Errata to “The Asymptotic Speed of Spread and Traveling Waves for a Vector Disease Model” **[*J. Dynam. Diff. Eq.* 18, 1001–1019 (2006)]**

Xiao-Qiang Zhao^{1,3} and Dongmei Xiao²

In our recent paper [2], the existence of traveling wave solutions $U(x + ct)$ with $c > c^*$ was proved via the method of upper and lower solutions. However, there is a mistake in the verification of the lower solution in the proof of [2, Theorem 3.1]. More precisely, the number M depends on the parameter $\xi \in \mathbb{R}$ and is unbounded as $\xi \rightarrow -\infty$. Thus, there may not exist a constant $M \geq 1$ such that $\underline{U}(\xi)$ is a lower solution for $\xi \leq \xi_0 := -\frac{\ln M}{\epsilon}$. The purpose of our current note is to correct this mistake. Instead of constructing an appropriate lower solution, we use the finite time-delay approximation and a limiting argument to prove the existence of traveling waves. It turns out that the theory developed in [1] can also be applied to establish both spreading speeds and traveling waves for monotone evolution equations with spatial structure and infinite time delay with the aid of the finite time-delay approximation.

The online version of the original article can be found under doi: [10.1007/s10884-006-9044-z](https://doi.org/10.1007/s10884-006-9044-z)

Supported in part by the NSERC of Canada and the MITACS of Canada.

Supported in part by the Program for New Century Excellent Talents in Universities of China.

¹Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NL, Canada A1C 5S7. E-mail: xzhao@math.mun.ca

²Department of Mathematics, Shanghai Jiaotong University, Shanghai 200030, China. E-mail: xiaodm@sjtu.edu.cn

³To whom correspondence should be addressed.

Proof of the existence of traveling waves in the case where $c > c^*$. We first point out that in the statement of [2, Proposition 2.1 (3)], $U(x - ct)$ was mis-typed as $U(x + ct)$ (see [1, Theorem 5.1]). In what follows, we use the notations in [2].

Let $c > c^*$ be fixed. Since $\lim_{\tau \rightarrow \infty} u_2^\tau = u_2 := 1 - \frac{a}{b}$ and $\lim_{\tau \rightarrow \infty} c_\tau^* = c^*$, it follows that there exists $\tau_1 \geq \tau_0$ such that $u_2^\tau > \frac{u_2}{2}$ and $c > c_\tau^*$ for all $\tau \geq \tau_1$. By [2, Proposition 2.1 (3)], for each $\tau \geq \tau_1$, Eq. (2.6) in [2] has a monotone traveling wave $U_\tau(x + ct)$ with $U_\tau(-\infty) = 0$ and $U_\tau(+\infty) = u_2^\tau$. By the spatial translation invariance of (2.6), we can further assume that $U_\tau(0) = \frac{u_2}{2}$ for all $\tau \geq \tau_1$. It is easy to see that $U_\tau(\xi)$ satisfies the following equation

$$\begin{aligned} cU'_\tau(\xi) &= dU''_\tau(\xi) - aU_\tau(\xi) + b(1 - U_\tau(\xi)) \int_0^\tau ds \\ &\quad \times \int_{-\infty}^\infty F(s, y)U_\tau(y + \xi - cs)dy. \end{aligned} \quad (1)$$

By similar arguments as in the proof of [2, Theorem 3.1 (1)] for the case where $c = c^*$, it follows that there exists a positive number B such that

$$|U'_\tau(\xi)| \leq B, \quad |U''_\tau(\xi)| \leq B, \quad |U'''_\tau(\xi)| \leq B, \quad \forall \xi \in \mathbb{R}, \tau \geq \tau_1.$$

Thus, three families of functions $\{U_\tau(\xi) : \tau \geq \tau_1\}$, $\{U'_\tau(\xi) : \tau \geq \tau_1\}$ and $\{U''_\tau(\xi) : \tau \geq \tau_1\}$ are uniformly bounded and equi-continuous on $\mathbb{R}$. Using the Arzela-Ascoli theorem and the standard diagonal method, we can choose a sequence $\tau_n \rightarrow +\infty$ such that $U_{\tau_n}(\xi)$, $U'_{\tau_n}(\xi)$ and $U''_{\tau_n}(\xi)$ converge uniformly on every compact subset of $\mathbb{R}$ as $n \rightarrow \infty$. It then follows that there exists a function $U \in C^2(\mathbb{R}, \mathbb{R})$ such that

$$\lim_{n \rightarrow \infty} (U_{\tau_n}(\xi), U'_{\tau_n}(\xi), U''_{\tau_n}(\xi)) = (U(\xi), U'(\xi), U''(\xi)), \quad \forall \xi \in \mathbb{R}.$$

By Lebesgue dominated convergence theorem, letting $\tau = \tau_n \rightarrow \infty$ in (1), we obtain

$$\begin{aligned} cU'(\xi) &= dU''(\xi) - aU(\xi) + b(1 - U(\xi)) \int_0^\infty ds \\ &\quad \times \int_{-\infty}^\infty F(s, y)U(y + \xi - cs)dy. \end{aligned} \quad (2)$$

This implies that $u(t, x) := U(x + ct)$ is a solution of (3.1) in [2]. Clearly, $U(\xi)$ is nondecreasing on $\mathbb{R}$ and $U(0) = \frac{u_2}{2}$. Then we have $U(-\infty) \in [0, \frac{u_2}{2}]$ and $U(+\infty) \in [\frac{u_2}{2}, u_2]$. It easily follows from Eq. (2) that both $U(-\infty)$ and $U(+\infty)$ are zeros of the equation $-au + b(1 - u)u = 0$ in $[0, u_2]$, and hence $U(-\infty) = 0$ and $U(+\infty) = u_2$. Consequently, $U(x + ct)$ is a monotone traveling wave solution of (3.1) connecting 0 and u_2 .

REFERENCES

1. Liang, X., and Zhao, X.-Q. (2007). Asymptotic speeds of spread and traveling waves for monotone semiflows with applications. *Commun. Pure Appl. Math.* **60**, 1–40.
2. Zhao, X.-Q., and Xiao, D. (2006). The asymptotic speed of spread and traveling waves for a vector disease model. *J. Dyn. Differ. Eqs.* **18**, 1001–1019.