Meccanica (2010) 45: 817–827
DOI 10.1007/s11012-010-9290-3

Viscoelastic MHD flow boundary layer over a stretching surface with viscous and ohmic dissipations

M. Babaelahi · G. Domairry · A.A. Joneidi

Received: 2 May 2009 / Accepted: 4 March 2010 / Published online: 11 June 2010

Abstract In this study the momentum and heat transfer characteristics in an incompressible electrically conducting viscoelastic boundary layer fluid flow over a linear stretching sheet are considered. Highly nonlinear momentum and thermal boundary layer equations are reduced to set of nonlinear ordinary differential equations by appropriate transformation.

Optimal Homotopy Asymptotic Method (OHAM) is used to evaluate the temperature and velocity profiles of the problem. Runge-Kutta numerical solution is used to show the validity of OHAM. Finally the effects of some important parameters such as Hartmann number, viscoelastic parameter and Prandtl number on boundary layer behaviour are discussed by several figures.

Keywords Boundary layer flow · Heat transfer · Viscoelastic boundary layer · Linear stretching sheet · Optimal Homotopy Asymptotic Method (OHAM)

M. Babaelahi
Department of Mechanical Engineering, K.N Toosi University of Technology, Tehran, Iran

G. Domairry (✉)
Department of Mechanical Engineering, Babol University of Technology, P. O. Box 484, Babol, Iran
e-mail: amirganga111@yahoo.com

A.A. Joneidi
Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, Netherlands

Nomenclature

$\vec{B}$	Uniform transverse magnetic fields
$\vec{E}$	uniform electric field
E	Eckert number
E_1	Local electromagnetic parameter
f	dimensionless stream function
Ha	Hartmann number
J	Joule current
k	Viscoelastic parameter
k_0	Elastic parameter
k_1^*	Thermal conductivity
l	Characteristic length
p	Embedding parameter
Pr	Prandtl number
U_0	Characteristic velocity
u, v	Velocity component
x	flow directional coordinate along the stretching sheet
y	Distance normal to the stretching sheet
μ	Limiting viscosity at small rate of shear
η	Similarity variable
γ	Kinematic viscosity
θ	non-dimensional temperature parameter

1 Introduction

The filed of boundary layer flow problem over a stretching sheet have many industrial applications such as polymer sheet or filament extrusion from a dye or long thread between feed roll or wind-up roll, glass fiber and paper production, drawing of plastic films, liquid films in condensation process. Due to the high applicability of this problem in such industrial phenomena, it has attracted the attentions of many researchers and one of the pioneering studies has been performed by Sakiadis [1]. Sheet extrusion is a technique for making flat plastic sheets. Thermoplastic sheet production is a significant sector of plastics processing. For producing such thin plastic film a cautious heat exchange with cooling media should be applied. The success of the whole process is depended to the rheological properties of the fluid above the sheet as it is the fluid viscosity which determines the (drag) force required to pull the sheet. The water and air are amongst the most-widely used fluids as the cooling medium. However, the rate of heat exchange achievable by above fluids is realized to be not suitable for certain sheet materials. To have a better control on the rate of heat exchange, in recent years it has been proposed to employ the fluids which are more viscoelastic in nature than viscous such as water with polymeric additives [2, 3]. Normally increment of such additives to the fluids leads to increasing of the fluid viscosity to alter flow kinematics in such a way that it leads to a slower rate of solidification compared to water. Recently, many researches have been studied on heat transfer of MHD and viscoelastic fluids on the various surfaces [4–9]. The electric and magnetic fields are also two of the important parameters to alter the momentum and heat transfer characteristics in a non-Newtonian fluid flow and should also be considered. Dandapat et al. [10] show that the magnetic field has stabilizing effect on the boundary layer flow as long as the wavelength of the disturbances does not exceed the viscoelastic length scale. The radiative heat transfer properties of the cooling medium may also be manipulated to judiciously influence the rate of cooling [11, 12]. There are extensive researches on sheet forming. Most of the related researches studied only momentum transfer aspects [13, 14], but there are also a few works directed to the heat or even mass transfer aspects [15, 16].

Although the above quoted theoretical studies are consequential, but they employed some simplifications. For example, the viscoelastic fluid models which are used in these works are simple models such as second-order and/or Walters' B model which are known to be good only for weakly elastic fluids subject to slow and/or slowly-varying flows [17]. It should be also added the fact that these two fluid models are known to violate certain rules of thermodynamics [18]. Another shortcoming of the above works is in the notion that virtually all of them are based on the use of boundary layer theory which is still incomplete for non-Newtonian fluids [19].

Therefore, the significance of the results reported in the above works are limited, at least as far as polymer industry is concerned. Obviously, for the theoretical results to become of any industrial significance, more realistic viscoelastic fluid models such as Maxwell or Oldroyd-B model should be invoked in the analysis. Indeed, these two fluid models have recently been used to study the flow of viscoelastic fluids above stretching and non-stretching sheets but with no heat transfer effects involved [20–22].

The most recent studies which have been carried out in the current subject are the work of Abel et al. which have studied viscoelastic MHD flow and heat transfer over a stretching sheet with present of magnetic field and solved highly nonlinear boundary layer and heat transfer equations using homotopy analysis method [23]. But the work has been neglected electric field which is also one of the important parameters to alter the momentum and heat transfer characteristics in a non-Newtonian boundary layer fluid flow.

Most of the problems in viscoelastic boundary layer have highly nonlinearity. It is very important to develop new effective method to surmount this nonlinearity as some researchers performed it [24–28]. Recently, an analytical tool for non-linear problems, namely the Optimal Homotopy Asymptotic Method (OHAM) which is proposed for the first time by Marinca and Herisanu [29, 30] is developed and examined appropriately by some authors [31–34].

The main goal of the present work is to use this method to obtain an analytical solution of the considerable problem. For this purpose, after brief introduction for OHAM and description of the problem, the highly non-linear momentum and heat transfer equations have been solved analytically using above mentioned method. Obtaining the analytical solution of the model and comparing with numerical solutions declare the capability, effectiveness, convenience and high accuracy of this method. Thereafter the effects of various physical parameters like viscoelastic parameter, Prandtl and Hartmann number on momentum and heat transfer characteristics have been reported.

2 Optimal Homotopy Asymptotic Method (OHAM) [29, 30]

Consider below differential equation:

$$L(u(\tau)) + N(u(\tau)) + g(\tau) = 0, \qquad B(u) = 0, \quad (1)$$

where L is a linear operator, τ denotes an independent variable, $u(\tau)$ is an unknown function, $g(\tau)$ is a known function, $N(u(\tau))$ is a nonlinear operator and B is a boundary operator. By means of OHAM one first constructs a family of equations:

$$(1-p)[L(\phi(\tau,p)) + g(\tau)] - H(p)[L(\phi(\tau,p)) + g(\tau) + N(\phi(\tau,p))] = 0, \quad (2)$$

$$B(\phi(\tau,p)) = 0$$

where $p \in [0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p \neq 0$ and $H(0) = 0$, $\phi(\tau,p)$ is an unknown function. Obviously, when $p=0$ and $p=1$, it holds that:

$$\phi(\tau,0) = u_0(\tau), \qquad \phi(\tau,1) = u(\tau). \quad (3)$$

Thus, as p increases from 0 to 1, the solution $\phi(\tau,p)$ varies from $u_0(\tau)$ to the solution $u(\tau)$, where $u_0(\tau)$ is obtained from (2) for $p=0$:

$$L(u_0(\tau)) + g(\tau) = 0, \qquad B(u_0) = 0. \quad (4)$$

The auxiliary function $H(p)$ is chosen in the form of:

$$H(p) = pC_1 + p^2C_2 + \cdots, \quad (5)$$

where $C_1, C_2, \ldots$ are constants which can be determined later.

Expanding $\phi(\tau,p)$ in a series with respect to p, one has:

$$\phi(\tau,p,C_i) = u_0(\tau) + \sum_{k\geq 1} u_k(\tau,C_i)p^k, \quad i = 1,2,\ldots. \quad (6)$$

Substituting (6) into (2), collecting the same powers of p, and equating each coefficient of p to zero, we obtain set of differential equation with boundary conditions. Solving differential equations by boundary conditions $u_0(\tau), u_1(\tau,C_1), u_2(\tau,C_2), \ldots$ are obtained. Generally speaking, the solution of (1) can be determined approximately in the form of:

$$\tilde{u}^{(m)} = u_0(\tau) + \sum_{k=1}^{m} u_k(\tau,C_i). \quad (7)$$

Note that the last coefficient C_m can be function of τ. Substituting (7) into (1), there results the following residual:

$$R(\tau,C_i) = L(\tilde{u}^{(m)}(\tau,C_i)) + g(\tau) + N(\tilde{u}^{(m)}(\tau,C_i)). \quad (8)$$

If $R(\tau,C_i) = 0$ then $\tilde{u}^{(m)}(\tau,C_i)$ is much closed to the exact solution to minimizing the occurred error for nonlinear problems, below phrase is supposed:

$$J(C_1,C_2,\ldots,C_n) = \int_a^b R^2(\tau,C_1,C_2,\ldots,C_m)d\tau, \quad (9)$$

where a and b are the values, depending on the given problem. The unknown constants C_i $(i = 1,2,\ldots,m)$ can be identified from the conditions:

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \cdots = 0. \quad (10)$$

With these known constants, the approximate solution (of order m) (7) is well determined.

3 Description of the problem

The steady state two-dimensional incompressible electrically conducting viscoelastic fluid flow over a continuous stretching sheet is investigated in this study. The flow is considered to be generated by stretching of an elastic boundary sheet from a slit with the application of two equal and opposite forces in such way that velocity of the boundary sheet is of linear order of the flow directional coordinate x (see Fig. 1). The frictional heating due to viscous dissipation as the fluid

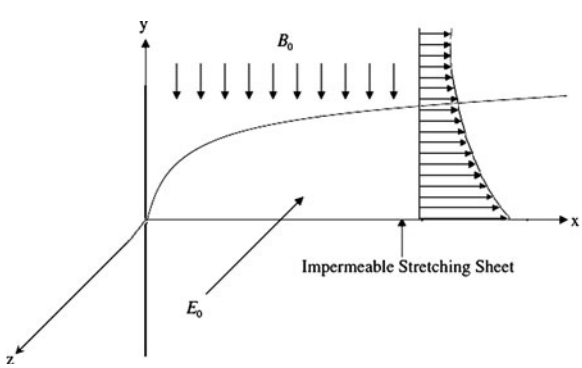


Fig. 1 Schematic view of considered problem

considered for analysis is of viscoelastic type which possesses viscous property has also taken into account. The flow region is exposed under uniform transverse magnetic fields.

For investigating of this problem, it is supposed that:

(1) Magnetic Reynolds number of the fluid is low and magnetic field and Hall effect may be neglected due to this assumption.
(2) Electric field as a result of polarization of charges has to be negligible.
(3) Presence of chemically inactive diffusive species in the boundary layer is low and hence Sorest–Dufour effects are negligible.
(4) Fluid is more viscous in nature than elastic and so we neglect elastic deformation effects.
(5) The wall must be electrically non-conducting.

We have from Maxwell's equation:

$$\nabla \vec{B} = 0 \quad \text{and} \quad \nabla \times \vec{E} = 0. \tag{11}$$

When magnetic field is not so strong, then electric and magnetic fields obey Ohm's law:

$$J = \sigma(\vec{E} + \vec{q} \times \vec{B}), \tag{12}$$

where J is the Joule current. The basic equations of considered problem are:

(1) Momentum and heat transfer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{13}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 y}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} + \frac{\sigma}{\rho}(E_0 B_0 - B_0^2 u), \tag{14}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K_1^*}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{(uB_0 - E_0)^2\sigma}{\rho c_p}. \tag{15}$$

(2) Velocity and temperature boundary layer conditions

$$u = U_W(x) = bx, \qquad v = 0, \qquad y = 0, \qquad u = 0, \qquad y \to \infty, \tag{16}$$

$$T = T_W = T_\infty + A_0\left(\frac{x}{l}\right)^2 \qquad y = 0, \qquad T \to T_\infty \qquad y \to \infty. \tag{17}$$

For simplicity of basic equations of considered problem, bellow transformation is used:

$$u = bxf_\eta, \qquad v = -\sqrt{b\gamma}\, f, \qquad \eta = \sqrt{\frac{b}{\gamma}}\, y, \tag{18}$$

$$\theta = \frac{T - T_\infty}{T_W - T_\infty}. \tag{19}$$

Applying the above transformations leads to the reduction of basic equations as below:

$$f'^2 - ff'' = f''' - k[2f'f''' - ff'''' - f''^2] + Ha^2(E_1 - f'), \tag{20}$$

$$f(0) = 0, \qquad f'(0) = 1, \qquad f'(\infty) = 0,$$

$$\theta'' + \Pr(f\theta' - 2f'\theta) = -\Pr E(f''^2 - Ha^2(f'^2 + E_1^2 - 2E_1 f')), \tag{21}$$

$$\theta(0) = 1, \qquad \theta(\infty) = 0.$$

4 Solution using OHAM

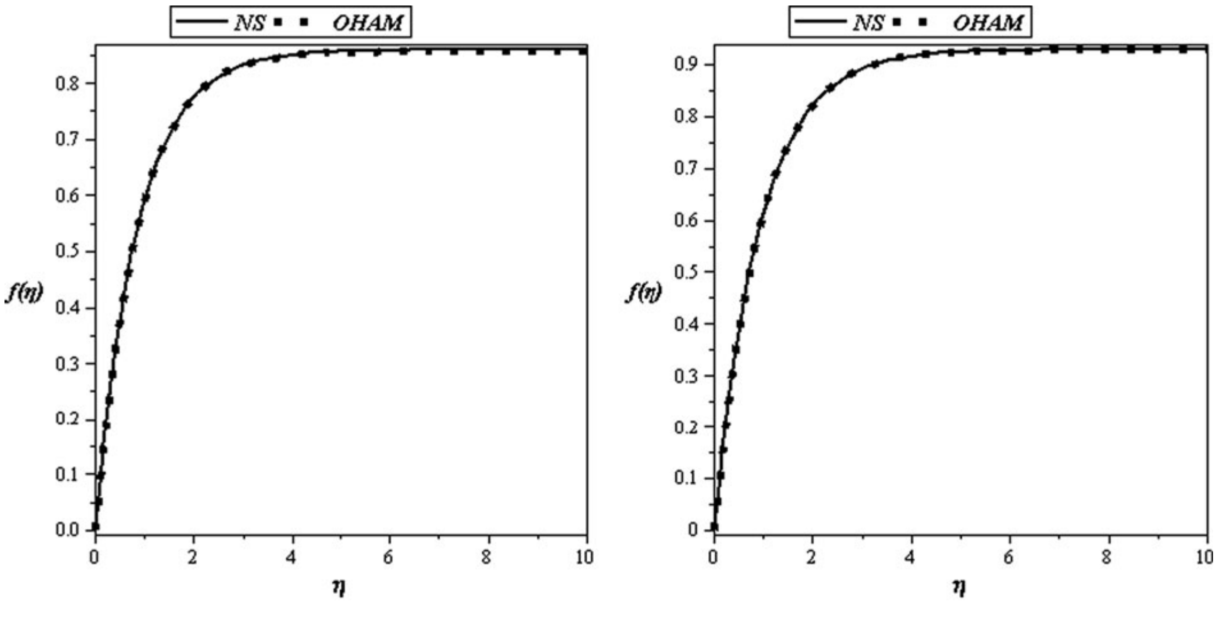
In this section, OHAM is applied to nonlinear ordinary differential equations (20) and (21). According to the OHAM, applying (2) into (20) and (21), gives:

$$\begin{aligned}&(1-p)[f'' + f'] - H_1(p)[-f'^2 + ff'' + f''' - k[2f'f''' - ff'''' - f''^2], + Ha^2(E_1 - f') - (f'' + f')] = 0,\\&(1-p)[\theta' + \theta] - H_2(p)[\theta'' + \Pr(f\theta' - 2f'\theta), + \Pr E(f''^2 - Ha^2(f'^2 + E_1^2 - 2E_1 f')) - (\theta' + \theta)] = 0,\end{aligned} \tag{22}$$

where primes denote differentiation with respect to η. We take $E = 0$ in our work.

We consider $f, \theta, H_1(p)$ and $H_2(p)$ as following:

$$\begin{aligned}f &= f_0 + pf_1 + p^2 f_2,\\ \theta &= \theta_0 + p\theta_1 + p^2\theta_2,\\ H_1(p) &= pC_{11} + p^2 C_{12},\\ H_2(p) &= pC_{21} + p^2 C_{22}.\end{aligned} \tag{23}$$



(a) (b)

Fig. 2 Comparison of the solutions via OHAM and numerical solution for $f(\eta)$ (**a**) for $Ha = 0.4$, $k = 0$, (**b**) for $Ha = 0.8$, $k = 0$

Substituting $f, \theta, H_1(p)$ and $H_2(p)$ from (23) into (22) and some simplification and rearranging based on powers of p-terms, we have:

$$\begin{aligned} p^0:\quad & f_0' + f_0'' = 0,\\ & f_0(0) = 0,\ f_0'(0) = 1,\\ & \theta_0 + \theta_0' = 0,\\ & \theta_0(0) = 1, \end{aligned} \tag{24}$$

$$\begin{aligned} p^1:\quad & C_{11} f_0' + C_{11} Ha^2 f_0' - f_0'' - C_{11} f_0'''\\ &\quad + f_1' - C_{11} k f_0''^2 - C_{11} k f_0 f_0''''\\ &\quad + f_1'' + 2C_{11} k f_0 f_0''' - f_0' + C_{11} f_0''\\ &\quad - C_{11} f_0 f_0'' + C_{11} f_0'^2 = 0,\\ & f_1(0) = 0, \qquad f_1'(0) = 0,\\ & -C_{21} \Pr f_0 \theta_0' + \theta_1 + \theta_1' - \theta_0\\ &\quad + 2C_{21} \Pr f_0' \theta_0 + C_{21} \theta_0'\\ &\quad - C_{21} \theta_0'' - \theta_0' + C_{21} \theta_0 = 0,\\ & \theta_1(0) = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} p^2:\quad & -f_1' + f_2' + C_{11} f_1' + C_{12} f_0'^2 + C_{12} f_0'\\ &\quad + C_{11} Ha^2 f_1' + C_{12} Ha^2 f_0'\\ &\quad + 2C_{11} f_0' f_1' + 2C_{11} k f_0' f_1''' + C_{11} f_1''\\ &\quad + C_{12} f_0'' - C_{11} f_1''' - C_{12} f_0'''\\ &\quad - f_1'' - C_{11} f_1 f_0'' - C_{11} f_0 f_1''\\ &\quad - C_{12} k f_0''^2 - C_{12} f_0 f_0'' + f_2''\\ &\quad + 2C_{11} k f_1' f_0''' - C_{11} k f_0 f_1''''\\ &\quad - C_{11} k f_1 f_0'''' - 2C_{11} k f_0'' f_1''\\ &\quad + 2C_{12} k f_0' f_0''' - C_{12} k f_0 f_0'''' = 0,\\ & f_2(0) = 0, \qquad f_2'(0) = 0,\\ & -C_{22} \theta_0'' + \theta_2 - \theta_1 + 2C_{22} \Pr f_0' \theta_0 + C_{21} \theta_1\\ &\quad - C_{21} \Pr f_1 \theta_0' - C_{21} \Pr f_0 \theta_1'\\ &\quad + C_{22} \theta_0 + C_{22} \theta_0' + C_{21} \theta_1\\ &\quad - \theta_1' + 2C_{21} \Pr f_0' \theta_1 + 2C_{21} \Pr f_1' \theta_0\\ &\quad - C_{21} \theta_1'' - C_{22} \Pr f_0 \theta_0' + \theta_2' = 0\\ & \theta_2(0) = 0. \end{aligned} \tag{26}$$

Solving (24)–(26) with boundary conditions yields:

$$\begin{aligned} & f_0(\eta) = 1 - e^{-\eta},\\ & \theta_0(\eta) = e^{-\eta}, \end{aligned} \tag{27}$$

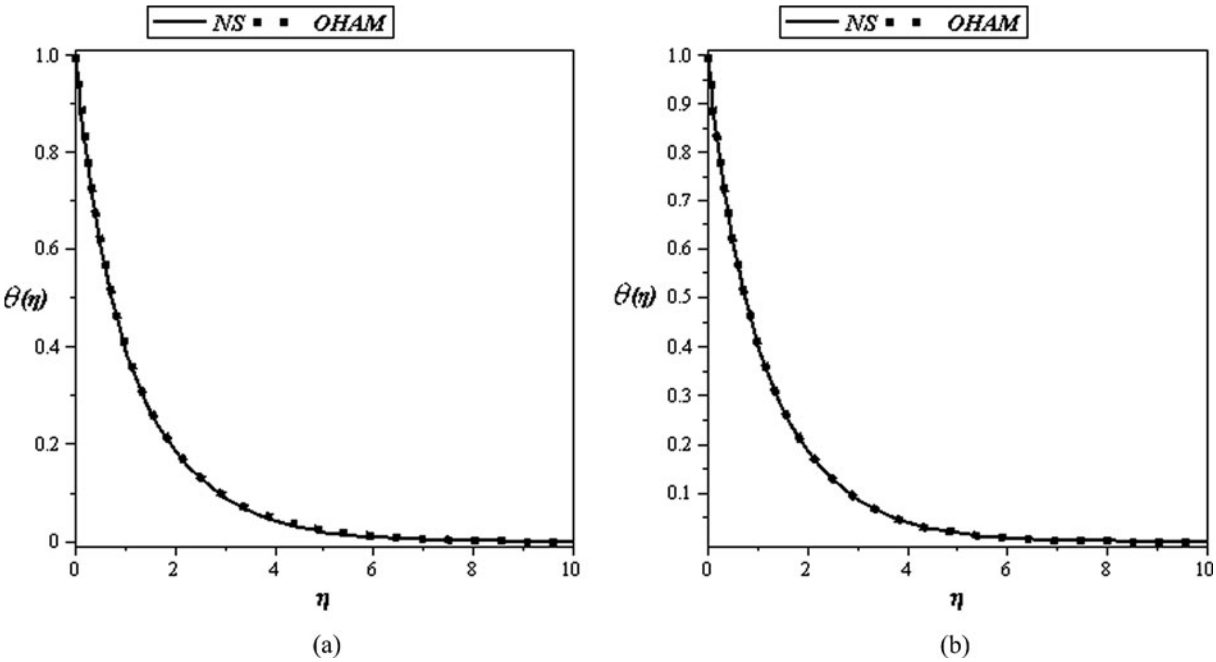


Fig. 3 Comparison of the solutions via OHAM and numerical solution for $\theta(\eta)$ (**a**) for $Ha = 0.4, k = 0, \Pr = 0.7$ (**b**) for $Ha = 0.8, k = 0, \Pr = 1$

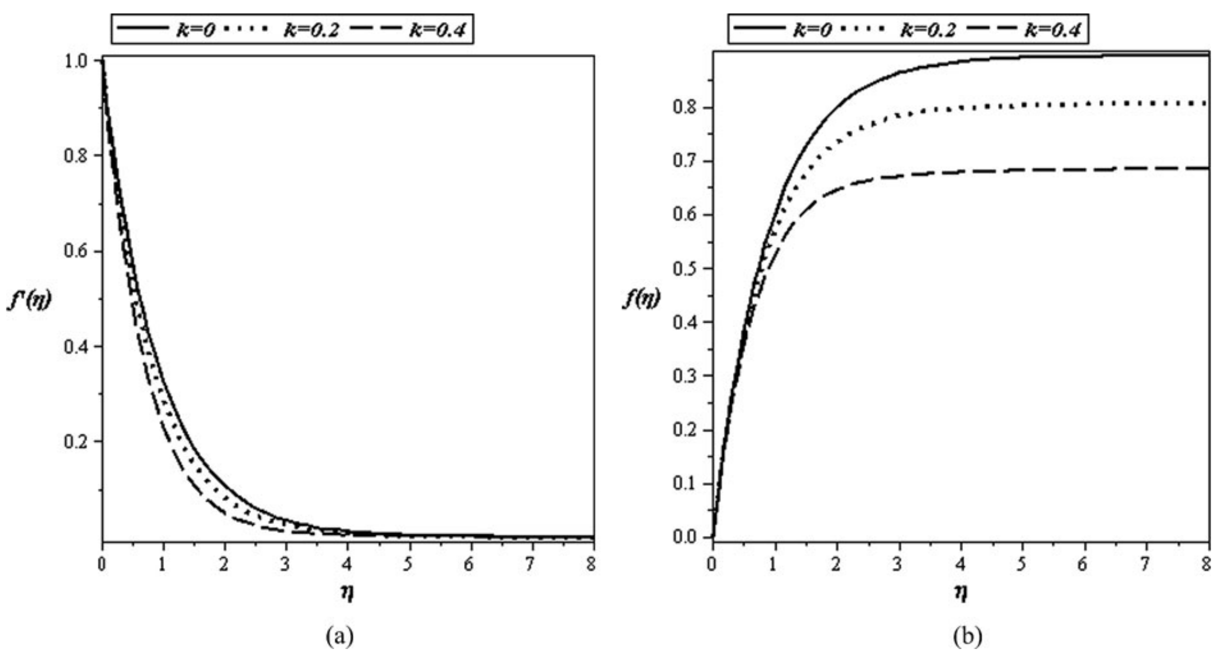


Fig. 4 Effect of viscoelastic parameter on velocity in (**a**) x direction, (**b**) y direction, for $Ha = 0.5$

$$\begin{aligned} f_1(\eta) = &-C_{11}Ha^2(-e^{-\eta}\eta - e^{-\eta}) \\ &- C_{11}k(-e^{-\eta}\eta - e^{-\eta}) \\ &- C_{11}Ha^2 - C_{11}k, \end{aligned} \tag{28}$$

$$\begin{aligned} &\theta_1(\eta) = (-C_{21}(\Pr\eta - \Pr e^{-\eta} - \eta) - C_{21}\Pr)e^{-\eta}, \\ &\vdots \\ &f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta), \end{aligned} \tag{29}$$

$\theta(\eta)=\theta_0(\eta)+\theta_1(\eta)+\theta_2(\eta).$

From (8) by substituting $f(\eta),\theta(\eta)$ into (20) and (21), $R_1(\eta,C_{11},C_{12})$ and $R_2(\eta,C_{21},C_{22})$ and subsequently J_1 and J_2 are obtained in the flowing form:

$$J_1(C_{11},C_{12})=\int_0^{\infty} R_1^2(\eta,C_{11},C_{12})d\eta, \tag{30}$$

$$J_2(C_{21},C_{22})=\int_0^{\infty} R_2^2(\eta,C_{21},C_{22})d\eta. \tag{31}$$

The constants C_{11},C_{12},C_{21} and C_{22} will be obtained from conditions (10). In the particular cases fol-

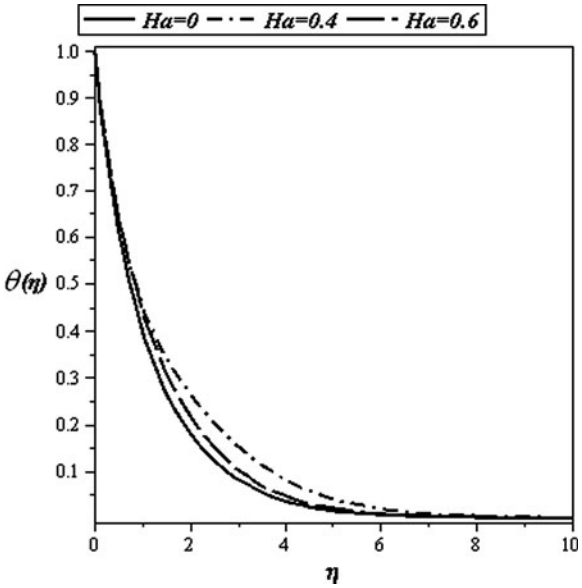


Fig. 5 Effect of Hartman number on temperature distribution for $\mathrm{Pr}=0.7, k=0.2$

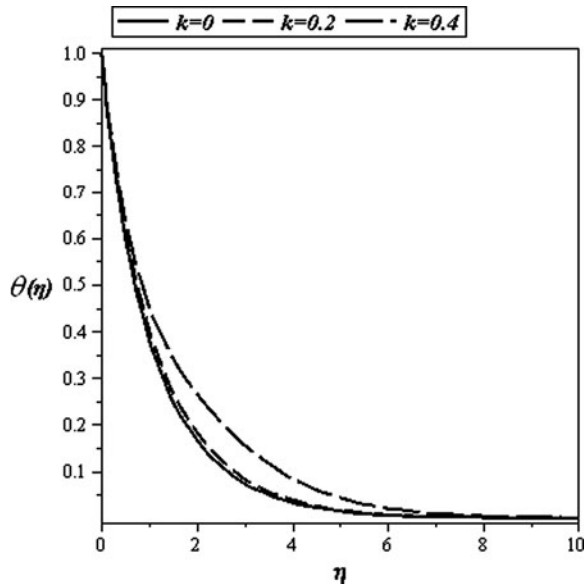


Fig. 6 Effect of viscoelastic parameter on temperature distribution for $\mathrm{Pr}=0.7, Ha=0.2$

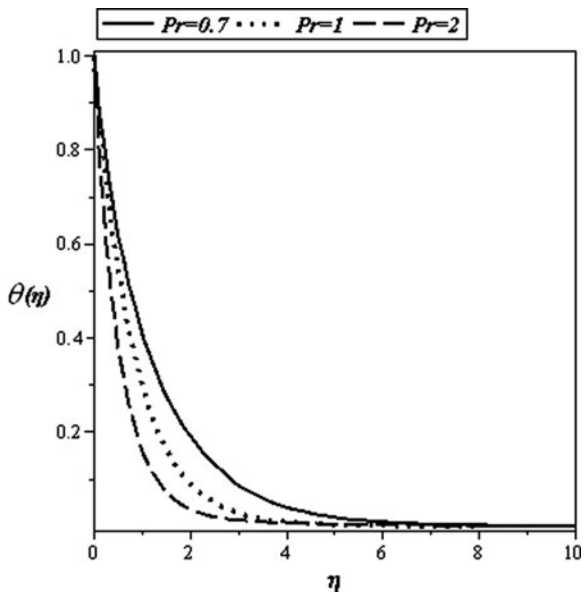


Fig. 7 Effect of Prandtl number on temperature distribution for $Ha=0.2, k=0.2$

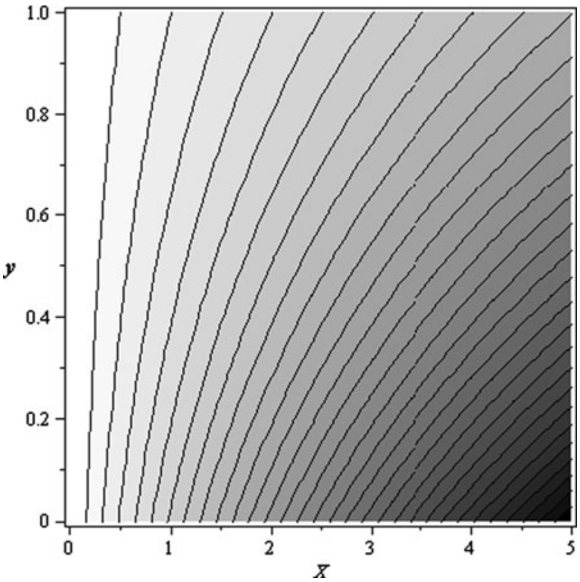


Fig. 8 Velocity distribution in x-direction for $Ha = 0.18$, $k = 0.2$, $\Pr = 0.7$

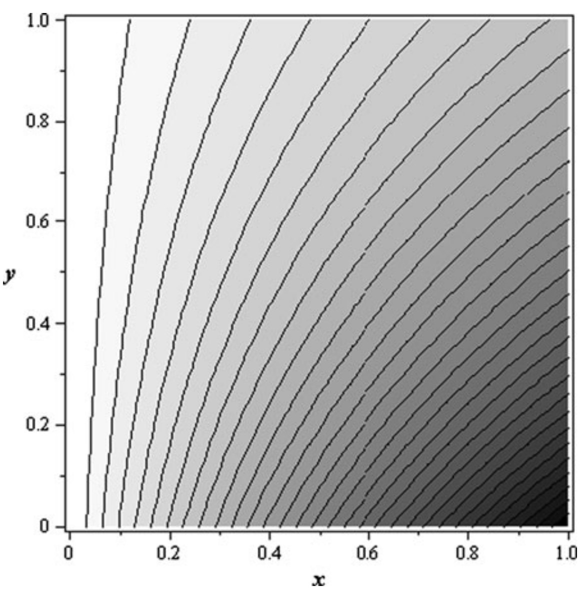


Fig. 9 Velocity distribution in x-direction for $Ha = 0.2$, $k = 0.4$, $\Pr = 0.7$

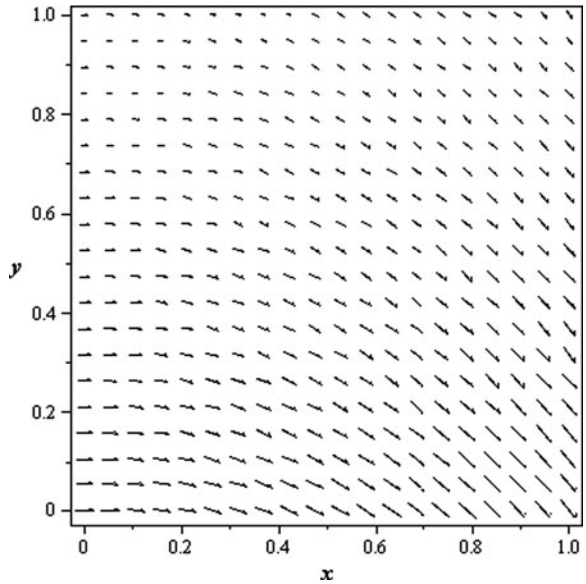


Fig. 10 Velocity vector in x-direction for $Ha = 0.18$, $k = 0.2$, $\Pr = 0.7$

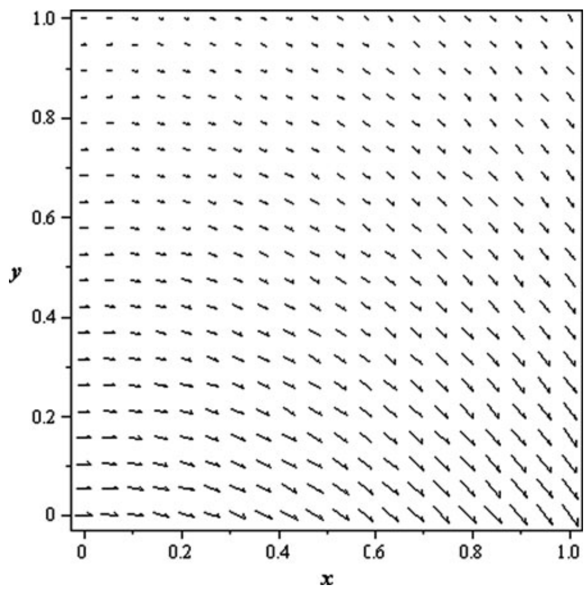


Fig. 11 Velocity vector in x-direction for $Ha = 0.2$, $k = 0.4$, $\Pr = 0.7$

lowing the procedure described above, it is obtained the convergence-control constants:

$$Ha = 0.2, \qquad k = 0.4, \qquad \Pr = 0.7,$$

$$C_{11} = -0.6725180330, \qquad C_{12} = 3.053499108,$$

$$C_{21} = 1.530462053, \qquad C_{22} = 8.006311650.$$

5 Results and discussion

For various values of Hartmann number, thermal conductivity and Prandtl number, results of the present analysis are compared with numerical solutions obtained by fourth-order Runge–Kutta in Figs. 2 and 3. In these cases, a very interesting agreement between

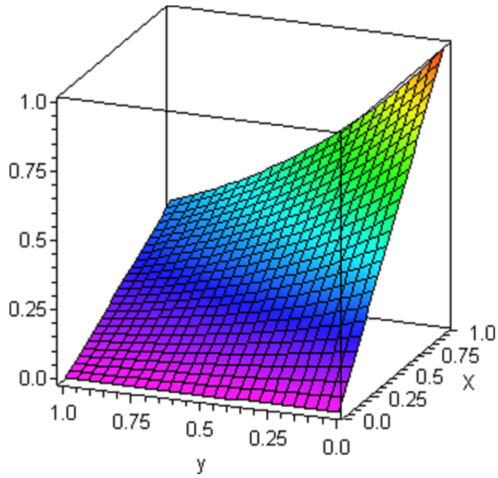


Fig. 12 Velocity profile in x-direction for $Ha = 0.18$, $k = 0.2$, $\Pr = 0.7$

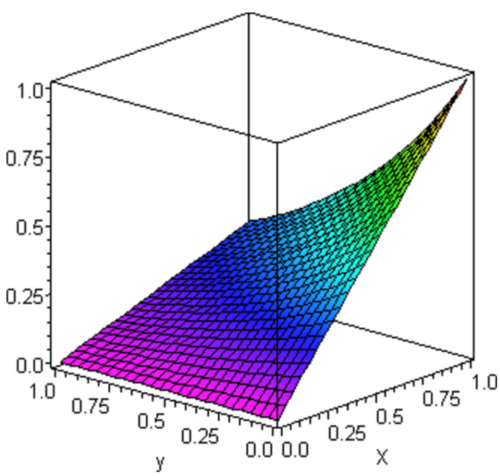


Fig. 13 Velocity profile in x-direction for $Ha = 0.2$, $k = 0.4$, $\Pr = 0.7$

the results is observed too, which confirms the excellent validity of OHAM.

Figure 4 depicts velocity in x and y-direction for various values of viscoelastic parameter (k) when Hartman number is fixed on 0.5. These figures display decreasing in velocity versus increasing in viscoelastic parameter.

Effects of Hartman number and viscoelastic parameter on temperature profile are shown in Figs. 5 and 6. Temperature increment occurs by increasing in Hartman number and viscoelastic parameter.

As it is obvious in Fig. 7, increasing in Prandtl number and decreasing in temperature values for specific

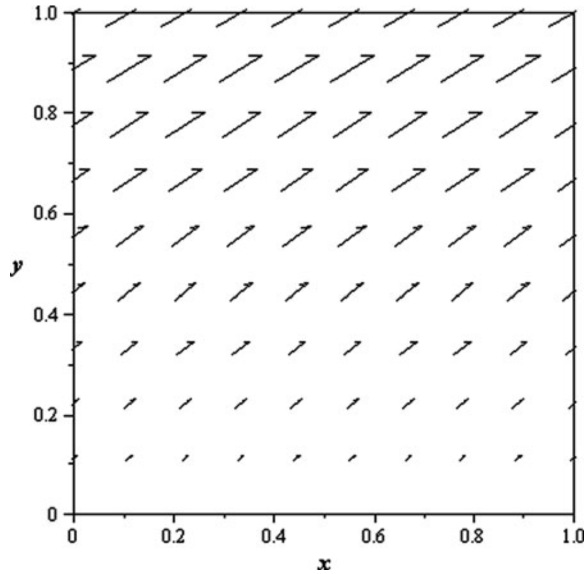


Fig. 14 Velocity vector in y-direction for $Ha = 0.18$, $k = 0.2$, $\Pr = 0.7$

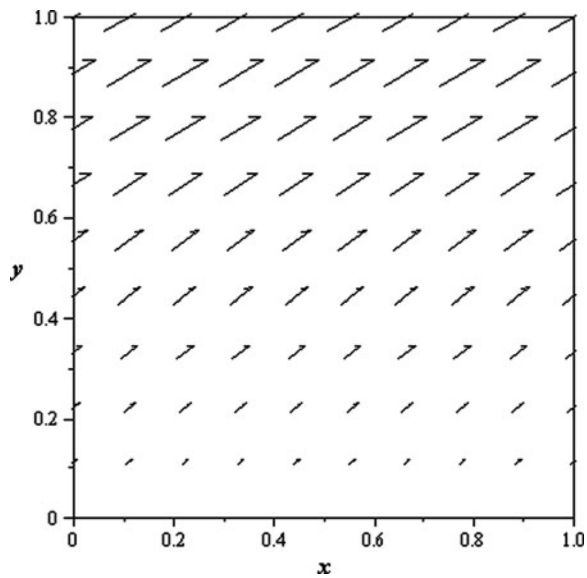


Fig. 15 Velocity vector in y-direction for $Ha = 0.2$, $k = 0.4$, $\Pr = 0.7$

values of Hartman number and viscoelastic parameter will occur at the same time.

Figures 8–13 depict velocity streams, vectors and sketches in x direction for two cases. Velocity concentration is far away from origin. It can be understood from both velocity streams and velocity vectors. By

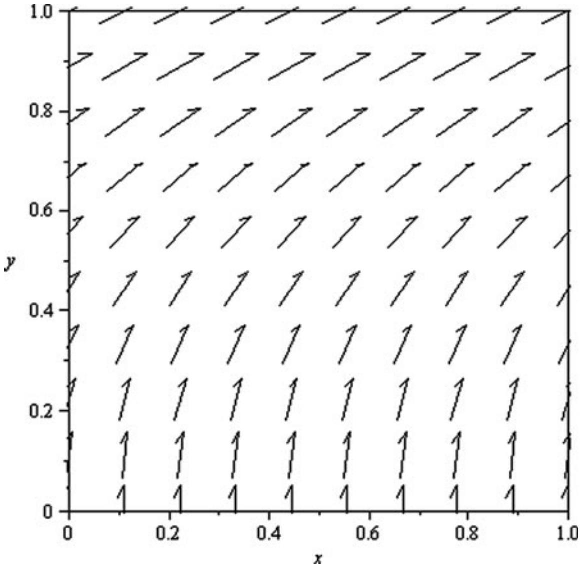


Fig. 16 Temperature vector when $Ha = 0.2$, $k = 0.2$, $\mathrm{Pr} = 0.7$

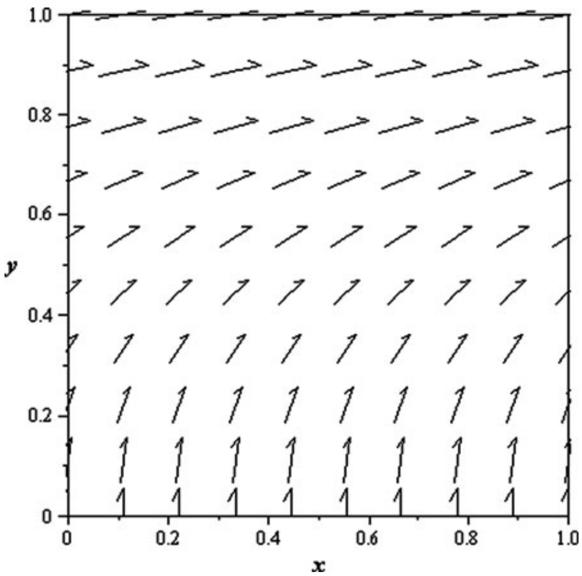


Fig. 17 Temperature vector when $Ha = 0.2$, $k = 0.2$, $\mathrm{Pr} = 2$

taking more distance from the origin, velocity vectors become greater.

As we know from Figs. 14 and 15, velocity streams in y direction become greater in monotonous form. Finally temperature vectors are plotted in Figs. 16 and 17.

6 Conclusions

In the present literature, the OHAM is successfully applied to obtain analytical solution of the temperature and velocity profiles of viscoelastic MHD flow over a stretching sheet. This exerting of OHAM is compared to fourth-order Runge–Kutta Numerical solution. The effect of flow characteristics such as Prandtl number, viscoelastic parameter and Hartman number are exhibited in several figures.

The minimum velocity in boundary layer flow is encountered if flow is viscous with higher values of Prandtl number (Pr) and Hartmann number (*Ha*). Another result is that in presence of magnetic field, the effect of electric field decreases the temperature near the stretching sheet.

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