On the development and parameter identification of Schapery-type constitutive theories

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Below equations (26) it is written that:

“where p_2 , p_3 or a_2 , $a_3 > 0$ to ensure that Ψ or G are at isolated minimums in the reference state. The same restriction must also apply to Ψ_0 and G_0 .”

In fact, p_2 , p_3 or a_2 , a_3 do *not* need to be strictly positive in order to ensure that Ψ or G are at isolated minima in the reference state. Since it is assumed that all the nonlinearizing functions (i.e. the p_z or the a_z) are equal to unity in the reference state, Ψ or G are at isolated minima in the reference state, by definition.

As pointed out by Schapery (1969b), the nonlinearizing functions p_2 , p_3 or a_2 , a_3 can have negative values. For the sake of discussion, consider the constitutive theory obtained in one dimension from (37b) where: a) the elastic part would be negligible; b) all constants are equal to unity and c) only one term in (38b) is used. Since $a_1 > 0$, assume that $a_1 = 1$. Define $\Theta = \frac{\partial a_3}{\partial \sigma} \sigma + a_3$ (this is the term in front of the integral in (37b)). It is worth

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considering the shape of constitutive theory (37b) as a function of the signs of a_2 , a_3 and Θ when subjected to a creep load σ_0 . Four different expressions for the strain ε are obtained:

1. If $(a_2 > 0, a_3 > 0, \Theta > 0)$ or $(a_2 > 0, a_3 < 0, \Theta < 0)$, then (37b) becomes

$$\varepsilon(t) = |\Theta|(1 - \exp[-|a_2|t]) \left| \frac{a_3}{a_2} \right| \sigma_0 \quad (\text{i})$$

which is consistent with many experimental observations.

2. If $(a_2 > 0, a_3 > 0, \Theta < 0)$ or $(a_2 > 0, a_3 < 0, \Theta > 0)$, then (37b) becomes

$$\varepsilon(t) = -|\Theta|(1 - \exp[-|a_2|t]) \left| \frac{a_3}{a_2} \right| \sigma_0 \quad (\text{ii})$$

which means that a negative strain is obtained if a positive stress is suddenly applied.

3. If $(a_2 < 0, a_3 > 0, \Theta > 0)$ or $(a_2 < 0, a_3 < 0, \Theta < 0)$, then (37b) becomes

$$\varepsilon(t) = |\Theta|(\exp[|a_2|t] - 1) \left| \frac{a_3}{a_2} \right| \sigma_0 \quad (\text{iii})$$

which means that the strain varies exponentially with respect to time.

4. If $(a_2 < 0, a_3 > 0, \Theta < 0)$ or $(a_2 < 0, a_3 < 0, \Theta > 0)$, then (37b) becomes

$$\varepsilon(t) = -|\Theta|(\exp[|a_2|t] - 1) \left| \frac{a_3}{a_2} \right| \sigma_0 \quad (\text{iv})$$

which means that the strain varies exponentially with respect to time and that a suddenly applied positive stress leads to a negative strain.

By examining all the experimental data that we either have generated or found in the literature, and for loadings that led to a viscoelastic behavior for t_v time units (see Sect. 4.2, item 1), it seems that only expression (i) above can adequately represent this creep data. The form of expression (iii) could be useful, for example, to represent creep behavior at the onset of failure, which is usually referred to as *Phase III* creep. Therefore, the restrictions detailed by (53b) are not all required by thermodynamics but are introduced as assumptions in order to represent experimentally observed behavior.

It should also be noted that Gibbs' free energy expansion in (26b) results from some simplifications of the expansion in (16), as initially introduced by Schapery (1969b) (where it is assumed that A_{rs} depends on σ). The expansion in (26b) leads to the same results when using expression (16) with the simplifications introduced by (16) and (37) from reference (Schapery 1997a). Therefore, using the expansions of (26) to derive constitutive theories will lead to a subset of all the constitutive theories that can be obtained using the general arguments introduced by Schapery. Further details can be found either in (Schapery 1969b) or (Schapery 1997a).

References

- Schapery, R.A.: Further Development of a Thermodynamic Constitutive Theory: Stress Formulation. Purdue University (1969b)
- Schapery, R.A.: Nonlinear viscoelastic and viscoplastic constitutive equations based on thermodynamics. Mech. Time-Depend. Mater. **1**, 209–240 (1997a)