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TEMPERATURE DEPENDENCE OF STRESS–FATIGUE LIFE

DATA OF FRP COMPOSITES

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Polymer-matrix composites are viscoelastic materials, and their mechanical properties are significantly influenced by temperature. The present study develops a power-law stress–fatigue life relation which takes into account the effect of temperature on the fatigue strength of fibre-reinforced polymer (FRP) composites. The validity of the relation is evaluated on the basis of various sets of experimental data found in the literature, and the estimated fatigue lives obtained are found to be in good agreement with the experiments.

1. Introduction

In the last three decades, the field of composite materials has experienced a rapid advancement. The vast use of these materials in aerospace, automotive, and electronic engineering and in many types of industrial equipment has turned them into one of the most interesting topics for research. Among the different research issues, investigating the fatigue behaviour of composites is a challenging and favourable topic. In these investigations, among the large number of factors affecting the fatigue behaviour of composites, the effect of temperature on the fatigue strength of laminated fibre-reinforced polymer (FRP) composites is mainly discussed.

This study develops a power-law stress–fatigue life relation that takes into account the effect of temperature on the fatigue strength of the composites. Both the coefficient and the exponent of the S – N relation are found to be temperature-dependent. The coefficient of the S – N diagram corresponded to the ultimate tensile strength. The variation in the ultimate tensile strength of several FRP composites at different temperatures (from the absolute zero temperature to the polymer melting

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Fig. 1. Simple tension (1) and tension-tension fatigue loading $R = \sigma_{\min}/\sigma_{\max} > 0$ (2).

point) was previously studied by the present authors [1, 2] to best describe this coefficient as a function of temperature. The exponent of the S – N relation as the slope of the S – $\log N$ curve is also found to be temperature-dependent, and this dependence is calibrated using several sets of experimental data available in the literature.

2. Fatigue Strength and Temperature

When a composite material is subjected to cyclic stresses, the fatigue damage is cumulated over loading cycles, and when the damage reaches 100%, the fatigue failure takes place. The relation between the cyclic stress σ and the number of cycles to failure N_f is mathematically defined as

$$\sigma = A(N_f)^m. \quad (1)$$

Equation (1) is known as the power-law stress–fatigue life relation, and the σ – N_f curve based on the equation is generally called the S – N curve. By plotting versus the logarithm of , a curve very similar to a straight line is obtained, where the coefficient A is the -intercept at $N = 1$ cycle, and the exponent , which is negative, represents the slope of the curve. When is considered as the maximum stress $\sigma_{\max}$, Eq. (1) is rewritten as

$$\sigma_{\max} = A(N_f)^m. \quad (2)$$

Figure 1 schematically presents a sinusoidal loading case at a stress ratio $R = \sigma_{\min}/\sigma_{\max} > 0$. If a specimen is subjected to a tension-tension loading and fails in the first quarter of the first cycle, this means that the stress has been equal to the ultimate strength. Then, the constant A corresponds to the ultimate tensile strength, and Eq. (2) is written as

$$\sigma_{\max} = \sigma_{\text{ult}}(4N_f)^m. \quad (3)$$

In the literature, Eq. (2) is also expressed as

$$\sigma_{\max} = \sigma_{\text{ult}}(2N_f)^m. \quad (4)$$

In fact, Eq. (4) specifies that if a specimen fails in the first half of the first cycle or in the first reversal, the maximum applied stress is equal to the ultimate tensile strength.

However, of the two Eqs. (3) and (4), Eq. (3) is recommended, because it correctly indicates that the maximum stress is equal to the ultimate tensile strength if and only if the specimen fails in the first quarter of the first cycle. Using the equation, with a know intercept A and slope m of the S – $\log N$ curve for a material, its ultimate tensile strength σ_{ult} can be calculated.

Fig. 2. Typical S -log N curves at different temperatures.

2.1. Temperature effect on intercept (A) at $N = 1$. By comparing Eq. (3) with Eq. (2), it is concluded that the intercept A can be expressed as

$$A = \sigma_{\text{ult}}(4^m).$$

The term A as a function of temperature T is then specified as

$$A(T) = \sigma_{\text{ult}}(T)(4^{m(T)}). \quad (5)$$

The ultimate tensile strength is calculated from Eq. (5) as

$$\sigma_{\text{ult}}(T) = \frac{1}{4^{m(T)}} A(T). \quad (6)$$

The effect of temperature on the ultimate tensile strength of laminated FRP composites has been formulated in an earlier study by the present authors [1, 2]

$$\sigma_{\text{ult}}(T) = \sigma_{\text{ult}}(T_0) \left[1 - \frac{\frac{\sigma_{\text{ult}}(0)}{\sigma_{\text{ult}}(T_0)} - 1}{\ln\left(1 - \frac{T_0}{T_m}\right)} \ln \frac{1 - \frac{T}{T_m}}{1 - \frac{T_0}{T_m}} \right], \quad (7)$$

where $\sigma_{\text{ult}}(T)$ is the ultimate tensile strength at an arbitrary temperature T , $\sigma_{\text{ult}}(T_0)$ is the ultimate tensile strength at a reference temperature T_0 , normally at room temperature (RT), $\sigma_{\text{ult}}(0)$ is the tensile strength at the absolute zero temperature (0 K = -273°C), and T_m is the polymer melting temperature of the composites. All temperatures in Eq. (7) are given in kelvins.

Rearranging Eqs. (6) and (7), we obtain

$$\frac{1}{4^{m(T)}} A(T) = \frac{1}{4^{m(T_0)}} A(T_0) \left[1 - \frac{\left(\frac{1}{4^{m(0)}} A(0) \right) / \left(\frac{1}{4^{m(T_0)}} A(T_0) \right) - 1}{\ln\left(1 - \frac{T_0}{T_m}\right)} \ln \frac{1 - \frac{T}{T_m}}{1 - \frac{T_0}{T_m}} \right]. \quad (8)$$

To simplify Eq. (8), the variation of slope m over $N = 0$ -1/4 fatigue life cycles is assumed to be the same at different temperatures T_1 , T_2 , and T_3 (schematically shown in Fig. 2):

$$m(T_1) \cong m(T_2) \cong m(T_3) \dots \cong m(T).$$

Fig. 3. Illustration of an intercept-temperature curve $A-T$ generated by using Eq. (9).

Therefore, based on the assumption that $m(T) = m(T_0) = m(0)$, the effect of temperature on A in Eq. (8) is reduced to

$$A(T) = A(T_0) \left[1 - \frac{\frac{A(0)}{A(T_0)} - 1 - \frac{T}{T_m}}{\ln \left(1 - \frac{T_0}{T_m} \right)} \right], \quad (9)$$

where $A(T)$ is the intercept at an arbitrary temperature, $A(T_0)$ is the intercept at the reference temperature (or room temperature), and $A(0)$ represents the intercept at 0 K, as illustrated in Fig. 3. In Eq. (9), the term $(A(0)/A(T_0) - 1)$ is designated by C_A and is called the sensitivity of intercept A to temperature variation.

2.2. Temperature effect on the slope m . The exponent in Eq. (2) determines the slope of the S -log N curve. Since the fatigue life decreases if the stress increases, m has a negative value. Therefore, the greater the absolute value of m , the higher the fatigue strength and vice versa. The value of m is found by performing at least three fatigue tests. To increase the precision of m , the number of fatigue tests should be increased even up to 30, covering a full range of fatigue lives, from 1 cycle to 10^6 or 10^7 cycles. This is mainly due to the high scatter and statistical nature of the results of fatigue tests.

The variation in the intercept A with temperature on laminated FRP composites shows a decreasing trend, almost the same as that seen in Fig. 3, which is based on Eq. (9). However, the variation of m with temperature is usually more complex. To simplify the description of effect of temperature on m , several sets of experimental S - N fatigue data at various operating temperatures given in [3-10] were investigated and analyzed. The investigations resulted in the relationship

$$m(T) = m(T_0) \frac{\ln \left(1 - \frac{T}{T_m} \right)}{\ln \left(1 - \frac{T_0}{T_m} \right)}, \quad (10)$$

where $m(T)$ is the value of m at an arbitrary temperature T , and $m(T_0)$ is its value at the reference temperature T_0 ($=$ RT). Figure 4 depicts the graph of Eq. (10). As seen from the figure, the absolute value of m increases with temperature.

With considering the temperature-dependent parameters A and m , Eq. (2) results in

$$\sigma_{\max} = A(T)(N_f(T))^{m(T)}. \quad (11)$$

Introducing Eqs. (9) and (10) into Eq. (11), the temperature-dependent S - N relation is expressed as

Fig. 4. Typical m – T curve given by Eq. (10).

TABLE 1. Experimental Information Used for Predicting the Temperature-Dependent S – N Relation

Composite	Reference	Plies/Laminates	A (RT = 298 K)	m (RT)	$A(0)$, MPa (extrapolated)	C_A	T_m , K (extrapolated)
AS-4/PEEK	[9]	$[0/90]_{4s}$	1207.2	–0.043	1690.1	0.4	616 (reported)
AS-4/PEEK	[9]	$[0/+45/90/-45]_{2s}$	787.6	–0.031	945.1	0.2	616 (reported)
T300/ Epoxy#2500	[10]	Plain-Woven (15°)	267.2	–0.049	454.2	0.7	603
T300/ Epoxy#2500	[10]	Plain-Woven (30°)	204.4	–0.068	378.1	0.85	603
T300/ Epoxy#2500	[10]	Plain-Woven(45°)	186.8	–0.080	355	0.9	603

$$\sigma_{\max} = A(T_0) \left[1 - \frac{\frac{A(0)}{A(T_0)} - 1}{\ln\left(1 - \frac{T_0}{T_m}\right)} \ln \frac{1 - \frac{T}{T_m}}{1 - \frac{T_0}{T_m}} \right] (N_f(T))^{m(T_0) \frac{\ln\left(1 - \frac{T}{T_m}\right)}{\ln\left(1 - \frac{T_0}{T_m}\right)}} \quad (12)$$

3. Evaluation of the Fatigue S – N Relation

The experimental data found in [9, 10] were used to evaluate the fatigue S – N relation (12). The materials tested, the types of plies and laminates, and the values of A and m found experimentally are listed in Table 1.

Jen et al. [9] conducted tension-tension fatigue tests on graphite/PEEK prepregs by using an MTS 810 machine under load control conditions, with a stress ratio $R = 0.1$ and a sinusoidal waveform of frequency 5 Hz. The glass-transition temperature T_g of PEEK is reported to be 416 K. The tests, up to 10^6 cycles, were performed at RT (25), 75, 100, 125, and 150°C.

Kawai and Taniguchi [10] tested carbon/epoxy woven fabric laminates. Tension-tension fatigue tests, up to 10^6 cycles, were performed at room temperature (25°C) and at 100°C under load control conditions, with a stress ratio $R = 0.1$ and a sinusoidal waveform of frequency 10 Hz.

3.1. Calculated fatigue S – N results. To evaluate Eq. (12), sixteen different fatigue S – N data sets obtained in [9, 10] at various temperatures were examined. All the parameters needed are listed in Table 1. First, the temperature-dependent

Fig. 5. The coefficient A (a) and the exponent m (b) as functions of temperature T for various FRP composites at $R = 0.1$: experimental data (dots) [9, 10] and calculated values (lines) according to Eqs. (9) (a) and (10) (b). ● — cross-ply AS-4/PEEK, $f = 5$ Hz; ■ — quasi-isotropic AS-4/PEEK, $f = 5$ Hz; ○ — woven 15° T300/epoxy # 2500, $f = 10$ Hz; ◆ — woven 30° T300/epoxy # 2500, $f = 10$ Hz; ▲ — woven 45° T300/epoxy # 2500, $f = 10$ Hz.

Fig. 6. S – N – curves calculated by Eq. (12) and experimental data [9] for $[0/90]_{4s}$ (a) and $[0/+45/90/-45]_{2s}$ (b) AS-4/PEEK composites at $f = 5$ Hz, $R = 0.1$, and $T = 298$ (◆), 348 (□), 373 (▲), 398 (●), and 423 K (■).

exponent m and coefficient A were calculated (see Fig. 5a,b). Then, Eq. (12) was used to assess the fatigue life of various composites. The calculation results are illustrated in Figs. 6 and 7.

4. Discussion of Results

To formulate and calibrate the temperature dependence of fatigue strength, the experimental data found at various temperatures were taken from [9, 10]. The types of composites investigated include $[0/90]_{4s}$, $[0/+45/90/-45]_{2s}$, and plain-woven 15, 30, and 45° graphite/epoxy and graphite/PEEK laminates.

To construct the fatigue S – N diagrams for composite samples, both the intercept A and the exponent m as functions of temperature T must be known. Figures 6 and 7 show the predicted fatigue S – N curves. It is seen that the fatigue strength

Fig. 7. $S-N$ – curves calculated by Eq. (12) and experimental points [10] for plane-woven 15 (a), 30 (b), and 45° (c) T300/Epoxy # 2500 composites at $f=10$ Hz, $R=0.1$, and $T=298$ (◆) and 373 K (▲).

Fig. 8. Comparison of experimental N_f^{exp} (dots) [9, 10] and predicted N_f^{calc} (lines) fatigue lives based on Eq. (12): ◆ — cross-ply AS-4/PEEK; □ — quasi-isotropic AS-4/PEEK; ▲ — woven 15° T300/epoxy; ● — woven 30° T300/epoxy; ■ — woven 45° T300/epoxy.

of the laminated FRP composites decreases with growing temperature, which is associated with the decreasing coefficient A and/or the increasing exponent m .

A comparison between the experimentally obtained fatigue lives and the predictions based on S - N relation (12) is presented in Fig. 8. As seen, of all the experimental points, about 80% fall between the upper and lower bounds, shown by dashed lines. The remaining ones, which deviate from the 3σ bounds, can be attributed to temperatures near the glass-transition temperature T_g of the matrix.

5. Conclusions

Based on the power-law S - N relation, both the coefficient A and the exponent m of the relation are found to be temperature-dependent. With account of this dependence, the predicted fatigue lives were found to be in a very close agreement with experimental data obtained for five different composite materials.

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