Erratum

On the Number of Records in an iid Discrete Sequence

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Correction:

Corollary 4 is incorrect as stated and needs an additional hypothesis. The correct hypotheses are the following.

For $d \geq 1$ put $r_d = f_d / (1 - F_{d-1})$, $r = \sup\{r_1, r_2, \dots\}$ and $R_d = r_1 + \dots + r_d$. Suppose that $r < 1$ and

$$\sum_{d=1}^{\infty} \frac{r_d^2}{R_d} < \infty.$$

The key estimate in the proof of the corollary (line 6 of the proof) is missing a factor of $1/(1-r)^2$ and should read:

$$(1-r) \sum_{d=1}^{\infty} \frac{r_d^2}{R_d} \leq \sum_{d=1}^{\infty} \left(\frac{f_d}{1-F_d} \right)^2 \frac{-1}{\log(1-F_d)} \leq \frac{1}{(1-r)^2} \sum_{d=1}^{\infty} \frac{r_d^2}{R_d}.$$

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These omissions render the second part of Remark 2 in the Discussion on page 105 moot. This material should be replaced by the observation that (9) in Theorem 3 may hold even if $r=1$ in Corollary 4.

To see this, let $\mathbf{N}$ be the positive integers and suppose that x_k and y_k are strictly increasing, divergent sequences with $x_1 > 0$ and $y_0 = 0$. Furthermore, suppose that D is an infinite subset of $\mathbf{N}$, $E = D^c \cap \mathbf{N}$ and

- x_k^2/y_k is summable over D and $y_k - y_{k-1} \leq x_k/(1+x_k)$ for $k \in D$;
- $(y_k - y_{k-1})^2/y_k$ is summable over E and $y_k - y_{k-1} \leq c < 1$ for $k \in E$.

For example, take $x_k = 3k^{1/12}$, $y_k = \sqrt{k/2}$ and $D = \{n^4, n \in \mathbf{N}\}$. Put $r_d = x_d/(1+x_d)$ if $d \in D$ and $r_d = (y_d - y_{d-1})$ if $d \in E$. Then $r = 1$ and (9) holds since $R_d \geq y_d$ for all positive integers d and

$$\begin{aligned} \sum_{d=1}^{\infty} \left(\frac{f_d}{1-F_d} \right)^2 \frac{-1}{\log(1-F_d)} &\leq \sum_{d=1}^{\infty} \left(\frac{r_d}{1-r_d} \right)^2 \frac{1}{R_d} \\ &\leq \sum_{d \in D} \frac{x_d^2}{y_d} + (1-c)^{-2} \sum_{d \in E} \frac{(y_d - y_{d-1})^2}{y_d} \\ &< \infty. \end{aligned}$$