Erratum to: Improved Complexity Analysis of Full Nesterov-Todd Step Feasible Interior-Point Method for Symmetric Optimization

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Received: 3 April 2016 / Accepted: 18 April 2016 / Published online: 30 March 2017
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Erratum to: J Optim Theory Appl
DOI 10.1007/s10957-014-0696-2

1 Introduction

We correct a typo in [1, Corollary 3.1] and an error in the proof of [1, Lemma 2.3].

2 Mathematical Details

Corollary 3.1 in [1] says, for $x, s \in \text{int}\mathcal{K}$,

The online version of the original article can be found under doi:[10.1007/s10957-014-0696-2](https://doi.org/10.1007/s10957-014-0696-2).

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$$\|(x \circ s)^{-\frac{1}{2}}\|_F \leq \|(P(x)^{\frac{1}{2}}s)^{-\frac{1}{2}}\|_F.$$

A correct statement of Corollary 3.1 should be:

Corollary 2.1 *Let $x, s \in \text{int}\mathcal{K}$. Then*

$$\|(x \circ s)^{-\frac{1}{2}}\|_F \geq \|(P(x)^{\frac{1}{2}}s)^{-\frac{1}{2}}\|_F.$$

Lemma 2.3 in [1] asserts that, for x and y in an Euclidean Jordan algebra V ,

$$\sum_1^r |\lambda_i(x \circ y)| \leq \frac{1}{2}(\|x\|_F^2 + \|y\|_F^2). \quad (1)$$

The proof of (1) given in [1] relies on the implication

$$-(x \circ y) \leq \frac{1}{2}(x^2 + y^2) \Rightarrow -\lambda_i^\downarrow(x \circ y) \leq \frac{1}{2}\lambda_i^\downarrow(x^2 + y^2),$$

which is wrong, as shown by the following example.

Example 2.1 Let $\text{rank}(V) = 2$. If $x = e_1 - e_2$ and $y = -e_1 + 2e_2$, then it is easy to verify that

$$2 = -\lambda_2^\downarrow(x \circ y) > \frac{1}{2}\lambda_2^\downarrow(x^2 + y^2) = 1.$$

A correct proof of (1) is given by the first three authors in [2] (see Theorem 3.1). For the sake of completeness, we recall the proof below.

Proof of (1): Let $x \circ y = \sum_{i=1}^k \lambda_i(x \circ y)e_i$. Without loss of generality, we assume that $\lambda_i(x \circ y) \geq 0$, for $i = 1, 2, \dots, k$ and $\lambda_j(x \circ y) < 0$ for $j = k+1, k+2, \dots, r$. Now, let $w = \sum_{i=1}^r \varepsilon_i e_i$, where $\varepsilon_i = 1$ for $i = 1, 2, \dots, k$ and $\varepsilon_j = -1$ for $j = k+1, k+2, \dots, r$. Writing the Peirce decomposition of y as $y = \sum_{i \leq j} y_{ij}$, where $y_{ii} = y_i e_i$, we have $y \circ w = \sum_{i \leq j} \frac{\varepsilon_i + \varepsilon_j}{2} y_{ij}$. Thus,

$$\|y \circ w\|_F^2 = \sum \left(\frac{\varepsilon_i + \varepsilon_j}{2} \right)^2 \|y_{ij}\|_F^2 \leq \sum_{i \leq j} \|y_{ij}\|_F^2 = \|y\|_F^2.$$

Now,

$$\begin{aligned} \|x \circ y\|_1 &= \sum_{i=1}^r |\lambda_i(x \circ y)| = \text{trace}(|x \circ y|) \\ &= \langle |x \circ y|, e \rangle \\ &= \langle (x \circ y) \circ w, e \rangle \\ &= \langle x, y \circ w \rangle \end{aligned}$$

$$\begin{aligned}
&\leq \|x\|_F \|y \circ w\|_F \\
&\leq \|x\|_F \|y\|_F \\
&\leq \frac{1}{2} \left(\|x\|_F^2 + \|y\|_F^2 \right).
\end{aligned}$$

□

Acknowledgements This work was supported in part by National Natural Science Foundation of China (11471211, 11431002, 11171018), Shanghai Natural Science Fund Project (14ZR1418900), and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry.

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