Editorial note to: Ludwig Flamm, Contributions to Einstein's theory of gravitation

Gary W. Gibbons¹

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In 1916, shortly after Einstein's paper [1] of November 18th of the previous year, there appeared in the *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* two remarkable papers by Karl Schwarzschild. The first, in which he derived the static spherically symmetric so-called exterior vacuum solution [2] was submitted on January 13. In the second (submitted on February 24) Schwarzschild obtains the so-called interior [3] solution for a static spherically symmetric incompressible fluid ball according to General Relativity. On 3rd September of the same year Ludwig Flamm submitted the paper under current consideration [4] to the *Zeitschrift für Physik*. Flamm's aim was to render the conclusions of Einstein and Schwarzschild clearer, more transparent and, in particular those relating to photon orbits, more accurate.

Flamm starts in Section 1 by recalling Schwarzschild's observation that the spatial sections of the interior solution have the geometry of a portion of a round three sphere and in his Fig. 1, he illustrates this by means of a sketch of any planar section, e.g. for $\theta = \frac{\pi}{2}$. In Section 2 Flamm goes on to consider a planar section of the exterior solution and shows that it, like the sphere, is isometric to a surface of revolution but now the meridional curve is a parabola. Hence, by contrast with the interior case any

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✉ Gary W. Gibbons
gwg1@damtp.cam.ac.uk

¹ Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge, UK

planar section has negative Gauß curvature. The parabola is shown in his Fig. 2 but his Fig. 3, where it is drawn smoothly joined on to a portion of a circle shows, contrary to what some subsequent authors seem to believe (see e.g. [5, 6]), that while the parabola of revolution illustrated in his Fig. 2 has two asymptotically flat sheets or ends Flamm did not contemplate the possibility of what we now call an Einstein–Rosen bridge to another region of spacetime. Indeed it is interesting to note that Flamm’s construction, which is described in Pauli’s magisterial 1921 survey of Relativity [7], is not used, nor is his work referred to, in the famous paper of Einstein and Rosen [8] which can be said to have initiated the modern study of “wormholes”. Neither does Flamm’s work appear to have been cited by the many papers it inspired on “Geometrodynamics” by the Wheeler group in the later 50’s and early 60’s. A common difference between Flamm’s figures and many subsequent illustrations, and models (see e.g. [9–11]) is that the two ends of the paraboloid never become asymptotic to flat planes. This is even true in the illustration attached to Section 23.8 of [12] which reproduces Flamm’s results, with a citation to him!

This latter fact is a feature of the $3 + 1$ spacetime dimensional nature of the solution being considered. The Flamm embedding for the Tangherlini metric [13] is obtained by acting with $SO(n - 1)$ on the curve

$$z = \int \frac{dr a^{\frac{1}{2}(n-2)}}{\sqrt{r^{n-2} - a^{n-2}}} \quad (1)$$

where a is the radius of the horizon. For $n = 4$ the curve is a *catenary* and as for $n = 3$ the separation between the two sheets or “flanges” goes to infinity as r tends to infinity. For $n > 4$ however the separation tends to a constant.

A similar phenomenon occurs for higher dimensional minimal surfaces of revolution. One has

$$z = \int \frac{dr}{\sqrt{\left(\frac{r}{a}\right)^{2(n-2)} - 1}} \quad (2)$$

where a is a constant.

For $n = 3$ we get the standard catenoid obtained by action with $SO(2)$ on the catenary. In this case the separation between the two sheets or “flanges” goes to infinity as r tends to infinity. For $n > 3$ the separation tends to a constant.

Another question one may ask is whether Flamm’s isometric embedding of the planar section into three dimensional Euclidean space is induced from Fronsdal’s isometric embedding [14] of what is now known as the full Kruskal manifold [9] into six-dimensional Minkowski spacetime. Fronsdal was certainly aware of Flamm’s work, which he cites, and although he does not say so directly his wording suggests that he was also aware that the answer is no. In fact, setting $t = 0$ and $\theta = \frac{\pi}{2}$ in his formulae gives an isometric embedding into four-dimensional Euclidean space.

Although not widely appreciated from the 1920’s to the late 1960’s, by the mid 1970’s the images conjured up by Flamm’s work were ubiquitous in introductory and popular texts on neutron stars, black holes, wormholes, etc. However from the evidence above it is doubtful that many of the authors had read Flamm’s essentially elementary paper at first hand but were relying on indirect sources. It is also possible

that artistically inconvenient facts about the geometry were ignored to make their presentations more palatable to their target audience.

Ludwig Flamm (1885–1964)—a brief biography

By Hubert Goenner and Malcolm A.H. MacCallum

Ludwig Flamm was born on 29 January 1885 and died on 4th December 1964. Starting life as the oldest son of a clockmaker in Vienna, he became a distinguished member of the Austrian academic community, and was given an obituary in the Austrian State Archives¹ [15]. The Flammweg in Wien-Simmering is named after him.

He became part of a prominent family in Austrian science. In 1920 he married Elsa (1891–1965), the youngest of the three daughters of Ludwig Boltzmann. Havas quotes Einstein as saying “one can see now clearly who among the new blood of the Viennese physicists is most lovingly concerned with Boltzmann’s works” [16, p. 167]. Their son Dieter (1936–2002), one of four children, was a Professor in Theoretical Physics at Vienna University and their grandson Christoph is a professor at the Institute for Theoretical Chemistry there. Dieter Flamm was at the same time an outstanding extreme climber: his most famous achievement in the mountains is the first ascent (in 1959, together with Walter Philipp) of the north-west face of Punta Tissi in the Civetta group in the Dolomites.

As well as the paper discussed here, and other papers concerned with relativity, Ludwig Flamm worked in many areas and was considered an “allround-physicist” by his colleagues. He did research on radioactivity and combustion processes, as well as in the areas of world geometry, wave mechanics, wave groups and wave packets.

It may have been for this reason that he was nominated in 1937 as the Physics department’s member of the commission to investigate the dispute between colleagues Fillunger and von Terzaghi concerning soil mechanics (see [17], chapter 4). The obituary [15] reports that Flamm contributed to the theory of the settlement of clay strata. Flamm became a leading member of the commission. von Terzaghi later alleged his membership was on Fillunger’s nomination, but in fact von Terzaghi himself had requested Flamm’s membership, which Flamm accepted in a hand-written note. After the commission’s decision in favour of von Terzaghi, Fillunger and his wife sadly committed suicide (For more details see [17]).

During his boyhood, Ludwig was unable to read for a considerable time due to an inflammation of the cornea, so his mother, although busy with four children, read the schoolwork to her son so that finally, after many hard years and a retake of the third grade, he passed his final exam (Matura) with excellent results. After finishing school, he studied physics and mathematics at the University of Vienna. Because his father died in 1905, he had to earn his living by giving private lessons. He was a near contemporary of Erwin Schrödinger and of Hans Thirring (some of whose own papers have appeared in this series) at Vienna, all three of them students of Friedrich Hasenöhl. Flamm’s work, and its relation to Schrödinger’s, is discussed in [18] (In

¹ We have relied on this obituary for much of the following information.

Schrödinger's own contributions to relativity, he refers to "repeated oral discussions" with Flamm [19]).

In 1909 he obtained his doctorate with a dissertation on "Real wave systems", became assistant in the Austrian Geodetical Institute (Gradmessungsbüro) and was sent to the port of Pola [since 1918, Pula in Croatia] for measurements of astronomical distances. The next year he was at the physics laboratory of the Technical University in Vienna; there, together with Heinrich Mache, he engaged in experimental research on radioactivity, which formed the basis for his later independent theoretical research in this field.

During the first world war, Flamm was ordered to measure the burning rate of gunpowder at low barometric pressure. With the simple means available at the time, it was easiest to do this at a greater height above sea level. Thus, Flamm and Hans Thirring joined in an expedition to the Hochschwab, a mountain in the Alps with a peak of 2277 m above sea level. There they observed and documented the speed of the combustion's advance using a stop watch and measuring tape.

In 1917, in *Physikalische Zeitschrift*, Flamm published an important critical study of the characteristic numbers of the proton and electron, and their relationship with the radiation constants. This publication gave the most exact numerical values of fundamental atomic constants at the time and provided physicists with a solid base for the numerical calculation of quantum effects.

In 1916, Flamm had obtained his habilitation in physics (and hence his right to teach in a university) at the University of Vienna, and in the same year he transferred to the Technical University in Vienna, where he remained until his retirement in 1956. He was promoted to associate professor in 1919 and professor in 1922. He was then appointed as director of the II. Physical Institute (*Physikalische Lehrkanzel*), served in 1929–1931 as Dean, and in 1950–1951 as rector of the university. In 1928 he was elected a corresponding member, and in 1940 a full member, of the Austrian Academy of Sciences. In 1960 he became an honorary member of the Austrian Physical Society and in 1963 (jointly with Karl Przibram) he was awarded the Erwin-Schrödinger prize "in particular for his research in the field of theoretical physics". Wolfgang Kummer was his student in the 1950s.

In communicating ideas, his obituary comments that Flamm distinguished himself rather by his precise writings than by a talent for brilliant speeches although he gave numerous lectures.

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