Fenchel Problem of Level Sets

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In Ref. [1], the correct form of the solution of the Fenchel problem of level sets is as follows:

Theorem 3.1 *Let $f \in H_{1L}$ be a real-valued function defined on an open convex set $A \subseteq R^n$. Then, f is convex image transformable by a one-to-one increasing function $\phi \in C^2(\text{Im}_f(A), R)$ iff, for every $\mathbf{x} \in A$, there exists a convex neighborhood $U(\mathbf{x}) \subseteq A$ such that, for every pair $(\mathbf{x}, \mathbf{y} = \mathbf{z} - \mathbf{x})$, $\mathbf{z} \in A$, the single variable function*

$$f(\mathbf{x} + \varphi_{(\mathbf{x}, \mathbf{y})}(t)\mathbf{y}), \quad \mathbf{x} + \varphi_{(\mathbf{x}, \mathbf{y})}(t)\mathbf{y} \in U(\mathbf{x}), \quad t \in [0, 1], \quad (31)$$

is convex, where

$$\varphi_{(\mathbf{x}, \mathbf{y})} : [0, 1] \rightarrow R, \quad \varphi_{(\mathbf{x}, \mathbf{y})}(0) = 0, \quad \varphi'_{(\mathbf{x}, \mathbf{y})}(0) = 1,$$

is a strictly increasing function given by the following differential equation:

$$\varphi'_{(\mathbf{x}, \mathbf{y})}(t) = \frac{1}{\phi'(f(\mathbf{x}))} \phi'(f(\mathbf{x} + \varphi_{(\mathbf{x}, \mathbf{y})}(t)\mathbf{y})), \quad t \in [0, 1]. \quad (32)$$

Moreover, if $\phi : A \rightarrow R_+$ and

$$\nabla f(\mathbf{x})\mathbf{y} > 0, \quad (33)$$

then $\varphi_{(\mathbf{x}, \mathbf{y})}$ is strictly convex.

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In the proof, the formulas (11) and the ones after (13), (14), (25), (26), (39), (40), (41) contain misprints.

References

1. Rapcsák, T.: Fenchel problem of level sets. *J. Optim. Theory Appl.* **127**, 177–191 (2005)