Teachers' self-representations in teaching mathematics

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Like any construct in education, “representation” can mean different things from different theoretical perspectives. In mathematics education, it is generally related to ways of expressing mathematics. In this editorial, influenced by articles in this issue, I address representation in this mathematical context, but my intent is more about offering teacher self-representation as a potential area of research and topic of teacher education.

Mathematical representation and the teacher

Mathematical representation plays important roles in mathematics education. These roles are interrelated and can be categorized as a way of thinking (i.e., making sense through what is represented), a way of recording (i.e., documenting one's thinking through representations), and a mode of communication (e.g., as an explanation tool). Representation allows students to think and communicate mathematically; “to model and interpret physical, social, and mathematical phenomena” (National Council of Teachers of Mathematics [NCTM] 2000); and to explore and interpret meanings of mathematical concepts, relationships, and procedures. Thus, representation also provides a basis of assessment of students' thinking.

As Vergnaud (1998) noted, we all experience representation as a stream of internal images, gestures, and words. More specifically, Bruner (1966) classified representation in terms of three sequential developmental stages of supporting learning: enactive representation, iconic representation, and symbolic representation. However, various types of representations are used in mathematics education, which can be classified as symbolic (mathematical); verbal (written or oral); pictorial (including graphical); concrete (physical); and real-world situations. Unlike Bruner's view of sequential stages, which tends to apply to preschool or primary/ elementary school children, the interrelationships among these types of representations in teaching and learning mathematics are of general importance. For example, having students explore multiple modes of representations of an

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algebraic function in an interrelated way is important for their development of conceptual understanding of the concept and mathematical thinking in general.

The articles in this issue of JMTE, directly or indirectly, deal with some form or aspect of representation with a focus on the teacher, in particular, the teacher's thinking or actions. Two of them relate to representation as described above. Despina Stylianou explicitly addressed mathematical representation. She examined middle-school mathematics teachers' conceptions of representation as a process in doing mathematics and their perspectives on the role of representations in the teaching and learning of mathematics. It is within expectation that the participants held narrow conceptions of representation that limited its use in instruction. However, this work implies it is important for teacher education to address possible limitations in how teachers think about and use mathematical representations.

Levenson, Tsamir, and Tirosh examined elementary school teachers' preferences for mathematically based and practically based explanations. Although their intent was not to address mathematical representation, the work has an implicit focus on verbal representations in the context of even and odd numbers. The study explored the types of explanations the teachers generated on their own as well as the types of explanations they preferred after reviewing various explanations of these concepts. While the teachers generated explanations that included more mathematically based than practically based explanations, more of them chose to use mostly the latter in their classrooms.

In the context of representation, these teachers' thinking for this choice can be related in part to their view of modes of representation embodied in the explanations and the relationship to the developmental stage of the students. For example, they viewed the practically based explanations as a form of concrete, tangible, or visual representation and thus more meaningful, convincing, and simpler for the elementary students. This view is less about Bruner's (1966) perspective of taking a student from concrete to pictorial to symbolic in learning a concept and more of a Piagetian (Piaget 1950) perspective in terms of age-dependent stages of cognitive development with younger students being at a concrete stage and older students (in particular high school students) being more symbolic or formal operational thinkers. As one teacher in the study noted, which one (mathematical or practical) is used depends on the grade and the mathematics concept involved. Thus, the implication of this study by Levenson, Tsamir, and Tirosh for teacher education is not simply about making teachers aware of or choosing to use both of these types of explanation. There are other complicating factors such as limitation in their conceptions of representations, as the Stylianou's article suggests. Consideration should be given to their understanding of such explanations as a form of representation. Another concern is how the teachers' preferred explanations shape their instructional approach (or vice versa) and consequently, support or restrict students' learning of the concept. The issue is not only what they know or choose, but how they engage the students with the explanations or how they use their knowledge to influence students' explanations.

The other two articles in this issue of JMTE offer different ways of thinking about representations in relation to the mathematics teachers and their education or teaching that do not involve representing mathematics concepts. For example, in the article by Tatto, Lerman, and Novotna, representation can be considered as the structures, approaches and general characteristics of the mathematics preparation, and development of teachers in over 20 country-regions. In the case of the article by Andrew Tyminski, on the surface it may not seem to be about representation at all. This article addresses the construct of teacher lust and identifies two working aspects of it: (i) enacted teacher lust, that is, an observable action that may remove an opportunity for students to think about or engage in

mathematics for themselves, and (ii) experienced teacher lust, that is, an internal impulse to act in the manner described. However, it is the empirical examples of each of these aspects of teacher lust that inspired me to think about representation in the teaching of mathematics in a different and humanistic way, that is, not in relation to the mathematics but in relation to the teacher and his or her mathematical self that gets represented during his or her teaching.

Mathematics teachers' self-representation

Self-representation can be interpreted in relation to different contexts (e.g., Levine 2003). Mathematics teachers' self-representation is considered here in terms of the selves teachers intentionally represent in their teaching of mathematics, in particular, their *mathematical self-representation* and *personal self-representation*. These representations, then, are from the perspective of the teachers and are conveyed to students through language and actions.

Mathematical self-representation relates to the mathematical self the teacher perceives or believes to be relevant or necessary to support students' learning of mathematics. The mathematical self includes the teacher's mathematics beliefs, mathematics knowledge, mathematics knowledge for teaching, and knowledge of mathematics instruction and learners. The teacher could hold this knowledge in ways that allow her or him to draw on what seems most appropriate for a particular learning context, such as the ability or grade level of students. Thus, the teacher represents a self that he or she perceives will serve the mathematical needs of his or her students. For example, in the article by Levenson, Tsamir and Tirosh, the teachers held knowledge of both mathematically based and practically based explanations for even and odd numbers. When given a choice between them, there was no consensus among the teachers of a personal preference for one type of explanation over another. However, the self they chose to represent to students was predominantly practically based, even in cases where they were personally convinced by the mathematically based explanations. This aspect of their practically oriented mathematical self resonated with the grade level of the students in terms of providing them with those explanations that would be easier to grasp. In my work with practicing high school mathematics teachers, I have found this tendency for those who leaned toward a teacher-centered instructional approach to present a simplified version of their mathematical self. As one explained, after engaging in a process of self-reflection on route to changing her practice, "For most of my teaching career, I felt my job was to simplify mathematics. Cover the curriculum in consumable bits that could easily be delivered and tested."

In Andrew Tyminski's article, the examples of teacher lust based on the thinking and practice of teacher educators are also examples of self-representation and the consequences of teachers' commitment to it in relation to their teaching and students' learning. These representations for the participants, Samantha and Elizabeth, were in the form of specific mathematical tasks and interactions that Tyminski identified as involving expounding, explaining, and expressing. For example, Samantha's representation was a "cool thing" (i.e., a task) to allow students to see something she considered "really neat." Thus, she controlled the nature of the task (i.e., a marshmallow problem to develop a closed formula for the sum of the first n square numbers) and consequently the nature of the teaching and learning by paying less attention to how students engaged in learning the mathematics and removing sense-making opportunities for students. Elizabeth represented self in terms of a particular task and mode of interaction to allow students to have agency or control in making sense of dividing by zero. In spite of the tensions and challenges these

representations created for her, she remained committed to them in guiding students toward an understanding of the concept. These examples show the commitment to the mathematical self-representations used even when they may remove opportunities for students to think about or engage in mathematics for themselves.

Personal self-representation, while related to mathematical self-representation, is more about the ways the teacher wishes to be perceived by her or his students. Thus, it is related to teacher identity. It can represent the image a teacher has of him or herself based on his or her interpretation or perception of what students consider as an accepted teacher self for a given classroom situation. Teachers can receive direct or indirect messages from students about the kind of teacher self they (students) may value, or become concerned about the kinds of judgments students can potentially make about their mathematical-selves, which can influence this form of self-representation. They can present particular selves that they think will be successful in a class or allow them to deal with various situations by engaging in them, reproducing them, or resisting them. In general, they can choose to represent a personal self that is consistent or in conflict with how their students potentially may perceive their mathematical-selves by embracing, rejecting, or dismissing the students' perceptions, whether real or imagined.

The articles in this issue of JMTE do not provide evidence of this type of representation, although it may have played a role in the teachers' behaviors. However, I have worked with secondary school teachers in my research who have displayed this form of representation. For example, I have encountered teachers who expressed concern about not appearing knowledgeable to students if they did not provide direct answers to students' mathematical questions. Thus, although they claimed to see the merit of helping students to become autonomous learners, they chose to represent a self that held expert knowledge and agency of mathematics. In contrast, I have encountered teachers who represented self as committed to autonomous learning in spite of initially being challenged directly or indirectly by students about whether their (teachers) behaviors were a result of not knowing the answer or how to respond to their (student) mathematical questions. This occurred when the teacher was trying to adopt a learner-centered approach, was encountering a new group of students who had not previously experienced this approach, or was requested by students to intervene when they were struggling with a problem or wanted validation of their solutions. Some teachers chose to present an empathetic self by repeatedly clarifying their approach until the students saw the value of it and accepted it. In the case of a high school mathematics teacher who was in the process of adopting an inquiry-teaching approach, she presented a reformed self in terms of her changed conceptions of mathematics and learning of mathematics in order to deal with the opposition she encountered from students. She helped her students to understand what it means to learn and do mathematics differently from their previous experiences. She explained,

I am not getting shaken off my mark. But it's a constant dialectic conversation in our class, about what does it mean to be a mathematician. [I say to them], if this is what we believe, then why do we continue to work that [instrumental] way. If it is the way you have learned things is that really serving you in math, you know, writing a bunch of notes over and over every day, doing a bunch of text book questions and then regurgitating them on testing. Do you really know and understand math? So the conversation is ongoing all the time. ... And so through conversation, journaling, reflecting, I am continually trying to keep the conversation about what it means to learn and understand mathematics alive. ... So, yes, there is resistance, and yes I just continue and continue and continue to work on it.

Other teachers chose to present a legalistic self that established the rules students should follow in terms of teacher–student interactions and sometimes reinforced them through behaviors that conflicted with students' expectation. For example, one teacher would tell her students that she will give them incorrect answers if they insisted on asking her if their solutions were correct. The students initially will not take her seriously and instead of finding ways of justifying their answers will eventually request the correct answer from her as a check for theirs. The teacher would follow through with giving them an incorrect answer but not indicating that it was incorrect. This created dilemmas for the students who then closely examined their solutions until finally realizing that they were correct, or why they were not, and that the teacher had given them an incorrect answer. This approach generally worked to get the students to take ownership of justifying their solutions and to acknowledge the self represented by the teacher.

Impact of self-representation

Self-representation can have both positive and negative impacts on students' learning of mathematics depending on what it represents and the message it conveys. The examples discussed above provide evidence of this. The teacher can represent self in an unproductive way for the learner in a variety of situations. Other examples of this are presenting a self that (1) saves students if they seem to be struggling with a problem or not to be getting what the teacher intends by telling them what to do or think; (2) can simplify mathematics for students; and (3) listens to help students focus on the mathematics topic predetermined by the teacher. As a teacher in one of my projects explained, "If a student asked a question that seemed off topic or confusing to me, I would seldom really listen, often dismissing it."

Unproductive self-representations are likely to be connected with teacher-centered pedagogy and situations to foster students' confidence and respect for the teacher's mathematics knowledge. However, whether productive or unproductive, for the teacher, the self-representation is the right thing to do in order to be helpful to students' learning. It takes into consideration the teacher's pedagogical and personal goals or intent which could be liberating or restrictive for the teacher and students. For example, based on the resulting personal self-representations, when teachers adopt a goal to avoid perceived failure instead of to pursue success, attribute their performance to external or uncontrollable forces, use self-handicapping representations, or set inappropriate goals, they are likely to undermine their own teaching and students' learning. Self-representation, then, can help describe the ways a teacher approaches mathematics teaching, apply instructional strategies, monitor his or her performance, and interpret the outcomes of his or her efforts.

Implications for teacher education

Self-representation is a relevant and an important construct in mathematics teaching and teacher education given that it can be productive or unproductive as a tool in facilitating students' learning of mathematics. As such, teacher education could benefit from explicitly addressing it and its impact on teaching. Self-representation as a topic for discussion and exploration would allow teachers to address its nature, to understand its implications in their teaching, and to attend to the alternative messages it conveys. Teachers could also reflect on the kind of selves they invite in their teaching and the kind of selves they dismiss, their personal contexts/experiences that are complexly wound into the selves they

represent, and the beliefs and abilities that interact with their actions and with how they perceive and relate to their environment. This could provide a way to empower teachers to assess and respond to self-representations.

In the context of research, it is worthwhile to investigate: what accounts for the difference in how teachers engage in self-representations; how teachers “see” themselves versus how they represent themselves (e.g., when and why are they the same or not); the role of self-representation in supporting or inhibiting change in practice; how to help teachers to critically engage with self-representation; the impact of teacher’s awareness of self-representation on their teaching; and what counts as acceptable self-representation in teaching mathematics.

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