Phase Transitions in *Quasi*-One-Dimensional System with Unconventional Superconductivity

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Abstract The paper is devoted to a study of superconducting properties of population-imbalanced fermionic mixtures in *quasi*-one-dimensional optical lattices. The system can be effectively described by the attractive Hubbard model with the Zeeman magnetic field term. We investigated the ground-state phase diagram of the model as a function of the chemical potential and the magnetic field. The ground

state of the system exhibits the conventional BCS-type superconductivity as well as the unconventional so-called Fulde-Ferrell-Larkin-Ovchinnikov state, in which the total momentum of Cooper pairs is non-zero. We determine the orders of transitions as well as the behavior of order parameters with a change of the model parameters.

Keywords Attractive Hubbard model · FFLO superconductivity · Ground state · Phase diagram · Phase transition

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1 Introduction

The unconventional superconductivity is still very fascinating but unsolved problem. It involves such issues as superconductivity with extremely short coherence length [1–4], the BCS-BEC crossover [4–6], unconventional pairings (particularly those with non-zero total momentum of Cooper pairs, $\mathbf{Q} \neq 0$, e.g., the so-called Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [7–9]). The phenomenon of the FFLO superconductivity can be realized possibly in many physical systems: (i) in condensed matter physics, e.g., iron-based superconductors [10–14], heavy-fermion compounds [14–20], and organic conductors [21–23], as well as (ii) in ultra cold atomic gases on optical lattices [24–34]. In the latter group of systems, the interaction parameters can be changed in a wider range [35–39]. Thus, it makes that group better candidate to experimentally investigate the model Hamiltonians.

Motivated by the experimental feasibility of such systems with ultracold gases loaded on a *quasi*-one-dimensional lattice, we studied the unconventional superfluid phases of the attractive Hubbard model, in the presence of an external magnetic field [40]. In that paper, we have shown that the

system evolves from the BCS-type superconducting state (at small field) to the FFLO phase (for sufficiently large field). In an extremal case, the momentum of Cooper pairs can lie on the vertex of the first Brillouin zone and the so-called η phase emerges. In this work, we present and discuss in details the dependence (as a function of chemical potential and magnetic field) of the following quantities: (i) an amplitude of superconducting order parameter, (ii) a total momentum of Cooper pairs, and (iii) the particle concentration.

The rest of the paper is organized as follows. In Section 2, the investigated model is presented and the method of its solution is briefly discussed. Section 3 is devoted to a discussion of the ground-state phase diagram of the model, particularly focussing on changes of order parameters with the model parameters. Finally, in Section 4, we summarize the results of the present work.

2 Model and Methods

In this paper, we study a one-dimensional chain with a BCS-type superconducting pairing term (i.e., *s-wave* one). The system is described by the attractive Hubbard model ($U < 0$) in a magnetic field [40], which in the real space can be written in the following form:

$$\hat{\mathcal{H}} = \sum_{\langle i,j \rangle \sigma} (-t - (\mu + \sigma h) \delta_{ij}) \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}, \quad (1)$$

where $\hat{c}_\sigma^\dagger$ ($\hat{c}_\sigma$) denotes an operator of creation (annihilation) of the electron with spin $\sigma \in \{\uparrow, \downarrow\}$ at site i and $\hat{n}_{i\sigma} = \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma}$ is particle number operator. $t > 0$ is the hopping between the nearest-neighbor sites, and $U < 0$ is the on-site pairing interaction. μ is the chemical potential, which determines the average number of particles $n = (1/N) \sum_{i\sigma} \langle \hat{n}_{i\sigma} \rangle$ in the system (filling) (N is the total number of sites in the lattice). Finally, h is a Zeeman field, which can originate from an external magnetic field (in $g\mu_B/2$ units) or from population imbalance in the context of the cold atomic Fermi gases. Moreover, we can introduce $\mu_\sigma = \mu + \sigma h$ as the effective chemical potential of atoms with (pseudo) spin σ . The second term of Hamiltonian (1) is decoupled within the mean-field approximation, which takes into account only superconducting averages:

$$\hat{n}_{i\uparrow} \hat{n}_{i\downarrow} = \Delta_i^* \hat{c}_{i\downarrow} \hat{c}_{i\uparrow} + \Delta_i \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger - |\Delta_i|^2, \quad (2)$$

where $\Delta_i = \langle \hat{c}_{i\downarrow} \hat{c}_{i\uparrow} \rangle$ is the site-dependent on-site superconducting order parameter (SOP). Then, the mean-field Hamiltonian in the real space is written in the form:

$$\hat{\mathcal{H}}^{MF} = \sum_{\langle i,j \rangle \sigma} (-t - (\mu + \sigma h) \delta_{ij}) \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_i (\Delta_i^* \hat{c}_{i\downarrow} \hat{c}_{i\uparrow} + H.c.) - U \sum_i |\Delta_i|^2. \quad (3)$$

Without loss of generality, one can write down the SOP as follows: $\Delta_i = \Delta_0 \exp(i \mathbf{Q} \cdot \mathbf{R}_i)$, where $\Delta_0 > 0$ is the spatially oscillating amplitude of the SOP and $\mathbf{Q}$ is the total momentum of the Cooper pairs. We assume that the lattice constant is equal to one, i.e., $a = 1$. In a one-dimensional case considered here, $|\mathbf{Q}| = Q_x$, where Q_x is the (absolute value of) coordinate (the only one) of the vector $\mathbf{Q}$ (with the largest allowed value $Q_x^{\max} \equiv \pi$). The procedure of numerical solving of the system and final equations for the grand canonical potential and order parameters are presented in Refs. [40, 41]. Notice that all found solutions correspond to minimal value of the grand canonical potential (with respect to Δ_0 and $\mathbf{Q}$) at fixed model parameters [40, 41]. Below, we just discuss the behavior of the quantities in the system for some exemplary value of the on-site attraction.

3 Numerical Results and Discussion

In this section, we consider the system with a value of the pairing interaction set as $U/t = -3$. The ground-state phase diagram of model (3) is presented in Fig. 1. In the absence of an external Zeeman field, the usual superconducting BCS-type *s-wave* state (with $\Delta_0 \neq 0$ and $Q_x = 0$) is stable. As the magnetic field increases, superfluidity is destroyed due to paramagnetic effects or by population imbalance. Hence, the unpolarized BCS-like superconducting phase undergoes a first-order phase transition to the polarized normal (NO) state (with $\Delta_0 = 0$, Q_x – undetermined, and $|n - 1| \neq 1$) or to the FFLO phase (with $\Delta_0 \neq 0$ and $Q_x \neq 0$). Inside the FFLO phase, for large fields and near half-filling, the

Fig. 1 The phase diagram of the model for $U/t = -3.0$ as a function of $|\mu|/t$ and h/t . BCS denotes the usual superconducting BCS-type *s-wave* state, FFLO labels the polarized superconducting state with non-zero momentum of Cooper pairs, whereas NO corresponds to normal (non-ordered) phase. Additionally, inside the FFLO region above the dashed line, there is a region, where the η phase is distinguished. Inside the NO area, the NO phase with $n = 1$ (for small $|\mu|/t$ and large h/t) and the NO phase with $|n - 1| = 1$ (for large $|\mu|/t$ and small h/t) are indicated by the dotted lines

η -FFLO superconducting phase (i.e., the FFLO phase with $Q_x = Q_x^{\max}$) is also identified. Increasing higher the field and close to half-filling, the FFLO- η -pairing superconducting phase undergoes a first-order phase transition to the normal state (this transition can be also second-order for larger $|\mu|/t$, see the text below). Moreover, for magnetic fields larger than the top of the band (i.e., $2t$ for one-dimensional case studied here), one can find the magnetic Lifshitz transition [40, 42]. As a consequence of the relatively strong pairing interaction, the superconductivity still exists in the system, even if the Fermi surface for one type spin disappears.

Now, we discuss the evolution of (i) the amplitude Δ_0 of the superconducting order parameter, (ii) the component Q_x of the total momentum of Cooper pairs, and (iii) electron concentration n in the system as a function of the model parameters μ and h .

Figure 2 presents the amplitude Δ_0 of the superconducting order parameter. In the BCS phase, Δ_0 is monotonously decreasing function of $|\mu|/t$ and it is independent of h/t . At the transition between the BCS phase and the NO phase with $n = 0$, the Δ_0 vanishes continuously as it should behave at continuous transition. At $h/t \neq 0$, the BCS–NO transition is discontinuous (with discontinuous change of Δ_0). Similarly, at the BCS–FFLO boundary, Δ_0 exhibits a discontinuous change to a lower value. Inside the FFLO region (including also η phase), Δ_0 decreases monotonously with increasing $|\mu|/t$ and h/t . At the FFLO–NO boundary, parameter Δ_0 vanishes continuously to $\Delta_0 = 0$. The transition between the η -FFLO phase and the NO phase is continuous only in some range of model parameters. For high magnetic field and near the half-filling, it is associated to a discontinuity of Δ_0 (it is hardly visible in Fig. 2).

The ground-state values of Q_x are presented in Fig. 3. As one can expect, $Q_x = 0$ at the BCS phase, where the total momentum of Cooper pairs is zero. In the FFLO phase, $Q_x \neq 0$ and it increases with increasing of the magnetic field to its maximal value $Q_x^{\max} = \pi$. In the whole

Fig. 2 The amplitude Δ_0 of the superconducting order parameter for $U/t = -3.0$ as a function of $|\mu|/t$ and h/t . The white planes denote discontinuous changes of Δ_0 at the boundaries between phases (notice the small region at the η -FFLO–NO boundary near $|\mu|/t = 0$)

Fig. 3 The momentum Q_x of superconducting pairs for $U/t = -3.0$ as a function of $|\mu|/t$ and h/t . At the η -FFLO phase, $Q_x = \pi$. In the NO region, Q_x is undetermined (not well defined), but in the figure, we have adopted the convention that in the NO phase, $Q_x = 0$. The white planes denote discontinuous changes of Q_x at the boundaries between phases

region of η -FFLO phase, the total momentum of the pairs does not change and equals $Q_x = Q_x^{\max}$. Notice that at the BCS–FFLO boundary Q_x changes discontinuously. Q_x is continuous at the boundary between the η -FFLO phase (with $Q_x = Q_x^{\max}$) and the FFLO phase (where $0 < Q_x < Q_x^{\max}$). At the NO region, Q_x is undetermined (i.e., it is not well defined, no Cooper pairs in the system), but in Fig. 3, we have adopted the convention that in the NO phase $Q_x = 0$.

The very important feature is the behavior of the particle concentration n , particularly in the context of the phase separations (cf. Refs. [43–46] and references therein). If n changes discontinuously from n_- to n_+ at the boundary line between two phases in the diagram for fixed μ , the phases can co-exist on the phase diagram as a function of n . The dependence of particle concentration n (precisely the value of $|n - 1|$) with changing the model parameters is shown in Fig. 4. At fixed h/t , the value of $|n - 1|$ is increasing function of $|\mu|/t$. It turns out that the discontinuous changes of n occur at the same phase boundaries where Δ_0 exhibits discontinuity. Namely, the BCS–FFLO, BCS–NO (only to the NO phase with $n \neq 0$) and η -FFLO–NO (only for high magnetic field and near the half-filling) transitions are associated to abrupt changes of n . The value of $|n - 1|$ in the FFLO phase is smaller than those in the NO phase (at the FFLO–NO boundary). Similarly, at the BCS–FFLO boundary, the value of $|n - 1|$ changes discontinuously to higher value in the FFLO phase. At the discontinuous BCS–NO boundary the relative values of concentrations depend on the value of μ/t . For the discontinuous η -FFLO–NO transition, the lower value of $|n - 1|$ is in the NO phase. As a result, one can distinguish three different phase-separated states, which can occur in the phase diagram as a function of n : phase separation between the BCS and FFLO phases, phase separation between the BCS and NO phases, and finally phase separation between the η -FFLO and NO phases (cf. also [40]).

Fig. 4 The average concentration $|n - 1|$ of the particles for fixed $U/t = -3.0$ as a function of $|\mu|/t$ and h/t . The discontinuities of $|n - 1|$ occur at the same phase boundaries as those for Δ_0 , but they are less visible in the figure. Notice that z -axis of the plot (corresponding to $|n - 1|$ quantity) has inverted direction

The behavior of Δ_0 , Q_x , and n should not strongly depend on low and intermediate values of $U < 0$ qualitatively since it only determines the magnitude of the pairing potential in the system. Notice also that the ground-state phase diagrams for different attractive U presented in Ref. [40] do not modify qualitatively with changing U in this range of U .

4 Summary

In this work, we have studied the ground state of the attractive Hubbard model focusing on the behavior of order parameters. We found that in the ground state of the system the following phases can occur: the NO phase, the BCS phase, and the FFLO phase (with its extreme case—the η -FFLO phase). The phase transition between the BCS and FFLO phase is discontinuous one, whereas the FFLO–NO and FFLO– η -FFLO transitions are continuous. The BCS–NO and η -FFLO–NO transitions can be of both types, depending on the region of the phase diagram.

Notice that the mean-field approximation is generally valid only for small U and high-dimensions. It overestimates usually critical temperatures and the range of stability of the phases with long-range order. However, the mean-field approach gives at least qualitative description of the system in the ground state, even in the strong coupling limit [4]. Nevertheless, we used this approach to study *quasi*-one-dimensional model describing fermionic ultracold gas on the optical lattice. According to the Mermin-Wagner theorem, a one-dimensional superfluid system cannot support superfluidity and would possess, at best, algebraically decaying long-range order at zero temperature [47] (cf. also with Refs. [48–50]). The real systems of atoms on the optical lattices are *quasi*-one-dimensional systems which means

that the one-dimensional cylindrical-shaped regions are weakly coupled with each other, what makes the mean-field approximation more appropriate.

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