Erratum to “On a Model of a Currency Exchange Rate – Local Stability and Periodic Solutions” [*J. Dynam. Diff. Eq.* 16, 393–432 (2004)]

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The main error which is corrected below concerns the proof of Proposition 3.1. Further errors were caused by the production process. Of such errors we list below only those which result in mathematically incorrect, misleading and irritating statements. Other minor corrections, concerning spelling, syntax, etc, are not included here.

Page and line numbers below refer to the electronic version of the paper.

Page, line

406,7 Read: $\lim_{\xi \rightarrow \infty} f(\xi) = -\infty \dots$

406,14 Read: defines ...

408,5 Read: ... $x(s) = 2a$ ($x(s) = -2a$).

408,31 and 32: Read m_j instead of M_j .

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409,18 Read: Segments of slowly oscillating functions ...

409,27 (line 2 from bottom): ... for $0 \leq t \leq 5$, ...

409,28 (bottom): $t \in [0, 5]$, and $y_t \in Z$ for $t \geq 5$.

410,22 until 411,6: Replace the text by the following:

1.3. Let

$$t_0 = \inf\{t \in [0, z+1] : y(t) < 0\}.$$

Then $0 \leq t_0 < z+1$ and $y(t) = 0$ on $[0, t_0]$, or $t_0 = 0$ and $y(t_0) = y(0) < 0$.

Claim: $y(t) < 0$ on $(t_0, z+1]$.

Proof. There is a sequence $(s_j)_0^\infty$ in $(t_0, z+1)$ converging to t_0 , with $y(s_j) < 0$ for all $j \in \mathbb{N}_0$. It suffices to show $y(t) < 0$ on $[s_j, z+1]$ for all $j \in \mathbb{N}_0$. Let $j \in \mathbb{N}_0$ be given. On $[s_j, z+1]$, $y(t) < w(t)$ for the solution $w : [s_j, z+1] \rightarrow \mathbb{R}$ of the IVP

$$\dot{w} = h(t, w) - ay(t-1), \quad w(0) = 0.$$

As in part 1.1 above one finds $w(t) \leq 0$ in $[s_j, z+1]$.

1.4. In case $y(t) < 0$ on $(t_0, t_0+1]$, we obviously have $y_t \in S_1$ on $[0, 1]$, and for $\epsilon > 0$ sufficiently small, y has no zero in $[t_0 + \epsilon, t_0 + 1 + \epsilon] \subset [0, 2]$. In particular, $y_{t_0+1+\epsilon} \in Z$.

1.5. In the remaining case, y has a smallest zero $z_1 \in (z+1, t_0+1]$. Then $y \leq 0$ on $[z_1-1, z_1]$, and $y < 0$ on (t_0, z_1) ; $z_1-1 \leq t_0 < z_1$. Arguing as in part 1.1 above one gets $0 \leq y(t)$ in $[z_1, z_1+1]$. As in part 1.2 one finds $t \in (z_1, z_1+1]$ with $0 < y(t)$. For

$$t_1 = \inf\{t \in (z_1, z_1+1] : 0 < y(t)\},$$

we have $z_1 \leq t_1 < z_1+1$, $y = 0$ on $[z_1, t_1]$, and by arguments as in part 1.3, $0 < y(t)$ on $(t_1, z_1+1]$.

In the subcase $z_1 = t_1$, obviously $y_t \in S_1$ for $0 \leq t \leq 1 \leq z_1+1$, and for $\epsilon > 0$ sufficiently small, y has no zero in $[z_1 + \epsilon, z_1 + 1 + \epsilon]$. Hence $y_{z_1+1+\epsilon} \in Z$, with $z_1 + 1 + \epsilon < t_0 + 2 + \epsilon < 3$.

In the subcase $z_1 < t_1 < z_1 + 1$, arguments as in parts 1.1–1.3 (for $z = 0 < t_0$) yield $0 < y(t)$ on $(t_1, t_1 + 1]$. This implies $y_t \in S_1$ for $0 \leq t \leq 1 < t_1 + 1$. For $\epsilon > 0$ sufficiently small, y has no zero in $[t_1 + \epsilon, t_1 + 1 + \epsilon]$. Hence $y_{t_1+1+\epsilon} \in Z$, with $t_1 + 1 + \epsilon < z_1 + 2 + \epsilon < t_0 + 3 + \epsilon < 4$.

411,9 Read: $t \in [0, 4]$.

411,14 Read: $\dots t_* \in [0, 5] \dots$

411,15 Read: $0 \leq t \leq 5$.

411,25 Read: $\dots t \geq 5$.

413,13 Read: The order of a solution λ of Eq. (4.2) ...

413,23 Read:

$$\zeta + \alpha e^{-\zeta} = 0 \quad (4.3)$$

414,1–6 Read (text of Corollary 4.1 in italics, spacing):

For all $a > 0$, $\dim L = 2$ and $C_0 \subset L$. For $0 < a \neq 1$, $\dim C_0 = 1$, while for $a = 1$, $\dim C_0 = 2$ and $C_0 = L$. For $a > 1$, $\dim C_+ = 1$ and $L = C_0 \oplus C_+$. For $0 < a \neq 1$, $C_0 = \mathbb{R}\eta_0$ where $\eta_0(t) \equiv 1$. For $a = 1$, $L = C_0 = \mathbb{R}\eta_0 \oplus \mathbb{R}\sigma$, with $\sigma(t) = t$. For $a > 1$, $C_+ = \mathbb{R}\eta_u$ where $\eta_u(t) = e^{ut}$. For $0 < a \neq 1$, $L = \mathbb{R}\eta_0 \oplus \mathbb{R}\eta_u$.

414,9 Read: ... of the linearized Eq. (4.1) is

414,25 Read:

$$\langle \psi, \phi \rangle = \langle \psi(0), \phi(0) \rangle - a \int_{-1}^0 \psi(\xi + 1) \phi(\xi) d\xi. \quad (4.4)$$

414,31 (bottom) Read:

$$\Pi(\chi) = \langle \Psi, \chi \rangle \Phi, \quad (4.6)$$

415,1 Read: ... and $\Phi = (\eta_0, \sigma)$ for $a = 1$

416,11–14 Read: ... $C \setminus H_0$. Consequently, $P^{-1}(PH_0) = C$, which ... – It follows that ... assertions for H_- are proved analogously.

416,21 Read: ... employ the standard ...

416,27 Read: ... we have $F(t, \phi) \in W_c$.

417,5 Read: ... from Section 4 and h a C^1 -map ...

417,6 Read: ... such that $h(z) = o(z)$.

417,14 Read:

$$\dot{x}(t) = \Lambda x_t + q(x_t),$$

417,27 Read: $C_0^* \subset C([0, 1], \mathbb{R})$ is ...

418,21 Read: ... by the system $\dot{z} = Bz$ with

420,11 Read: $[0, \infty) \rightarrow \dots$

421,2 Read: ... Should ... for some $t \geq t_1$,

421,21 Read:

$$\dot{\omega}(t) \leq \cos^2 \omega(t) + \kappa(t)$$

423,4 Read: ... Proposition 5.1 imply ...

423,13 Read: ... the (positive definite) C^1 -function ...

423,20 Read:

$$E = \{(z_1, z_2) \in \mathbb{R}^2 : \dot{V}(z_1, z_2) = 0\} = \{(z_1, z_2) \in \mathbb{R}^2 : z_1 = 0\}.$$

423,28 Read: ... with $a, q(\phi)$ from (5.3) replaced by

423,29 Read: 1 and $q(\phi, \alpha) = -|\phi(0)|\phi(0) + \alpha(\phi(0) - \phi(-1))$, respectively. One obtains

423,31 (line 2 from bottom) Read:

$$\dot{z}_2 = -2(|z_1|z_1 + \alpha z_2) + o\left((|z_1| + |z_2| + |\alpha|)^2\right)$$

424,4 Read:

$$\dot{u}_2 = -2|u_1|u_1 - \frac{4}{3}|u_1|u_2 - 2\alpha u_2$$

425,31 (bottom) Read:

$$x_s^{(\phi)} = x_{t+s}^{(\chi)} = F(-r, x_{t+s+r}^{(\chi)}) \in W.$$

427,7 Read: $\dots(-\infty, -7] \dots$

428,7 Read:

$$F(s, x_{s_k}^{(\psi_1)}) - F(s, x_{s_k}^{(\psi_2)}) = x_{s+s_k}^{(\psi_1)} - x_{s+s_k}^{(\psi_2)}$$

428,15 Read: given. By Corollary 6.1,

428,16 Read: $\dots x_{-5}^{(\phi_j)} \dots$

428,20 Read: $\dots F(5, \psi_1) - F(5, \psi_2) \dots$

429,20-22 Read (text of Corollary 6.5):

$\mathbb{R}$, contains a zero of $x^{(\phi)}$. In case $0 \neq \phi \in W$ the zeros of $x^{(\phi)}$ form a strictly increasing sequence $(z_j(\phi))_0^\infty$. In case $0 \neq \phi \in \overline{W} \setminus W$ the zeros of $x^{(\phi)}$ form a strictly increasing sequence $(z_j(\phi))_{-\infty}^\infty$. In both cases,

429,24 Read: $\dots$ has no zero in

429,27 Read: segment Px_s is $\dots$

429,29 Read: $\dots$ in $N_{cu} \setminus \{0\}$ must leave N_{cu} .

429,31 Read: $\dots$ zero $z_0 = z_0(\phi)$ of $x = x^{(\phi)}$. Proposi-

429,32 Read: $\dots$ for $z_0 < t \leq z_0 +$

430,6 Read: $\dots$ for all $t \leq t_0$. As $\dots$

430,7 Read: $\dots$ we have that Px_t avoids $\dots$

430,16 Read: $\dots$ flowlines $\mathbb{R} \ni t \mapsto Px_t^{(\phi)} \in L$ wind $\dots$

430,24–25 Read (text of Corollary 6.6):

$$Px_t \in (0, \infty)\phi_0 + (0, \infty)\phi_- \quad \text{for } z_j + 1 < t < z_{j+1},$$

$$Px_{z_{j+1}} \in (0, \infty)\phi_0.$$

431,2 Read:

$$(\overline{W} \setminus \{0\}) \cap H_0 \ni \phi \mapsto \zeta_2(\phi) \in \mathbb{R}.$$

431,29 (line 2 from bottom) Read: Notice that $Pp_1 \notin PW$. Our ...

432,2 Read: ... the arc $\{Pp_t : 0 < t < 1\}$, and ...

432,28 (line 3 from bottom) Read: Let $\epsilon_* = \epsilon(\rho_*)$. Using $Px_1 = Px_0^* = \rho_*\phi_-$ and continuity we obtain that for

433,17 Read:

$$F(s, x_0) = F(s, F(s' - s, p_0)) = F(s, p_{s' - s}).$$

434,6 Read: ... We have $P\mathcal{O} \cap (0, \infty)\phi_0 =$

435,6 Read: ... The set $\{Px_t : t < z_0\} \cup [r, 0]\phi_0$ is the

435,19 Read: ... so that (7.1) holds. We have

438,9 Read: ... solutions $x : \mathbb{R} \rightarrow \mathbb{R}$ the spiraling motion