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Mathematics teacher educator knowledge: What do we know and where to from here?

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Abstract
The knowledge that mathematics teacher educators need has attracted limited but increasing attention in recent years. The papers in this special issue build on emerging themes from that work and raise additional questions that contribute towards a future research agenda in the field. Several of the articles develop conceptualisations of teacher knowledge to apply to mathematics teacher educators, introducing new aspects and drawing attention to unique characteristics of mathematics teacher educators in the process. To some extent the questions these papers raise reflect similar questions in mathematics education more generally but for which the implications for mathematics teacher educators warrant attention. In this paper we review the articles in this special issue and use them as a springboard to examine contemporary developments in the field and posit ways forward for research on mathematics teacher educators' knowledge.

Keyword Mathematics teacher educators · Mathematics teacher educator knowledge · Mathematics teacher educator learning

Introduction

Mathematics teacher educator (MTE) knowledge remains an under-researched area but one of increasing importance in the context of growing emphases in many countries on teacher quality (Dinham 2013) driven, at least in part, by a desire to improve performance in international testing programmes. Governments have turned to regulatory approaches to enhancing teacher quality that have led to increasing accountability requirements for teachers, schools, and initial teacher education programmes (e.g. Australian Institute for Teaching and School Leadership 2015; National Council for the Accreditation of Teacher Education 2008). Just as understanding and being able to articulate the unique knowledge of teachers, and mathematics teachers in particular, was important in previous decades, it

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behoves us now to focus upon just what it is that MTEs need to know, and in what ways this knowledge is distinct from that of teachers, and other teacher educators including mathematicians. It is also timely to consider the ways in which MTEs become knowledgeable and to develop research-based models and processes that move the development of expertise beyond incidental on-the-job acquisition that has prevailed in many contexts.

As a preliminary consideration we note that MTEs are considered in this special issue to be anyone engaged in the education or development of teachers of mathematics. Papers in this issue thus focus on research mathematicians (Leikin, Zazkis and Meller), and mathematics education researchers (Chen, Lin and Yang; Chick and Beswick; Masingila, Olanoff and Kimani; Zazkis and Mamolo) in their roles as MTEs. All but Masingila et al. concern MTE's work with secondary teachers, and all but Chen et al. consider their work with prospective teachers. In the following sections we examine a range of threads in current research on mathematics teacher educators' knowledge. In each case we use contributions to this special issue as a starting point from which we make connections to other work in the area. We conclude each section by positing questions and directions for future research in the area.

The nature of mathematics teacher educators' knowledge

MTE knowledge has been thought of as a kind of meta-knowledge that includes at least some of the knowledge that mathematics teachers require (Beswick and Chapman 2012). Just as teachers of mathematics need to know more than the mathematics they teach, MTEs need to know more than the knowledge that teachers require to help school students learn mathematics. An analogous argument can be made that, just as teachers of mathematics need a particular kind of knowledge that is different from that required by anyone not engaged in teaching mathematics (e.g. Ball et al. 2008; Shulman 1987), MTEs need a particular form of knowledge about teaching mathematics and they need to hold this knowledge in a way that is rather different from the way that teachers know it. With this in mind it is unsurprising that conceptualisations of MTE knowledge have been influenced by models of school mathematics teachers' knowledge. Some researchers, including authors in this special issue, have drawn upon or adapted particular conceptualisations of teachers' knowledge, or aspects of these conceptualisations, while others have taken a more eclectic approach drawing upon a range of earlier work (e.g. Appova and Taylor 2017; Zazkis and Mamolo, this issue). Various other theoretical constructs that have been developed with school mathematics teachers in mind and applied in such contexts have also been adopted by those considering MTEs [for example, Amador (2016) used Mason's (2011) idea of professional noticing].

In this special issue, Leikin et al. extend an earlier adaptation of Jaworski's (2002) teaching triad (Zaslavsky and Leikin 2004) to incorporate their findings concerning the role in teacher education that mathematicians, as MTEs, perceive for the advanced mathematics that they teach, while Chick and Beswick present an adaptation for MTEs of Chick's (2007) framework for teaching the pedagogical content knowledge (PCK) needed by school teachers to teach mathematics to school students. Two of the papers (Masingila et al.; Zazkis et al.) draw from or build upon aspects of Ball and colleagues' (e.g. Ball et al. 2008) Mathematical Knowledge for Teaching (MKT) in considering the knowledge needed by MTEs. For Masingila et al. MKT is the content that MTEs were aiming to help elementary preservice teachers to learn, whereas for Zazkis et al. it is Ball et al.'s (2008) notion of Horizon Content Knowledge (HCK) that is most relevant to their consideration of the importance of Advanced Mathematical Knowledge (Zazkis and Mamolo 2011) to MTEs' work with prospective teachers. They adapt HCK in their construct of Knowledge at the Mathematical Horizon (KMH) and argue that it is KMH that enables MTEs to respond appropriately to contingent (Rowlands et al.) moments in order to exercise the level of awareness (Mason 1998) required to assist prospective teachers to develop their mathematical knowledge and awareness. The notion of contingency also resonated with other authors in this issue (i.e. Chick and Beswick; Zazkis et al.). Whereas Masingila et al. use Zaslavsky's (2008) framework of task-based knowledge, Chen et al. adapt that framework in their consideration of the learning of a novice MTE engaged in facilitating the professional development of in-service mathematics teachers.

In Leikin et al.'s extended teaching triad model, Challenging Content for Mathematics Teachers comprises Mathematical Challenge for Teachers (MCT) in addition to Didactical Challenge for Teachers (DCT) [called Challenging Content for Mathematics Teachers by Zaslavsky and Leikin (2004)]. Whereas the original MTE triad (Zaslavsky and Leikin 2004) was based on the views of MTEs who were not research mathematicians, the addition of MCT in the extended triad reflects the mathematicians' view that teachers need to know considerably more mathematics than that included in the curriculum that they teach, and that the study of advanced mathematics allows teachers to develop an appreciation of the history and beauty of mathematics as well as deeper and more complete understandings of mathematical concepts.

The extended MTE triad (Leikin et al.) thus provides a conceptualisation of MTE knowledge grounded in earlier work on mathematics teachers' knowledge and able to accommodate the differing perspectives of at least two categories of MTE: mathematics education researchers and research mathematicians. That the views of the different MTE groups differ is illustrative of Li and Superfine's (2016) observation that MTEs' use of instructional strategies, curriculum materials, and content is mediated by their mathematics knowledge and beliefs about teaching and learning. Despite differences, as Leikin et al. point out, there is also considerable commonality between the views of mathematicians, mathematics education researchers, and school mathematics teachers regarding the role of advanced mathematics content in teaching with all sharing an emphasis on the development of what Leikin et al. describe as meta-mathematical ideas (e.g. proof, beauty). Nevertheless, Leikin et al. were motivated by a belief, shared by Zazkis and Mamolo, that advanced mathematics learned at university should have a stronger impact on school mathematics teaching than is currently the case. In addition to the need for research to support or counter that view, it and the mediating effect of MTE content knowledge acknowledged be Li and Superfine (2016), raise the question of the extent and nature of the mathematics knowledge that MTEs require.

A move towards considering affective constructs in relation to MTEs is also emerging in the literature. Forays into this territory have included MTEs' goals (Li and Superfine 2016) and purposes (Appova and Taylor 2017) in designing mathematics content courses for preservice teachers. Two of the four goals of MTEs identified by Li and Superfine (2016) include affective aspects. These concerned MTEs' desires to provide preservice elementary teachers with positive experiences of learning mathematics, and to provide a safe learning environment. The data used to illustrate these goals included concern about the potential negative impact of criticism on preservice teachers' engagement with mathematical problem solving, and the importance of overcoming any embarrassment at making mathematical mistakes since these provide an important vehicle for learning. Implicit in these and other data provided by Li and Superfine (2016) are beliefs held by the MTEs about such things as the ways in which mathematics is best learned by preservice teachers, the likely affective reactions of preservice teachers, and the importance of pedagogical modelling as a strategy for teaching preservice teachers how to teach mathematics. They also explicitly referred to the challenge that MTEs reported in influencing their preservice teachers to believe that learning to teach mathematics for conceptual understanding was a worthwhile endeavour. Appova and Taylor (2017) similarly acknowledged that the MTEs in their study had "personal and professional intentions and purposes" (p. 20) that informed their design of courses for preservice teachers. Other authors who have considered the beliefs of MTEs include Aydin et al. (2009), Callingham et al. (2012) and Lovin et al. (2004).

In this issue, Chick and Beswick present an amended version of Chick's (2007) framework for teaching the pedagogical content knowledge (PCK) needed by school teachers to teach mathematics to school students and present a parallel framework for describing the PCK needed by MTEs to teach PCK to teachers—a kind of meta-PCK. Unlike Chick (2007) these frameworks include assessment approaches and, of most relevance to this discussion, three elements related to affect. Specifically, they conceptualise the PCK of MTEs as including Student (i.e. teachers for MTEs) affect in relation to mathematics, the MTEs' beliefs about the nature of the content they are teaching (i.e. PCK for teaching mathematics), and Student (i.e. teacher) affect in general. Chick and Beswick, thus, address MTE's knowledge of teachers' affective states and responses, and their own beliefs about the nature of PCK for teaching mathematics (i.e. SMTPCK). Chick and Beswick's conceptualisation of MTEs' PCK does not extend to MTEs' beliefs about the nature of mathematics (the content that school mathematics teachers teach) or of school student affect relevant to learning mathematics. Of course, MTEs' beliefs about the nature of SMPCK may or may not include a belief that SMTPCK encompasses beliefs about the nature of mathematics and knowledge of school student affect. The conceptualisation of SMPCK that they include incidentally includes both of these aspects.

In Jaworski's (2002) teaching triad *Sensitivity to students* included both cognitive and affective sensitivity with the latter referring to the provision of encouragement and support based on awareness of students' confidence and concern for their wellbeing. In this issue, Leikin et al. reported little evidence of either type of *Sensitivity to mathematics teachers* among the four mathematicians (who were MTEs) that they interviewed. What evidence there was related to recognition of the possibility of providing additional scaffolding for prospective teachers by proving more, and more interesting, examples, and attending to aesthetic qualities of mathematics if they were to teach a course designed especially for prospective teachers. While these ideas are relevant to prospective teachers' affect, they do not address it explicitly.

Given the close connection between studies of MTEs' knowledge and earlier work on the knowledge of school teachers it would be reasonable to expect that lessons learned in relation to teachers, including the important role of affect and its relationship with knowledge, might expedite analogous research concerning MTEs. At this stage there is evidence that at least some MTEs are not aware or convinced of the need to know about teachers' affective characteristics. In addition, we argue that research on MTEs' knowledge should extend to their own beliefs about the nature of the content that they are teaching to teachers. There is evidence in the extant literature (Aydin et al. 2009; Callingham et al. 2012; Lovin et al. 2004) and in this special issue (Chick and Beswick) that this is beginning to happen. In so doing, MTE researchers would be wise to avoid the conceptual muddle that has attended teacher beliefs research (Furinghetti and Pehkonen 2002) by being careful to define the constructs that they use and to distinguish them from one another.

Similarly, considerations of the mathematical content knowledge needed by MTEs and the role that it plays in their work with mathematics teachers could be informed by similar considerations regarding the mathematical content knowledge needed by teachers. Zazkis and Mamolo and Leikin et al. argue for the importance of MTEs having strong mathematical content knowledge in the form knowledge of advanced mathematics taught at universities. In both cases they considered prospective secondary mathematics teachers. Zazkis and Mamolo argue that advanced mathematical knowledge of the sort learned at university is essential to KMH. Beyond contributing to greater appreciation of the aesthetics of mathematics and other meta-mathematical issues, knowledge of advanced mathematical content allows an MTE to situate particular mathematics within a broader context, to better appreciate pertinent aspects of the knowledge at hand, and to more effectively respond to contingencies in working with mathematics teachers flexibly with a view to their future mathematical learning (Zazkis and Mamolo). While much remains to be understood about these phenomena, the question of the nature and extent of mathematics knowledge required by MTEs working with prospective or in-service elementary teachers has received even less attention. For example, in what ways might it be problematic that these MTEs may not have studied any university mathematics content (Callingham et al. 2012)?

The acquisition/development of mathematics teacher educators' knowledge

How mathematics teacher educators acquire or develop the necessary professional knowledge to fulfil their role is a question that seems to have received less attention than research on the nature of this knowledge. Three papers in this issue shed light on how MTEs learn and develop their knowledge. Zazkis and Mamolo present a story of an experienced MTE that is an amalgamation of their own experience as MTEs. They examine how disturbances to the hypothetical MTE's personal mathematical knowledge influenced her teaching of prospective mathematics teachers. Chen et al. analyse what was learned by a novice MTE through the design and delivery of a professional development course for practising teachers. Masingila et al. describe how two novice MTEs worked with a more experienced colleague and reflected on the process of learning to teach via problem solving in a preservice teacher education course. In addition, although Chick and Beswick focused on the nature of knowledge required by MTEs, the illustration from the second author's teaching indicate that she also learned from examining her practice.

Together these papers illustrate possible areas of focus for examining research on mathematics teacher educators' learning. One focus relates to the *career stage* of the MTE, with two papers involving novice MTEs and two concerning experienced MTEs. The paper by Masingila et al. combines these two possibilities by asking how novices can learn with, and from, an experienced colleague. A second focus encompasses the *processes through which MTE knowledge is developed*. A common approach here is to conceive of MTE learning as reflective practice. For example, Tzur (2001) and Krainer (2008) have provided reflective self-studies of their own developmental trajectories, tracing their experiences as mathematics learners, mathematics teachers, mathematics teacher educators, and mentors of fellow mathematics teacher educators to identify critical events and experiences that advanced their professional knowledge and practice. Similarly, Zaslavsky and Leikin (2004) have presented a three-layered hierarchical model of learning, where each successive layer contains the knowledge of mathematics learners, mathematics teachers, and mathematics teacher educators, respectively. A recursive relationship exists between the layers as each form of knowledge operates and reflects on knowledge in the layer beneath. There is also space for a fourth layer representing the knowledge of educators of MTEs. In these recursive models of mathematics teacher and MTE development, learning through teaching the level below (e.g. MTEs teaching teachers) is facilitated by mentoring from the layer above (e.g. MTE educators mentoring novice MTEs).

In this special issue, the paper by Masingila et al. illustrates recursive relationships in developing MTE knowledge, with the two novice MTEs reflecting on their practice in teaching prospective teachers while being mentored by the more experienced MTE (Masingila). A similar recursive and reflective flavour is evident in the paper by Zazkis and Mamolo through their use of Mason's (1998) notion of levels of awareness. The first level, awareness-in-action, refers to a teacher's ability to act in the moment, responding instinctively to lesson events but without being able to explain his/her choices. At the second level, a teacher displays awareness-in-discipline through awareness of awareness-in-action: such teachers can articulate and justify their pedagogical choices. A mathematics teacher educator, on the other hand, needs to develop the third level of awareness, labelled by Mason as awareness-in-counsel—a sensitivity to how one might develop prospective or practising teachers' awareness-in-discipline.

Recursive models of learning through practice blend the individual and the social by incorporating reflection on one's own actions and interactions with other people. However, an alternative model that brings these processes into sharper contrast is suggested by Krainer's (2001) framework that identifies four dimensions of teachers' professional practice: action, reflection, autonomy and networking. While each of these dimensions is important, he explains that it is necessary to achieve a balance between action and reflection, and also between autonomy and networking. Such a framework, if adapted to the practice of MTEs, might prove useful in stimulating and organising research into how MTEs learn through practice, not only by directing attention to reflection-on-action methodologies but also by prompting researchers to consider how networking with colleagues can promote this reflection (as in the paper in this issue by Masingila et al.).

In addition to learning through reflecting on their own practice, MTEs are in a position to learn from their research with teachers, even though this learning is often left unacknowledged and unarticulated (Jaworski 2001). Chapman (2008) suggested that an explicit goal of MTEs' research of their practice should be self-understanding and professional development—a goal that is made explicit in the paper by Chen et al. in this special issue. Chen et al. highlighted an important distinction between MTEs who are working as school teachers but who contribute to teaching preservice or professional development courses, and MTEs who are university academics with additional research responsibilities. They claim that the latter MTEs are more likely to invoke theory when designing teacher education courses and also when reflecting on their own knowledge development through practice. They drew on Engeström's (1987) Activity Theory to analyse the professional development course as interacting systems, one involving the activity of the mathematics teacher educator-as-researcher (MTE-R) with the aim of carrying out academic research, and the other involving the activity of mathematics teachers with teaching practice as their object. Analysing the tensions between these two systems facilitated development of the MTE-R's knowledge as he designed tasks, implemented them with the mathematics teachers participating in the professional development programme, and modified the tasks based on his observations of the teachers' engagement. Thus, while this study, like those of Zazkis and Mamolo and Masingila et al., documented reflective practice as a learning process for MTEs, it was explicitly concerned with learning through research with teachers.

A third area of focus for examining research on mathematics teacher educators' learning concerns the supporting *theoretical perspectives* that researchers draw on to understand the development of MTEs' knowledge. These perspectives are additional to emerging frameworks specific to MTE learning, such as the recursive models described earlier. Activity Theory has already been mentioned as one such perspective; others include the Community of Practice approach taken by Masingila et al. in this special issue. The latter approach seems particularly fruitful for elaborating on Krainer's (2001) model of teacher professional practice to develop a model of MTE practice with the interacting dimensions of reflection-action and autonomy-networking. Drawing on Wenger's (1998) defining features of a community of practice, the processes of *mutual engagement* in such a community might illuminate key features of effective networking between novice and experienced MTEs; the *joint enterprise* of the community might focus on reflection on MTE practice; and the participants might develop a *shared repertoire* of language and resources for making meaning of their work as MTEs.

Issues in researching mathematics teacher educators' knowledge

Studies of MTEs' knowledge have, to date, been conducted by MTEs and have involved various forms of self-study. They have used action research, and narrative and autobiographical methods, and have drawn upon the reflections of the MTEs concerned (Schuck 2002). Where studies have involved other MTEs, the research subjects tend to be more junior MTEs (e.g. Speer et al. 2005). Together these conditions of MTE research present at least two major sorts of risks. First, questions can be raised about the validity or trustworthiness of findings and, second, there is a danger that important questions will not be asked.

In relation to the first of these, Schuck (2002) emphasised the importance of involving a critical friend as a means of militating against the danger of self-study becoming solipsistic, and driven by questions "whose answers will confirm existing beliefs" (p. 335). In addition to improving one's own teaching, Schuck (2002) argued the importance of having a second purpose; that of advancing knowledge about teacher education more broadly. Mueller (2003) attempted to overcome these dangers by having a graduate student interview her, and to probe for additional explanation. In this issue Masingila et al. explain their use of a community of practice in which participants reflect together as well as individually on their practice. By working with others in this way, individuals are afforded the benefit of insights from others that can inform their own reflections and can be pointed to pertinent aspects of their practice for consideration. Similarly, the novice MTE described by Chen et al. was supported by an experienced MTE whose input could inform his reflections. In these cases, novice MTEs benefited from the guidance of an experienced colleague acting as a mentor and the research was also strengthened by the inclusion of an external perspective. Although studies of this kind provide more robust findings than self-study without a critical friend or community, they raise the question of the impact of the difference in status of the research subjects (novice MTEs) and their mentors (experienced or expert MTEs) or of the interviewer and interviewee in the case of Mueller (2003). The question of how we can guard against power relations influencing findings of research about MTEs remains open. Even in contexts where the status of those involved might be considered essentially equal (e.g. Chick and Beswick) the dangers remain where the collaborators have a personal/collaborative relationship that they may be unwilling to risk by raising uncomfortable questions or offering frank critique, and where they are immersed in the same institutional culture that can blind both parties to questions that might occur to an outsider.

The discussion above illustrates how the second risk, that of failing to ask important questions, is related to the first as it arises from MTEs researching themselves. The danger can be reduced by broadening sources of input which can happen at range of levels from involving a critical friend, to working with groups of colleagues in the same context, to collaborating across institutions and perhaps across countries. Nevertheless, if all of those involved are MTEs it can still be argued that an external perspective is missing. A possible exception might be for MTEs from different backgrounds and working in differing disciplinary contexts, such as research mathematicians and mathematics education researchers. In this issue Leikin et al., as mathematics education researchers, were interested in the views of their MTE colleagues, research mathematicians. The research mathematician MTEs were participants in the research but not collaborators. There are, in fact, few examples of these groups of MTEs collaborating to research mathematics teacher education, much less MTEs. The challenges involved in forging fruitful collaborations between these groups have been documented (Goos and Bennison 2018), but it seems that the differing disciplinary cultures that underlie the struggle to arrive at shared understandings are also the drivers of productive research partnerships. That of Bass and Ball (e.g. Ball and Bass 2002) is a well-known example of a productive collaboration. Building on what we have learned about establishing collaboration between mathematics educators and research mathematicians to consider the knowledge required of MTEs seems worthwhile. The fact that many prospective teachers work with MTEs from differing disciplines gives rise to questions about the kinds of knowledge the different groups of MTEs require and what might be the optimal nature and balance of their knowledge and their involvement in mathematics teacher education.

There is no obvious answer to the question of who, other than MTEs, should research MTEs. It might be possible, however, to involve others who are not MTEs in this research. It is worth considering the value that, for example, mathematics teachers, teacher educators other than MTEs, or educational administrators might add to the endeavour, and what questions they would they ask.

New areas for research

Research into the knowledge of MTEs is beginning to consider finer grained issues and contexts in a way that is analogous to other areas of mathematics education that have shifted progressively from broad concepts to more nuanced issues. Hill (2007), for example, provide a useful account of this progression in relation to the knowledge of school mathematics teachers. While there is more to do in relation to conceptualising the nature of MTE knowledge, the field is at a stage where more specific aspects of knowledge can be considered. The work of Li and Superfine (2016) on MTEs goals is an example of this move. In this special issue, particular aspects of MTE knowledge, although still quite broad, were the focus of three of the papers: Leikin et al. and Zazkis and Mamolo were concerned with MTEs mathematical knowledge, whereas Chick and Beswick considered their pedagogical content knowledge (MTEPCK). Similarly, in relation to how MTEs learn, Chen et al. considered how an MTE's knowledge of how to use his research knowledge to assist teachers to learn about task design developed, while Masingila et al. focussed on the development of MTEs' capacity to use a problem-solving approach to teaching mathematics to their prospective teachers. There is scope for increasingly fine-grained research on the specific knowledge that MTEs bring to bear on the various facets of their work.

Although some aspects of affect, principally beliefs, have begun to feature in work on MTE knowledge (e.g. Chick and Beswick, Leikin et al. in this issue) its role has been largely incidental. Understanding the beliefs that underpin the practice of MTEs must be at least as important as understanding those of that influence the work of mathematics teachers. There is evidence from work with school teachers that beliefs about the nature of mathematics, mathematics teaching and learning, the capacity of students to learn mathematics, and how students can be engaged in mathematics are particularly important to their practice (Beswick 2018). What categories of beliefs might be most salient for MTEs? Furthermore, what do MTEs need to know about teachers' and prospective teachers' affect? Although considerable research has been conducted on affect and elementary teachers [mainly on mathematics anxiety (Pekrun et al. 2002)] we know relatively little, beyond reports of particular interventions, about how MTEs' knowledge of their students' affect informs their teaching. The dearth, in comparison with those of elementary preservice teachers, of studies of prospective secondary teachers' affect could be indicative of MTEs' beliefs about the role of affect teaching at this level. This question, itself, warrants exploration.

Online teaching is a growing phenomenon in higher education including in mathematics teacher education, and research on MTE knowledge in this environment is needed. Kastberg et al. (2014) explored MTE practices in an online environment and particularly their struggles to adopt practices that aligned with their views about learning. Understanding the knowledge that underpins MTE practices in any context is crucial. In particular, it is not clear whether, to what extent, or how the knowledge needed by MTEs teaching in an online environment might be different from that needed in face-to-face contexts. Chick and Beswick illustrated their MTEPCK framework with excerpts from online discussions. This illustrates a particular affordance of the online environment in which discussions are text based and hence available for analysis. Although they acknowledge some pertinent differences between teaching online and face-to-face, including the asynchronous nature of the communication, implicit in Chick and Beswick's paper was that the framework was not specific to MTEs' online teaching. This remains to be tested.

Although more small-scale studies of the kind that have dominated research into MTE knowledge to date are needed, there is also a need to explore ways to broaden input into this work through collaborations across disciplines, institutions and countries. Large-scale studies in mathematics teacher education are relatively recent and have involved collaborations across institutions within a single country (e.g. Beswick and Goos 2012) and internationally [e.g. TEDS-M (Blömeke et al. 2008)]. Involving colleagues from diverse contexts to study MTEs' knowledge would help with avoiding some of the pitfalls described in the preceding section and are worth pursuing even though they come with their own complexities and challenges.

A further challenge yet to receive attention concerns the impact of increasingly regulated teacher education environments on the work of MTEs and hence the knowledge that they need. Hill (2007) and Beswick and Goos (2012) pointed to increasing accountability as reasons to be concerned with the knowledge of teachers and prospective teachers, respectively. With the implementation of processes aimed at ensuring the quality of initial teacher education graduates (e.g. Paulson and Marchant 2012) it seems inevitable that attention will turn eventually to the knowledge of those delivering teacher education, including MTEs. Efforts to ensure that the questions we ask about ourselves are those whose answers will be or interest beyond our own communities seems especially important in this context.

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