Erratum to: Minimizing total tardiness on parallel machines with preemptions

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Abstract In this note, we correct a mistake which was made in the paper “Minimizing Total Tardiness on Parallel Machines with Preemptions” (see J Schedul 15:193–200, 2012).

Keywords Parallel machines · Total tardiness · Preemptions · NP-hardness

1 Introduction

In Sect. 3 of paper (Kravchenko and Werner 2012), a mistake in the NP-hardness proof for the problem $P \mid \text{pmtn} \mid \sum T_j$ was made. A counterexample was given in Prot et al. (2012). In Sect. 2 of this note, we present a corrected proof. In Sect. 3, we give some further remarks.

2 Corrected proof

In this section, we show that problem $P \mid \text{pmtn} \mid \sum T_j$ is NP-hard, i.e., we consider the problem of scheduling n jobs on m machines $\{1, \dots, m\}$ with preemptions to minimize the total tardiness value $\sum_{j=1}^n T_j$. Thus, for each job j , we know in advance its processing time p_j and its due date d_j .

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The proof is obtained by a reduction from PARTITION: Given a set of positive integers $a_1, \dots, a_k, b$ with $\sum_{i=1}^k a_i = 2b$. Does there exist a subset $I \subseteq \{1, \dots, k\}$ such that $\sum_{i \in I} a_i = b$?

PARTITION is ordinary NP-hard (Karp 1972), and it can be shown that it remains so under the assumption that $3k < b/2$. To see this, it is sufficient to replace $a_1, \dots, a_k, b$ by $6ka_1, \dots, 6ka_k, 6kb$.

Given any instance of PARTITION, we define a corresponding instance of the problem $P \mid \text{pmtn} \mid \sum T_j$ as follows. Let $n = 3k - 1$ and $m = k$. We set $L = 10b^3$. There are three classes of jobs:

- the class of a -jobs comprises k jobs with processing times a_i and due dates $d_i = L$ for $i = 1, \dots, k$;
- the class of ba -jobs comprises k jobs with processing times $9b^3 + a_i b^2 - a_i$ and due dates $d_i = L - a_i$ for $i = 1, \dots, k$;
- the class of b -jobs comprises $k - 1$ jobs with processing times b^3 and due dates b^3 .

We claim that PARTITION has a solution if and only if there exists a schedule with $\sum T_j \leq b^3 + b$.

First, we show that, if PARTITION has a solution, then for the corresponding instance of the problem $P \mid \text{pmtn} \mid \sum T_j$ a schedule exists with $\sum T_j \leq b^3 + b$. Without loss of generality, suppose that $\{a_1, \dots, a_z\}$ is a solution for PARTITION, i.e., we suppose that $\sum_{i=1}^z a_i = b$ holds. Then:

1. we schedule each ba -job $i, i = 1, \dots, k$, with processing time $9b^3 + a_i b^2 - a_i$ for $9b^3 - a_i$ time units in the interval $[b^3, L - a_i]$ on machine i , and each ba -job $i, i = 1, \dots, z$, for $a_i b^2$ time units in the interval $[L, L + a_i b^2]$ on machine i ;

Fig. 1 Scheduling the jobs for the given partition

2. we schedule each a -job i , $i = 1, \dots, k$, with processing time a_i for a_i time units in the interval $[L - a_i, L]$ on machine i ;
3. we schedule each b -job i , $i = 1, \dots, k-1$, with processing time b^3 for b^3 time units in the interval $[0, b^3]$ on machine i ; and
4. we schedule the remaining parts of the ba -jobs i , $i = z+1, \dots, k$, in the interval $[0, b^3]$ on machine k , see Fig. 1.

Note that after steps 1–3, all a -jobs, all b -jobs, and all ba -jobs for $i = 1, \dots, z$ are completely scheduled whereas for $i = z+1, \dots, n$, each ba -job is scheduled for $9b^3 - a_i$ time units. Thus, before step 4, the only unscheduled jobs are the ba -jobs for $a_i b^2$ time units, $i = z+1, \dots, n$.

Since, before step 4, machine k is idle for the time interval $[0, b^3]$ and $\sum_{i=z+1}^n a_i b^2 = b^3$ holds, step 4 completes the construction of a feasible schedule.

Besides, for the constructed schedule we obtain

$$\sum_{b\text{-jobs}} T_j = 0, \quad \sum_{a\text{-jobs}} T_j = 0, \quad \sum_{\substack{ba\text{-jobs} \\ j \in \{1, \dots, z\}}} T_j = b^3 + b,$$

$$\sum_{\substack{ba\text{-jobs} \\ j \in \{z+1, \dots, n\}}} T_j = 0.$$

Thus, for the constructed schedule $\sum T_j = b^3 + b$ holds.

Next, we show that, if there exists a schedule for which $\sum T_j \leq b^3 + b$ holds, then PARTITION has a solution. Consider a schedule, say s , with $\sum T_j \leq b^3 + b$.

The b -jobs have to be scheduled within the time interval $[0, 2b^3 + b]$, since otherwise inequality $\sum T_j > b^3 + b$ holds. Thus, after L only a -jobs and ba -jobs can be scheduled.

All a -jobs have to be scheduled before their due dates, i.e., before the time $L = 10b^3$. To see this, consider a ba -job, say f , with the maximal completion time C_f among all ba -jobs. There are two cases:

Case 1. If there is an a -job scheduled (partially or completely) within the interval $[C_f, +\infty[$, one can switch this part of the a -job with the ba -job f scheduled within the interval $[2b^3 + b, 9b^3]$. The value $\sum T_j$ does not increase.

Case 2. However, if there are no a -jobs scheduled within

the interval $[C_f, +\infty[$, take any a -job processed within the interval $[L, C_f]$ for some time, say for Δ time units. Denote this part of the a -job by a_Δ . Now one can take the part of the ba -job f processed within the interval $[2b^3 + b, 9b^3]$ for Δ time units. Denote this part of the job by f_Δ . Finally, one can replace f_Δ by a_Δ , and schedule f_Δ in the interval $[C_f, C_f + \Delta]$. The value of $\sum T_j$ does not increase.

As a result, we obtain a new schedule, say s' , where all a -jobs are processed before L and the completion time of the ba -job f is $C_f + X$, where X is the total amount of the processing of a -jobs, scheduled after L in s . Since the completion times are known for all jobs, the schedule s' can be constructed without applying the described transformation, but using a network flow model (Horn 1974) with a vertex for each job and for each completion time.

Thus, after the time $L = 10b^3$, only ba -jobs can be scheduled. Denote this set of jobs by A , i.e., $A = \{i \mid i \text{ is a } ba\text{-job and } C_i \geq L\}$. Denote by l_i the total length of a ba -job i , scheduled after L .

Moreover, since the sum of all processing times is $10b^3 k + b^3$, we have $\sum_{i \in A} l_i \geq b^3$ and therefore, all b -jobs have to be scheduled not in the interval $[0, 2b^3 + b]$ but in the interval $[0, b^3 + b]$.

Thus, within the interval $[b^3 + b, 10b^3]$, only ba -jobs and a -jobs can be scheduled. Since the total length of the a -jobs is $2b$, it follows that each ba -job has to be processed in the interval $[b^3 + b, 10b^3]$ for at least $9b^3 - 3b$ time units and thus, inequality $l_i \leq p_i - 9b^3 + 3b$ holds, and since $p_i = 9b^3 + a_i b^2 - a_i$, we obtain the inequality $l_i \leq a_i b^2 + 3b - a_i$, i.e., $l_i \leq a_i b^2 + 3b$ holds.

Now, by assumption, we have $b^3 + b \geq \sum_{i \in A} T_i$. Since $\sum_{i \in A} T_i \geq \sum_{i \in A} a_i + \sum_{i \in A} l_i$ and $\sum_{i \in A} l_i \geq b^3$, we obtain

$$b^3 + b \geq \sum_{i \in A} T_i \geq \sum_{i \in A} a_i + \sum_{i \in A} l_i \geq \sum_{i \in A} a_i + b^3$$

and therefore, $\sum_{i \in A} a_i \leq b$ holds.

Moreover, since inequalities $\sum_{i \in A} l_i \geq b^3$ and $l_i \leq a_i b^2 + 3b$ hold, we get

$$b^3 \leq \sum_{i \in A} l_i \leq \sum_{i \in A} (a_i b^2 + 3b).$$

Now, we have $\sum_{i \in A} (a_i b^2 + 3b) = (\sum_{i \in A} a_i) b^2 + |A| 3b$. Since $|A| \leq k$, then

$$b^3 \leq \left(\sum_{i \in A} a_i \right) b^2 + 3kb.$$

Thus, one can see that, if $3k < b/2$, then inequality $\sum_{i \in A} a_i \geq b$ holds.

Hence, we obtain $\sum_{i=1}^z a_i = b$, i.e., the set of ba -jobs completed after time point L defines the solution for PARTITION.

3 Remarks

In Kravchenko and Werner (2012), the polynomial transformation from PARTITION to the problem $P \mid r_j, p_j = p, \text{pmtn} \mid \sum T_j$ substantially used the decision version of the problem $P \mid \text{pmtn} \mid \sum T_j$. In fact, the decision version of the problem $P \mid \text{pmtn} \mid \sum T_j$ was polynomially reduced to the decision version of the problem $P \mid r_j, p_j = p, \text{pmtn} \mid \sum T_j$. In the corrected proof, we changed the decision version of the problem $P \mid \text{pmtn} \mid \sum T_j$. However,

it is easy to see that the new decision version of the problem $P \mid \text{pmtn} \mid \sum T_j$ can be polynomially transformed to the problem $P \mid r_j, p_j = p, \text{pmtn} \mid \sum T_j$ using the same scheme given in Kravchenko and Werner (2012).

Thus, both problems $P \mid \text{pmtn} \mid \sum T_j$ and $P \mid r_j, p_j = p, \text{pmtn} \mid \sum T_j$ are NP-hard.

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