Special issue on symbolic methods in multidimensional systems theory

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In the seventies, the study of transfer matrices of linear time-invariant systems led to the development of the polynomial approach (Kailath, Rosenbrock, et al.). In this approach, univariate polynomial matrices play a central role (e.g., Hermite and Smith normal forms, Bezout equations etc.).

When generalizing linear systems given by ordinary differential/difference equations to differential time-delay systems, systems with parameters, 2-D or 3-D filters and circuits, one had to face the case of systems described by means of matrices with entries in multivariate polynomial rings. These new systems were called 2-D or 3-D and, more generally, multidimensional systems (Bose, Lin, Pommaret, Oberst, Youla, Wood, et al.). For such systems, no normal forms such as Hermite and Smith forms exist. In order to handle these problems, the concept of Gröbner bases (developed by Buchberger) was introduced in multidimensional systems theory. In many ways, the computation of these bases can be seen as an extension of both Gaussian elimination and the Euclidean algorithm. Gröbner bases have been implemented in nearly all modern computer algebra systems.

Another difficulty with multidimensional systems is the lack of a canonical first order representation. Several models (Roesser, Fornasini-Marchesini, reduced Spencer form etc.) have been studied extensively. The geometric approach to systems and control (Basile, Marro, Wonham, et al.) has recently been extended to these classes of multidimensional systems yielding promising results. The reduction to first order representations (realization problem) is of great importance to many applications of multidimensional systems theory such as uncertainty modeling for robust control. Special computer algebra packages have been developed for the effective manipulation of linear fractional transformations.

In his pioneering work, Kalman developed a module-theoretic approach to the realization problem. Independently, the Japanese school of Sato, Kashiwara, and Kawai proposed an

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algebraic analysis approach to linear systems of partial differential equations using module theory, homological algebra and sheaf theory. This school highly inspired the works of Fliess, Oberst and Pommaret while they were developing the algebraic analysis approach to multidimensional systems with constant or variable coefficients. One of the successes of this new approach to mathematical systems theory was to show that the behavioural approach to dynamical systems proposed by the school of Willems provides the right framework for tackling control problems for multidimensional linear systems from an algebraic point of view.

Even if the fine study of multidimensional linear systems requires the use of powerful constructive algebraic techniques such as (non-commutative) Gröbner bases, module theory, algebraic analysis, homological algebra etc., the implementation of the different results in symbolic softwares makes them accessible to a large audience by demonstrating their use on various explicit examples. On the other hand, symbolic computation may serve as a tool to achieve new theoretical insights into multidimensional systems theory.

The present Special Issue combines contributions by both developers and users of computer algebra software for multidimensional systems theory and control applications. Using module theory and homological algebra, Barakat and Robertz show in their respective papers, how built-in properties of multidimensional systems can be constructively characterized and thus checked by means of dedicated computer algebra packages. Oberst and Pommaret contribute mathematically oriented papers that make heavy use of algebraic and homological tools (equidimensional decomposition and relative parametrizations of multidimensional systems). Pal and Pillai, and Bisiacco and Valcher show new results in the behavioural approach to multidimensional systems (restriction and partial interconnection problems). Maleki et al. present a challenging application to a problem of high control theoretic significance (failure identification). Kareltsou and Karampetakis and Doan et al. discuss recent developments in the area of first order representations and minimal realizations of multidimensional systems. Thus the contributions cover a broad range from abstract theory to the concrete development and implementation of algorithms and computer algebra packages in the context of multidimensional systems, and from structural insights and algebraic machinery to relevant control problems such as fault detection, dead-beat control and observers, control by interconnection, uncertainty modeling, and multidimensional realization.

We believe that the different views on the topic presented in this Special Issue may stimulate further cooperation and discussion. We also hope that it will be of considerable interest for the MSSP audience.