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To the article “Bell-type inequalities and upper bounds for multiqubit states” by *Loran V. Akopyan and Vladimir I. Man’ko* [Vol. 30, No. 4, pp. 338-358 (July-August, 2009)].

Due to a glitch in computer typesetting, the entire page 345 was incorrectly printed in boldface type. In addition, some of the mathematical symbols at the top one-third of the page dropped out. These errors appeared in the print copy only.

The corrected page is reprinted below.

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upper bounds analogous to the Cirel’son bound in the lower dimension $d = 2$. With the implementation of quantum tomographic methods, we see that there exist large domain of parameters violating the Bell-type inequalities.

Our approach allows one to easily generalize the results obtained for the maximum entangled state (8) of the well-known Werner states for two qubits [23]. Indeed, for the state

$$\rho^W(p) = \left\| \begin{array}{cccc} \frac{1+p}{4} & 0 & 0 & \frac{p}{2} \\ 0 & \frac{1-p}{4} & 0 & 0 \\ 0 & 0 & \frac{1-p}{4} & 0 \\ \frac{p}{2} & 0 & 0 & \frac{1+p}{4} \end{array} \right\| = \frac{1}{4}\hat{1} + \frac{p}{2} \left\| \begin{array}{cccc} \frac{1}{2} & 0 & 0 & 1 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{array} \right\|,$$

the Bell-CHSH form $B_{2\otimes 2}^W(\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3, \theta_4, \varphi_4, p)$ assumes the expression presented in the Appendix.

Figure 2 shows the dependence of upper bounds of the Bell-type inequality for two qubits on the parameter p responsible for the entanglement of the system. As can be drawn from the curves, the upper bound grows linearly in the interval of entanglement $1/3 < p < 1$, reaching its maximum at $p = 1$.

The two-qubit system reviewed here provides an insight into studying higher-dimension multiqubit states at least with respect to the amount of information that the Bell-CHSH form of a multipartite system stores in it. First of all, the tomographic probability representation of multipartite states provides a universal tool both in classical and quantum realms. Procedure-wise the program of studying the system’s Bell-type inequalities and upper bounds can be formulated in the following way.

Each subsystem contributes to the resulting joint probability distribution of the composite system via a stochastic matrix. Information on correlations between various dynamic variables in the system (both in quantum and classical cases) is contained in the stochastic matrix of the composite state, which can be either simply separable, separable, or entangled (non-separable). This matrix enters a bilinear form named the Bell-CHSH form through the average value of the product of correlating observables. We calculate all local (Bell-type numbers) and global maxima (the upper bound) of the Bell-CHSH bilinear form. We establish first the Bell-type numbers in classical probability theory for multilevel systems. Then we calculate the Bell-type numbers through quantum tomograms and compare the results obtained. This recipe allows one to detect possible Bell-type inequalities, classify them by their corresponding upper bounds, and predict violation of these inequalities in quantum mechanics by observable angles rising in the measurement process of the quantum system. Moreover, given an entangled multiqubit state, our method allows one to find explicitly the domains in the parameter space where the Bell-type inequalities are preserved and the small quantum domain where they are maximum violated. We will now realize this program for several interesting multiqubit states.

Fig. 2. The upper bounds of the Werner states as a function of the parameter p .