Editorial note to:
Pascual Jordan and Wolfgang Kundt,
Geometrodynamics in the null case.
Exact solutions of the field equations of the general
theory of relativity III

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The editors of Golden Oldies have asked me to comment on “the importance of the Jordan–Kundt paper and its relation to earlier and later developments of the subject.” This I am happy to do as important aspects of the matter are not widely known. This Jordan–Kundt paper treats primarily a special case of a Rainich theory [1,2] which Wheeler [3] called the “Already Unified Theory”. The Rainich theory essentially claims that in a Universe made from only Einstein's gravity and Maxwell's electromagnetism, the gravitational field alone contains all the necessary information from which one can infer the values of the Maxwell field. This gravitational imprint is then argued to be so precise that equations equivalent to the combined Einstein–Maxwell equations can be written entirely in terms of gravity (i.e., the metric field). There is then no need to use or mention the Maxwell field except as an emergent concept that makes descriptions of some aspects of gravity more convenient. Maxwell's equations become theorems derived from the Rainich equations and from the definitions yielding Maxwell fields as extracted from the geometry.

The troublesome case in arguing this equivalence between the Rainich equations and the Einstein-Maxwell equations is the case where the Maxwell fields one wants to extract from their gravitational effects are algebraically null with both quadratic

The republication of the original paper can be found in this issue following the editorial note and online via doi:[10.1007/s10714-013-1659-x](https://doi.org/10.1007/s10714-013-1659-x).

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invariants of the Maxwell field vanishing – a condition easily restated in terms of the Ricci tensor. The Jordan–Kundt paper adds clarity and mathematical rigor to this null case where the Misner and Wheeler [3] presentation had left only the hope “that trivial changes in the statement of the theorem will cover the case of null fields, but this point remains to be investigated.” Jordan and Kundt here carry out such a further investigation for the case where the null condition is satisfied throughout a four dimensional domain. The cases left open by Misner and Wheeler included a concern for the more prevalent practical fields where the null condition only intrudes in subspaces of lower dimension. In the summary paragraph of this paper Jordan and Kundt find that the Already Unified Theory “cannot be formulated as an initial-value problem... on a spacelike hypersurface.” Sometime in late 1959 or early 1960 Roger Penrose, who was in Princeton at the time, informally communicated a similar result (described below) which remained unpublished for decades [4]. Neither Jordan–Kundt nor Penrose considered this failure peculiar to the null case.

This paper is part of an important series of five articles that were, until recently, accessible only in German and are now being re-published in English.

Early developments beyond Rainich began with Hugh Everett III as my recollections below describe. An inquiry through a colleague from Rainich’s daughter about my involvement with this theory was answered as follows:

“Hugh Everett III called to my attention a paragraph in Rainich’s textbook on GR [2] explaining how, in the absence of other fields, some aspects of the Maxwell EM field could be extracted from the spacetime curvature. He (Everett) thought correctly that this would be of interest to Wheeler. I then set to work with Wheeler’s encouragement to see whether the remaining details of the Maxwell EM field could also be extracted from the spacetime curvature. After I solved this problem, Wheeler mentioned it to Bergmann, who directed us to Rainich’s paper containing these same further results which he had not included in his text book. These are cited in our paper “Geometrodynamics” [3], where footnotes cite [1,2] and mention his work prominently in the abstract.

Wheeler called this the ‘already unified theory’, but Roger Penrose later showed that it did not admit standard causality. Two compactly supported bundles of EM waves which don’t overlap initially could evolve to give the identical spacetime metrics (via the Einstein-Maxwell equations) for as long a finite time as it took them to begin overlapping, but then quite different metrics could evolve later, depending on the “complexion” (E vs B duality mixtures) of the Maxwell fields in one initial patch relative to the other.”

Comment by the Golden Oldies Editor: Brief biographies of the authors were published with another Golden Oldie, namely;

Pascual Jordan: *Gen. Relativ. Gravit.* **41**, 2182 (2009),
doi:[10.1007/s10714-009-0868-9](https://doi.org/10.1007/s10714-009-0868-9)

Wolfgang Kundt: *Gen. Relativ. Gravit.* **41**, 2184 (2009),
doi:[10.1007/s10714-009-0868-9](https://doi.org/10.1007/s10714-009-0868-9)

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