k}_{33} \sin^2 \tau - \check{k}_{44} \cos^2 \tau) \theta_x] + \frac{\Omega_s^2}{\Omega^2} \theta_x \\
 & = 2 \frac{\Omega_1}{\Omega} \cos \vartheta
 \end{aligned}$$

where

$$\begin{aligned}
 (\cdot)' &= \frac{d}{d\tau}, & X &= \frac{x}{\delta}, & Y &= \frac{y}{\delta}, & \Omega &= \frac{\omega}{\omega_t}, \\
 \Omega_s &= \frac{\omega_s}{\omega_t}, & \check{k}_{ij} &= \frac{k_{ij}}{k_z}, & \tau &= \omega t, & & (10) \\
 U &= \frac{e}{\delta}, & R_1 &= \frac{r_1}{\delta}, & \Omega_1 &= \frac{\omega_1}{\omega_t}.
 \end{aligned}$$

The equations were wrong by our carelessness when we were writing another paper and the simulation results of the original publication are acceptable.

**References**

1. Yang, Y.F., Ren, X.M., Qin, W.Y.: Answering two questions concerning stiffness of a Jeffcott rotor with transverse crack. *J. Northwest. Polytech. Univ.* **24**, 778–781 (2006)