

Analytical Approach to Design Vegetative Crib Walls

Madhu Sudan Acharya 

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Abstract Crib walls can be described as a specialized form of gravity retaining structure made by using on-site fill material held within a constructed framework which may be of different materials. A crib retaining structure with live plants between crib layers is called vegetative crib wall. In this article, wooden logs and bamboos are considered as crib elements and live cuttings or rooted plants (vegetation) are laid between crib layers. A number of guidelines and manuals exist for construction of crib walls made of different materials based on practical experience, but there are no proper methods available for the analysis and design of vegetated log crib walls. This paper aims to fill this gap in designing or dimensioning vegetated crib walls. The paper describes the analysis, design and construction procedure of vegetated log crib wall in detail which may be useful for sustainable slope management practice.

Keywords Crib wall analysis and design · Vegetated crib · Bamboo crib

1 History of Vegetative Crib Wall

A crib retaining wall is a structure built up of longitudinal and transverse elements (Fig. 1) to form a series of rectangular cells into which infill is placed; the infill acts as an integral part of the wall. Crib elements may be of concrete, steel or wood but this paper deals mainly with a wooden log and bamboo crib wall (Fig. 2). The crib elements are bound, nailed or stacked together by making certain type of interlocking system to form crib cells or crib framework. Morgan et al. described the crib wall as “a specialized form of gravity-retaining structure using on-site fill material, held within a constructed framework, in order to provide most of the necessary mass to resist overturning by the weight of the slope” (Morgan and Rickson 1995) irrespective of crib or fill materials.

The wooden log crib wall system was originally developed in a place called “Kranj” in north Slovenia. This system of wall is popular under the name “Krainwand” in German speaking countries. This type of wooden log wall has been used in the eastern Alps for many years. In earlier time, dead vegetation, vegetative parts or other materials such as boulders or stones were used between crib layers to prevent the fill material from coming out of the open spaces between crib elements during construction. A disadvantage of this method is its lack of durability since the dead wood materials rots fast. In such construction, when the wooden materials start decaying, the fill materials will start to come out from open spaces

M. S. Acharya (✉)
University of Natural Resources and Life Sciences
(BOKU), Vienna, Austria
e-mail: madhu_sudan.acharya@boku.ac.at

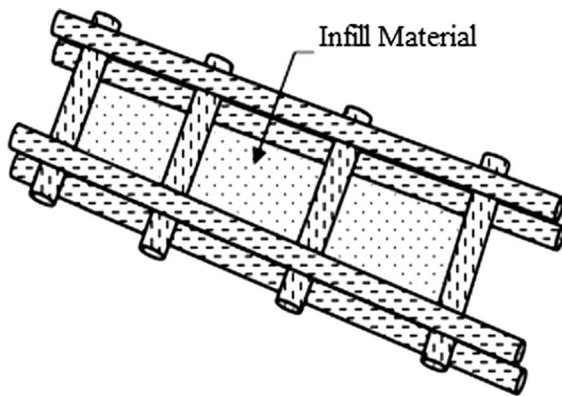


Fig. 1 Typical crib construction



Fig. 2 Wooden log crib wall for noise control in Lower Austria (March 2004)

under the influence of weathering agents (rainfall, sun, wind, snow etc.) and the whole structure will be collapsed. Therefore, Hassenteufel in 1934 used live willow branches between the crib layers instead of using boulders or stones. It is supposed that the growing plants gradually takeover the function of the rotting timber (Schiechtel and Bergmann 1994) through root reinforcement. To overcome the problem of decaying of dead plant materials placed between crib layers and to utilize reinforcing effects of plant roots, nowadays people have started to use live plants in the form of either rooted seedlings or vegetative cuttings of selective plants species. Depending upon the geographical locations, the species of plants used in vegetative crib wall varies widely. However, the commonly used plant species are willows, bamboos, Napier grass (*Pennisetum purpureum*), vetiver grass (*Vetiver zinzaniode*) etc. The crib retaining structure made of crib elements

with live plants inside are called vegetative crib wall (or live crib wall), which is a kind of soil bioengineering wall. In this paper, wooden logs and bamboos are considered as crib element and live cuttings are used as filling support between crib layers (see Figs. 1, 2).

2 Importance and Uses

Plants and plant materials can be used in soil bioengineering system with different techniques. It can be used as a single unit or in combination with other engineering system, either live or dead. There are various techniques in soil bioengineering construction used for slope stabilization which use living plant materials in combination with other dead materials. With the increasing awareness of the people on the ecological environment, there is a growing tendency to use vegetation and vegetative techniques for various engineering purposes. It may take decades of trying of different combinations of techniques and materials to establish an appropriate and sustainable technique for practice. A sustainable and cost effective method of slope stabilization should meet all the site specific requirements in terms of strength, durability, cost effectiveness and environment friendliness. Moreover, for the sustainability, the technology must be accessible and affordable.

As a result of many years of practical experiences, the development of vegetative crib wall system has taken its present state of art construction. In recent years, with the increase in environmental awareness, combinations of the civil engineering structures with soil bioengineering techniques have become a common construction practice in slope stabilization works. The soil bioengineering technique, sometimes also called “biotechnical method of slope stabilization or erosion control” will be more effective than the conventional civil engineering structures if the soil bioengineering system is well designed and properly placed.

The retaining walls made of dry stone, stone masonry, concrete or gabions are very popular in road and other infrastructure construction and also in slope stabilization works in developing countries. But due to the rigidity of the cement masonry structures, they are not suitable in landslide prone area where future

ground movement is obvious. Moreover, the conventional walls may be a costlier solution for developing and least developed countries. Under such circumstances vegetative log crib walls could be “an appropriate technique” and “a low-cost and sustainable alternative” to conventional retaining walls in landslide prone areas where future ground movements are expected. Furthermore, soil bioengineering walls are environment friendly, improve the ecological environmental of the surrounding areas and it may be a sustainable alternative for the management of slopes.

3 Theoretical Aspects

The different soil bioengineering techniques used for erosion control and slope stabilization are generally dimensioned based rather on the practical experiences than an actual analysis and design based on earth pressure theories. Even today, it is not in the practice to design soil bioengineering system based on the earth pressure theories considering the effects of surface and sub-surface water hydrology and effects of plants. In this context this paper highlights the need of analysis and design of soil bioengineering walls and presents a simple procedure for the analysis and design of a vegetated wooden or bamboo crib wall based on conventional crib wall theory and available guidelines and information.

Basically there are four different configurations for crib wall construction (see Fig. 3) used in practice. Depending upon the material used and type of crib elements, crib walls up to a height of 25 meters are possible (Brandl 1987). In this paper, an analysis and design of type 2 crib wall is presented.

Since a crib wall is a specialized form of gravity-retaining structure, as in other gravity walls, external and internal stability analysis are required to ensure the safety of the structure. For the external stability analysis of such walls, the analysis can be performed as for other gravity retaining walls by assuming it a monolithic construction, considering the composite body as a whole (Monolithic theory, Brandl 1987).

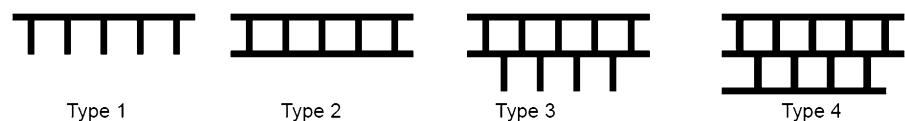
For internal stability analysis, the strength of crib elements, fill material and vegetative cuttings should be considered in the analysis and the conventional silo theory can be applied.

According to monolithic theory, the crib wall is taken as a single composite body like other gravity retaining wall and the loads acting on such walls are taken as external forces. The stability of this wall is then checked against sliding, slip failures and shearing of crib elements and overloading of joints. The safety check against overturning is generally not required because the center of gravity of such walls always falls on the inner half of its width. Different model experiments and observations on concrete cribs walls at the construction sites have shown that the crib retaining walls are not prone to overturning movements but more prone to bulging and tilting or sliding movements parallel to the slope (Brandl 1980). However, the safety check against overturning may be required in some special cases with highly cohesive material and subjected to water pressure and for high walls.

According to the silo theory, the crib wall cannot be taken as monolithic construction like other gravity walls. The wall is taken as a series of silo cells, which are loaded with different external forces (earth pressure, self-weights and other loads) from all sides. To analyze a crib wall under this theory, the crib cells are taken as “silo cells” and their internal stability will be calculated using silo theory. In such cells, the earth pressure will not increase linearly with depth due to the friction between crib elements and soil. However, depending upon the direction of relative motion of the fill material, there will be an active or a passive earth pressure mobilized in crib cells.

There is another theory, called “Frame-work theory” which can also be applied in the analysis of crib structure. In this theory, the crib cells are considered as hinged “rectangular cranks”. But the real behavior of crib wall is some-where between these two theories. For the external stability analysis and design of crib walls in practice, the assumption of monolithic theory will be enough and for internal stability analysis, silo theory is applied.

Fig. 3 Different ways of crib construction (ground plan)



4 Detail Analysis of Crib Construction

For the external stability, all forces (self-weight, earth pressure from the backfill and other loads like water pressure, traffic loads, loads from structures etc.) acting on the wall are taken into consideration and crib wall is considered as a monolithic construction and analysis is done similar to gravity retaining walls. For internal stability of crib construction, silo theory is applied in the analysis. In general, for crib constructions, the following safety checks should be performed for external and internal stabilities (Brandl 1987).

- Sliding and overturning of wall
- The overstress on foundation (bearing capacity failures)
- Sliding and overturning of crib elements
- Overloading of crib elements and shearing of joints

4.1 Sliding and Overturning of Wall

As mentioned above, for the safety check against sliding and overturning, the crib wall is assumed as monolithic construction and the analysis is carried out as follows:

It is assumed that, the crib wall will be subjected to an earth pressure (Fig. 4) from the backfill and the load will be transferred to the ground through the base of the wall. In such case, the coefficient of active earth pressure from the backfill is calculated by using

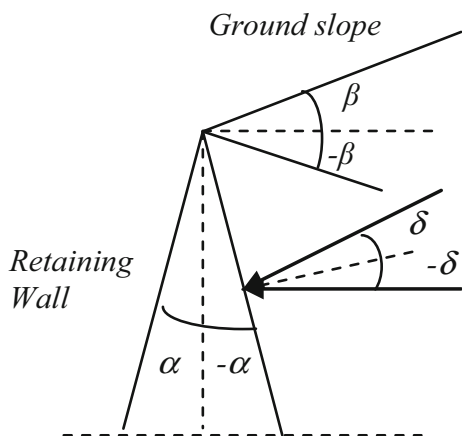


Fig. 4 Illustration of wall, slope and earth pressure resultant inclination angle

the following equation after Coulomb’s earth pressure theory:

$$\lambda_{ah} = \frac{\cos^2(\varphi + \alpha)}{\cos^2 \alpha \left[1 + \sqrt{\frac{\sin(\varphi + \delta) \cdot \sin(\varphi - \beta)}{\cos(\alpha - \delta) \cdot \cos(\alpha + \beta)}} \right]^2}$$

and $\lambda_a = \frac{\lambda_{ah}}{\cos(\alpha - \delta)}$.

Where λ_{ah} is a coefficient of horizontal active earth pressure. Then the resultant earth pressure will be given by $E_{ah} = \frac{1}{2} \cdot \gamma \cdot H^2 \cdot \lambda_{ah}$, which will act at 1/3 of height from the bottom of the wall.

The effects of uniformly distributed surface load (p) can be taken into calculation by superposition with the assumption of fictitious height z' calculated as $z' = \frac{p}{\gamma}$. The reduction of active earth pressure due to cohesion (c) of fill material or existing soils will be given by $\Delta e_{ah} = -2c \cdot \sqrt{\lambda_{ah}}$. The effects of cohesion will only be considered, if it exists for a longer period.

According to the monolithic theory, the external forces acting on the body of crib wall will be transferred to the ground (Fig. 5). The calculation is done by taking the average density (γ_w) of composite structure. If N is the resultant of the total normal force acting at the bottom of the wall, A is the total area of the wall at base, b' the effective width and e, the eccentricity, then the maximum and minimum

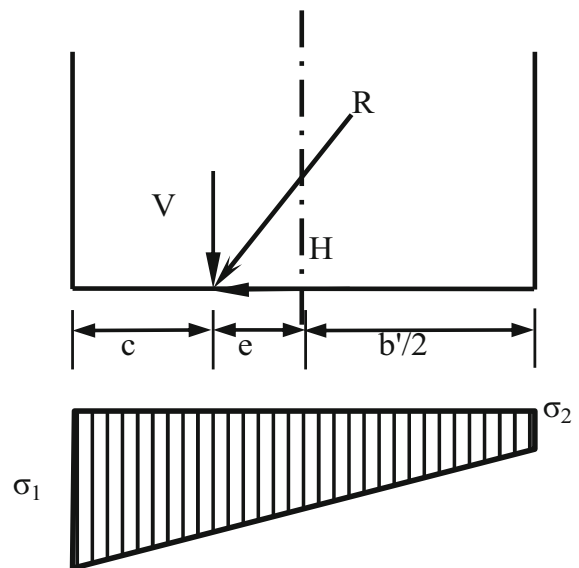


Fig. 5 Schematic diagram of forces and normal pressure for the calculation of safety against overturning and bearing capacity

pressures at wall base are given by $\sigma_{1,2} = \frac{N}{A} \cdot (1 \pm \frac{6e}{b'})$. For displaced horizontal joints situation (lifting up of front sides of header elements), $\sigma_1 = \frac{2N}{c}$ where c is the distance of resultant from outer edge of the wall. Then the safety factor against overturning is given by $F_O = \frac{M_{Passive}}{M_{Active}}$.

The safety factor against overturning for other gravity walls is generally taken as 1.5. As pointed out by Brandl (1984), for crib walls this factor of safety is too high and he suggested a reduced factor of safety of 1.1 will be enough for practice. He further suggested that the permissible eccentricity for crib construction shall not exceed $b'/4$, where b' is the effective base width of crib wall. However, for cemented/concrete crib cells or anchored crib walls, a factor of safety of 1.3–1.5 is suggested.

It is obvious, that the inclined crib walls are more stable than the vertical ones but due to the space problem and practical difficulty in transferring the lateral earth pressure on the ground through wall base, only limited amount of inclination of wall is possible. Vegetative crib wall with wooden elements are generally kept inclined to ease the plant growth. The optimal inclination of a concrete crib wall varies in the range of 10° – 12° i.e. 1H to 5V (Brandl 1980), whereas for wooden crib wall, it varies in the range of 10° – 35° (1:5–1:1.5) from the vertical depending upon the soil type and wall height.

The silo pressure on crib cells in an inclined crib wall will be different than on a straight wall. It varies in two ways:

- With increasing inclination, the sum of frictional forces G_1 (Fig. 6a) acting on the crib cells will decrease to 0 for a horizontal layered crib wall. Test results showed that for inclination range between $0^\circ \leq \alpha \leq 20^\circ$, a reduction coefficient (j) varies from 1 to 0.02 α linearly. Thus for inclined wall, $G_1 = j \cdot G_1$.
- With increasing inclination, the inside crib elements will be stressed more than the outside elements by the frictional forces from the fill material. Then the distribution of frictional forces acting on crib cells (G_1) will be calculated as follows:

The silo pressure on outside crib element:

$$\frac{G_1}{2} \rightarrow k_A \cdot G_1 \quad k_A \leq 0.5$$

The silo pressure on inside crib element:

$$\frac{G_1}{2} \rightarrow k_B \cdot G_1 \quad k_B \geq 0.5$$

In the above equation k_A and k_B are the reduction factors which depend on inclination (α), width (b) and height (z) of crib wall. In general, $k_A \leq 0.5$ and $k_B = \beta(\alpha) \cdot \frac{z}{b} + 0.5$, where $\beta = f(\tan \alpha)$, for $0^\circ \leq \alpha \leq 12^\circ$, β varies between 0 to 0.2 approximately (Brandl 1980).

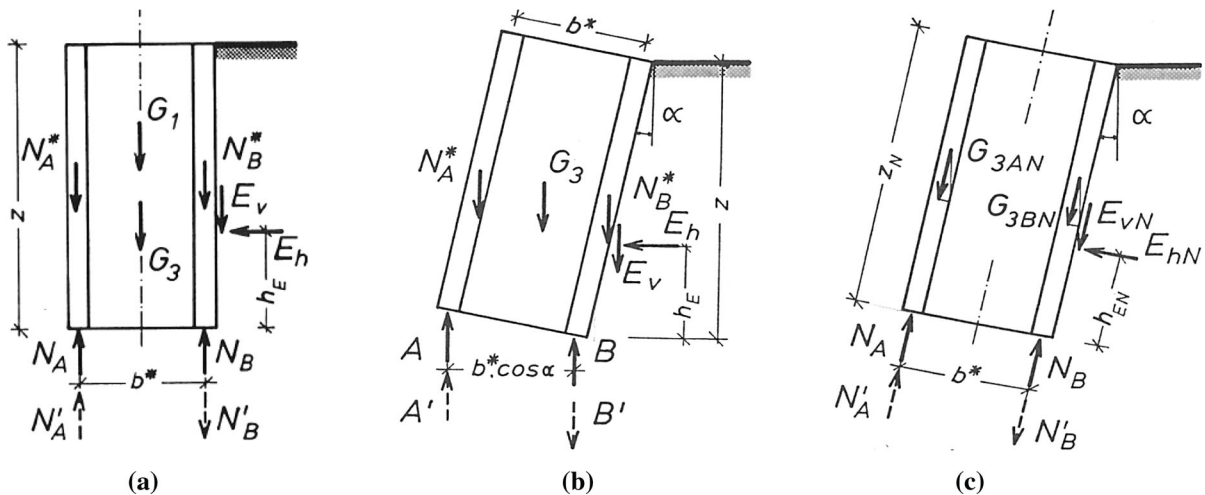


Fig. 6 Illustration of the forces acting on vertical and inclined crib walls made from precast concrete elements (Brandl 1980)

Moreover, from model tests and site observations, it has also been proved that upon loading of crib walls, the stress will be accumulated within the crib elements first and then on the fill material. When the stresses in crib cells increase further, there will be a reduction of forces on joints and edges of outer faces and increase in stress in inner faces joints and edges (Brandl 1980). Therefore, wall inclination angle in case of a wooden crib shall not be more than 35° from vertical.

4.2 Factor of Safety Against Sliding

For the safety against sliding, the resultant sliding force should be less than the resisting force and therefore fulfill the condition

$H \leq V \cdot \tan \delta_1$. Then the factor of safety against sliding will be given by:

$$F_s = \frac{V \tan \delta_1}{H} \geq 1.5$$

where H = Resultant of all horizontal components of forces, V = Resultant of all vertical components of forces, δ_1 = Friction angle between the base of crib wall and foundation, which can be taken as $2/3 \phi$ for concrete crib elements and ϕ for fill material in between (Brandl 1980). For wooden crib elements it varies from 0.3 to 0.75 ϕ depending upon surface roughness of the wooden logs used in crib construction.

4.3 Safety Against Foundation Failure

The safety against foundation failure or overstress on foundation should be checked in the similar way as for other gravity retaining walls according to prevailing norms of practice (e.g. Euro code 7, Part 1, §9). The foundation pressure shall not exceed the allowable limiting pressure specified in the respective norms. A global safety factor of 1.5 will be enough for this safety check.

4.4 Safety Against the Shear Failure

This safety check is for the internal stability of the crib structure. For this safety check, the crib wall is assumed either as one single composite body of fictitious density and fictitious shear strengths (γ_w ,

ϕ_w , c_w) or it is taken as simple construction made of two different materials having different densities and shear strengths properties (properties of fill materials and crib elements are taken separately) and a stability analysis calculation for shear failure can be done using slice or block method of stability analysis.

To check the safety against shear failure, calculations can be done using different software programs (e.g. GEOSLOPE) which uses methods of slices. The crib wall system can be analyzed by using Bishop's, Janbu's or Morgenstern and Price's methods of stability analysis. If the crib wall system is taken as a composite body, the fictitious angle of friction (ϕ_w) for the crib wall will be taken as: $\phi_w = k_w \cdot (\phi_c + \phi)$, where ϕ_c = friction angle between crib elements and ϕ = friction angle of fill material, k_w = system factor, which depends upon friction coefficient of crib elements and degree of compaction of fill. In general k_w varies between 0.3 and 0.5. A factor of 0.5 can be taken for fully compacted backfill with concrete crib elements (Brandal 1980). For wooden log crib wall with full compaction, a factor of 0.4 can be taken in the calculation.

According to Euro code 7, part 1, §9, partial safety factors for soil parameters for the STR and GEO limit states set M2 can be applied to calculate the safety against shear failure of the slope. A global factor of safety of 1.4 shall be used for the safety against shear failure.

4.5 Safety Against the Uplift of Crib Elements or Joint Displacement

In this safety check, the safety of a single crib element against the detachment from the crib system will be checked. Sometimes the earth side of crib wall can have tensile forces in vertical direction, which might cause a lifting up or displacements of joints. In this case it is required to check the strength of joints. In case of inclined walls, the lifting up of crib elements can be happened in two ways:

a. From vertical and horizontal forces (Fig. 6b):

If \bar{A} , \bar{B} = self-weights of crib elements and soil trapped between crib layers in case of inclined crib wall, then total pressure on each side of crib elements can be calculated as follows:

$$\begin{aligned} \bar{B} \cdot b^* \cdot \cos \alpha &= N_B^* \left(\frac{z}{2} \cdot \tan \alpha + b^* \cdot \cos \alpha \right) \\ &+ G_3 \cdot \left(\frac{z}{2} \cdot \tan \alpha + \frac{b^*}{2} \cdot \cos \alpha \right) + N_A^* \cdot \frac{z}{2} \cdot \tan \alpha \\ \bar{B} &= \frac{G_3}{2} + N_B^* + \frac{z}{2b^*} \cdot \frac{\tan \alpha}{\cos \alpha} \cdot (G_3 + N_A^* + N_B^*) \\ \bar{A} &= \frac{G_3}{2} + N_A^* - \frac{z}{2b^*} \cdot \frac{\tan \alpha}{\cos \alpha} \cdot (G_3 + N_A^* + N_B^*) \\ \bar{A} + \bar{B} &= G_3 + N_A^* + N_B^*, \text{ for vertical walls,} \\ \bar{A} &= G_{3A} \quad \text{and} \quad \bar{B} = G_{3B} \end{aligned}$$

Where G_3 = Weight of crib elements in kN/m; N_B^* = Weight of fill material between crib layers (earth side) in kN/m; N_A^* = Weight of fill material between crib layers (outer side) in kN/m; G_2 = Weight of the fill material inside a crib cell in kN/m; G_1 = Frictional force from silo pressure kN/m; E_{va} = Vertical component of earth pressure from backfill in kN/m; A, B = Vertical component of forces on the joints (for inclined wall) in kN/m; N_A, N_B = Normal component of forces on the joints (for inclined wall) in kN/m. N_A^*, N_B^* = The weights of the fill material between crib stretcher elements in kN/m. z, z_N = Vertical and inclined heights of wall in m. h_E, h_{EN} = Vertical and inclined heights from the base to the assumed point of action of the resultant in m.

The vertical pressure at joints will be given by $B = f(k \cdot j \cdot G_1, E_v, G_3, N_A^*, N_B^*, \alpha, b^*, z, h_E)$. Where k = a form factor which varies from 0.3 to 0.7, j = a reduction in mobilization of friction factor varies from 1 to $0.02 \cdot \alpha^\circ$ for $0^\circ \leq \alpha \leq 20^\circ$

Tensile force at the earth side crib element due to horizontal component of earth pressure will be

calculated as: $B' = \frac{E_h}{b^* \cdot \cos \alpha} \cdot (h_E - b^* \cdot \sin \alpha)$ Then the safety factor against the lifting up of crib elements at earth side will be given by $F_{LC} = \frac{B}{B'}$.

b. Lifting up from the forces parallel to wall inclination (Fig. 6c):

In this case, the forces are resolved in parallel and normal direction to the wall inclination and the forces at the joint are calculated as $N_B = k_B \cdot j \cdot G_{1N} + G_{3B,N} + E_{vN}$. Then the safety against uplifting will be given by $F_{LC} = \frac{N_B}{N_B^*}$.

In the above equation $k_B \geq 0.5$ and $j \leq 1$ and for practical purpose, $k_B \cdot j$ can be assumed as 0.5 (Brandl 1980). For each of these cases a global factor of safety of 1.5 will be required.

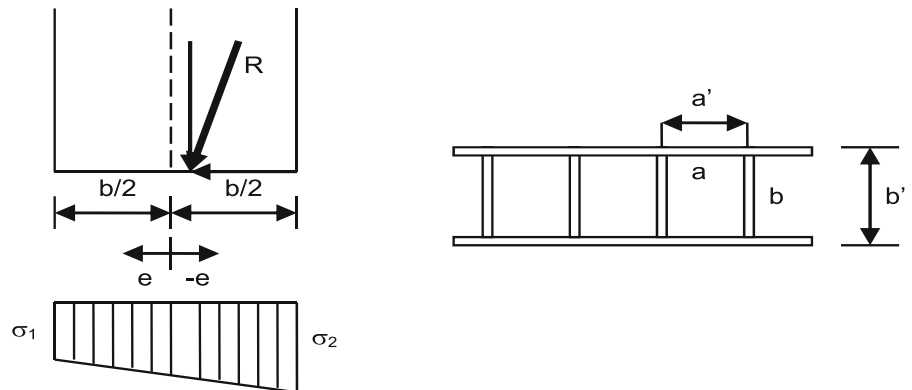
4.6 Safety Against Breaking of Joints

In crib walls, concentration of stresses will take place at the crossing points between stretcher and header elements. Therefore, care should be taken while selecting the size of crib elements so that the compressive and bending stresses at each element will not exceed the permissible limits. Based on the monolithic and silo theory assumptions, two safety checks are required in design practice. According to monolithic theory, the maximum compression at the outer side of crossing point is given by $N_A = a' \cdot N \cdot (0.5 \pm \frac{e}{b'})$. Where a and b are the length and width of the crib cells, a' and b' are the effective lengths and widths (Fig. 7).

$a' = a + \text{width of header element.}$

$b' = b + 2 \cdot \text{width of stretcher element.}$

Fig. 7 Illustration of the forces acting on the wall base and distribution of pressure and crib cell dimension



Using the above equations, from the force diagram (Fig. 6), the forces acting on the front and back joints (N_A , N_B) can be calculated. Then the safety factor is given by $F_{CJ,A} = \frac{N_{A,Break}}{N_{A,Available}} \geq 1.5$.

As mentioned above, according to conventional silo theory, for inclined wall, the forces on front side joints of a crib wall are lower than the forces on the earth side joints. According to the silo theory, the compressions at the joints are calculated as $N_B = \frac{1}{2} \cdot G_3 + N_B^* + \frac{1}{2} \cdot G_1 + E_v$ for inside joints and for outside joint, pressure is given by $N_A = \frac{1}{2} \cdot G_3 + N_A^* + \frac{1}{2} \cdot G_1$ and G_1 is given by $G_1 = a \cdot b \cdot \gamma_v \cdot \left[z - z_0 \cdot \left(1 - e^{-\frac{z}{z_0}} \right) \right] = a \cdot b \cdot (\gamma_v \cdot z - p_{vz})$.

Then the safety factor against the breaking of joints will be given by $F_{CJ,B} = \frac{N_{B,Break}}{N_{B,available}} \geq 1.5$.

5 Detail Design of a Log Crib Wall

As other gravity retaining walls, bamboo/wooden log vegetative crib walls shall be designed to withstand lateral earth and water pressures, the effects of surcharge loads, the self-weight of the wall and in special cases, earthquake loads in accordance with the general principles specified in design standards for gravity retaining walls. While designing a vegetative crib wall, it is required to consider the two extreme conditions:

1. Just after the construction (The role of vegetation is negligible/almost zero compared to crib elements);
2. After the decay of crib elements (The slope should be naturally stabilized, plant roots will support the slope with increasing shear strength of the fill materials).

1. Condition of vegetated crib wall just after construction

In the first condition, while calculating the internal stability of the crib walls, the total shear resistance of the vegetative crib wall will be: *Total shear resistance = Shear resistance by crib wall + shear resistance by vegetative cuttings.*

As mentioned in Sect. 1, the inserted cuttings will work as reinforcement in soil and prevent the local shear failure of the fill material which increases the

internal strength and stability of the crib structure. The shear resistance by vegetative cuttings can only be taken into considerations if the embedment length of cutting is more than 1 m and backfill is properly compacted. However, if these cuttings are placed in crib layers in the form of brush layers or fascines (as a bundle of vegetation cuttings), there will be no direct shear resistance by vegetative cuttings and it should be neglected. In general, the shear resistance by vegetative cuttings can be calculated by $R_p = \pi \cdot D \cdot L \cdot \tau_f \cdot \cos(\alpha + \theta)$ (Schuppener 2001). Where L , D = lengths and diameters of vegetative cuttings in m, τ_f = average bonding shear strength between vegetative cutting and fill material in kN/m^2 , θ = Slope angle with the critical failure plain, α = inclination angle of cuttings laid on crib layers.

In this case the analysis of a vegetative crib wall (for internal stability) can be done using monolithic theory and the wall should be designed as composite gravity retaining wall. For designing crib retaining walls, the “*Design manual for roads and bridges (Volume 2, Sect. 1, part 4 (BA 68/97), (1997))*” prepared jointly by The Highway Agency, The Scottish Office of Development Department, The Welsh Office, Road Department and The Department of the Environment for Northern Ireland in United Kingdom) can be used.

The standard in this manual follows a limit state approach with partial safety factor for design as expressed in *Euro code 7, Geotechnical design, Part 1* and in *Code of Practice for Earth Retaining Structures (BS 8002:1994)*. The earth retaining structure shall satisfy safety and serviceability requirements which should be derived through the application of partial safety factors to accommodate uncertainties in the applied loads, material strengths and model of analysis.

According to this manual, for economy, the dimensions of the crib cells should be selected such as to induce arching of the infill between the crib elements. Crib cells having a square cross-section may be a particularly efficient shape for promoting arching. To ensure an appreciable transfer of the weight of the infill to the crib structure, the ratio of the length (a) to width (b) of the crib cells should not be greater than 2.0. In ultimate limit state calculations in which the stability of a retaining wall depends on the ground resistance (passive earth pressure) in front of the structure, the level of the resisting soil should be

lowered below the nominally expected level by an amount Δa which should equal 10% of the distance between the lowest support and the excavation level, limited to a maximum of 0.5 m for a supported wall (EC-7-1,§9).

According to this manual, the assumption of design life does not necessarily mean that the structure will no longer be fit for its purpose at the end of that period, or that it will continue to be serviceable for that length of time without regular inspection and adequate maintenance. For the ultimate limit state, calculations will almost certainly be required to fulfill the stability. However, calculations may not be necessary for the serviceability limit state and the requirements may be satisfied by inspection, by reference to published data for similar structures, and by good construction practice. The following six limit modes of failure must be considered in design, although other limit modes may be appropriate in certain circumstances and should be checked accordingly.

1. Overturning failure
2. Sliding failure
3. Bearing failure of foundation
4. Slip failure of the soil
5. Failure of header and stretcher elements
6. Deformation (maximum horizontal and vertical displacements)

The maximum allowable deformation/displacements (tolerances in construction) of the crib wall construction as suggested in the *Design Manual* are presented in the

Table 1, which may be used as a guide for construction. The crib elements may fail in tension, compression, shear, bending and torsion, or by any combination of these. The designer must ensure that

Table 1 End of construction tolerances for a crib wall

Locations of deformation/displacements	Tolerances
Location of the plane of structure	±50 mm
Variation in front batter slope from design slope	±5 mm/m height
Bulging (vertical) and bowing (horizontal)	±20 mm in 4.5 m
Steps at joints	±5 mm
Alignment along top and bottom	±15 mm from reference

the most onerous combination of design load is checked.

For dimensioning such crib walls, the following parameters are required.

- The strength of crib elements in bending and tension
- The friction coefficients between fill and crib; fill and vegetation cuttings
- The cohesion and shear strength of fill material
- The strength of joints of crib elements
- The tensile, shear and bending strength of vegetative cuttings used in crib walls

2. Condition of vegetated crib wall after decay of crib elements:

To ensure the safety of the structure in the second situation, i.e. after the decay of wooden/bamboo crib elements, one should be able to calculate the factor of safety at that particular time taking into consideration of the increase in shear strength of soil due to existence of plant on the slope. There are various factors associated with living plant, which grows on the slope and has influence on the shear strength of soil and the slope hydrology. The net effects of all these factors should be considered in the calculation.

The presence of vegetation, mainly roots, results in an overall increase in the strength of soil. As mentioned earlier, the increase in the shear strength of soil is due to hydrological and mechanical effects of the plants. There are other factors like increase in surcharge, wind effects and anchoring effects of large roots, which also affects the safety factor of a vegetated slope.

Although it is not possible to quantify precisely the individual effects of vegetation on slope stability, it can be estimated to some extent from laboratory and field based measurements and tests and it can be then taken into consideration in the factor of safety calculation of a slope for a particular point of time. Some judgments and experience are required when assessing the physical effects of vegetation on slope stability. For the quantification of the hydrological effects one should be able to assess the role of pore water pressure on the shear strength of soil. To quantify the mechanical effects it is required to analyze influence of soil root matrix on the shear strength of soil. The shear strength of rooted soil

mass is enhanced due to the presence of a root matrix. The mechanical effect of the roots of the vegetation is to enhance the confining stress and resistance to sliding and increase the strength of the soil mass through the soil aggregate binding action of the roots in the fiber-soil composite. The soil friction angle remains unchanged during failure. If the slip surface passes through the root zone, failure occurs either by pullout or rupture. The magnitude of the mechanical reinforcing effects of vegetation is a function of the different root properties. As given by Coppin and Richards (1990), $R_r = f$ (density, tensile strength, tensile modulus, length/diameter ratio, surface roughness, alignment and orientation of roots).

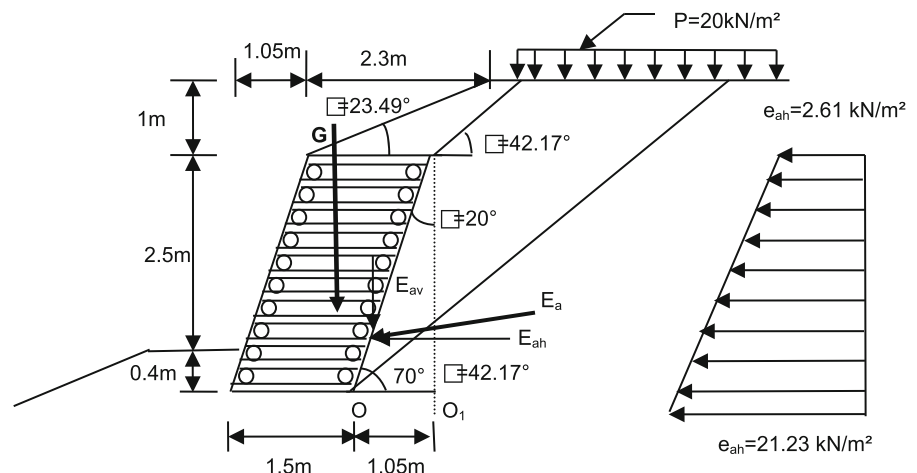
There are different analytical root model suggested by various researchers to calculate the effects of soil reinforcement by roots. Wu (2006) developed a simple theoretical model for predicting the shear strength increase due to the presence of roots. Similar models were developed by several other researchers (Gray and Leiser 1982). Some other researchers such as Daniels (1945), Hidalgo et al. (2002) and Pollen and Simon (2005) have suggested fiber bundle model (FBM) to calculate the root reinforcement. FBM takes into account the fact that roots within the soil matrix have different maximum strengths, and therefore break at different points as a load is applied to the soil (Pollen and Simon 2005). The results of the RipRoot model suggested by Pollen and Simon (2005) show that the use of fiber bundle theory can provide more accurate representations of shearing resistance due to roots compared to perpendicular model by Wu (2006). However, analytical results of

root shear resistance should be verified with field tests and experiments on each specific case before these results are used for future analysis.

In case of vegetated crib walls, the interaction between roots and soil can be quantified using simple perpendicular root model which allows the quantification of increased shear strength of the soil due to root reinforcement. Based on this perpendicular model the increase in shear strength of the soil composite is given by $\Delta s = t_R(\sin \theta + \cos \theta \cdot \tan \phi)$. Where Δs = Shear strength increase; ϕ = the angle of internal friction of the soil; θ = the angle of roots crossing the shear zone; and t_R = the mobilized tensile stress of the root fiber per unit area of soil.

The angle of shear rotation (θ) varies with the thickness of shear zone (z) and the amount of shear displacement (x). Wu et al. (1979) showed that the value of the bracketed term in above equation is fairly insensitive to normal variations in θ (40° – 90°) and ϕ (25° – 40°) with values ranging from 1.0 to 1.3. A value of 1.2 was therefore selected by Wu et al. (1979) to replace the bracketed term and then the simplified equation becomes $\Delta s = t_R(A_R/A) * 1.2$, which gives an average tensile strength of root or fiber per unit area of soil (Wu 1979; Pollen and Simon 2005). Where T_R = average tensile strength of root or fiber and A_R/A = root area ratio or fraction of soil cross sectional area occupied by roots. Preti and Giadrossich (2009) suggests a supplementary coefficient (0.39) as an empirical correction factor to reduce the overestimation of strength (Pollen and Simon 2005). In the above equation, A_R can be determined by counting the number of roots in

Fig. 8 Typical example of a slope with bamboo crib wall and illustration of the forces acting on the wall and earth pressure distribution



different size classes in a given soil cross sectional area (A) and determining the mean cross sectional area for that size class (Coppin and Richards 1990).

6 Typical Design Example

A typical crib wall of 2.9 m high made of bamboo elements is analyzed here as a typical example (Fig. 8). It has an average layer thickness of 0.29 m and a uniformly distributed surcharge of 20 kN/m². Because of a small diameter of bamboo, usually 3 bamboos in a bundle is used either as header or stretcher elements of the bamboo crib wall. In the example below, 3 bamboos in bundle in stretcher element is assumed. But in the analysis and calculation, a single stretcher element having a diameter equals to two-times the diameter of a header element is assumed.

Wall geometry and soil properties:

- Height of slope: 3.5 m, total height of wall: 2.9 m
- Slope inclination β = 23.49° and wall inclination angle (α) = 90°–70° = 20° from vertical
- Cohesion (c) = 2 kN/m², angle of internal friction (φ) = 30°, bulk density (γ) = 18 kN/m³, assumed angle of inclination of earth pressure (δ) = 2/3 φ = 20°

6.1 Assumption

In the following calculations, the cohesion of the fill material and the effects of vegetation or vegetative cuttings inside crib cell are neglected. Although the base of foundation generally made inclined inwards, a horizontal base is assumed here.

6.2 External Stability Analysis

Active earth pressure coefficient after Coulomb’s earth pressure theory: $\lambda_{ah} = \frac{\cos^2(\varphi + \alpha)}{\cos^2 \alpha \left[1 + \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi - \beta)}{\cos(\alpha - \delta) \cos(\alpha + \beta)}} \right]^2} = 0.258$ and $\lambda_a = \frac{\lambda_{ah}}{\cos(\alpha - \delta)} = 0.258$.

The effects of uniformly distributed surface load (p) can be regarded as a fictitious height z’ according to $z' = \frac{p}{\gamma} = \frac{20}{18} = 1.1m$. Then total height from wall base to the top of the ground can be obtained as

$3.5 + 0.4 + 1.11 = 5.01$ m. The earth pressure at the base of wall $e_{ah} = \gamma \cdot z \cdot \lambda_{ah}' = 18 \cdot 5.01 \cdot 0.258 = 23.26$ kN/m². A reduction of the active earth pressure due to cohesion (c) of fill material or existing soils is given by: $\Delta e_{ah} = -2c \cdot \sqrt{\lambda_{ah}} \cdot \sqrt{\cos(\delta - \alpha)} = -2.03$ kN/m².

Then the effective earth pressure at the base of wall = 23.26 – 2.03 = 21.23 kN/m². The earth pressure at the top of wall = 18*1*0.258 = 4.64 kN/m². The effective earth pressure at the top of wall = 4.64 – 2.03 = 2.61 kN/m². The total horizontal earth pressure acting on the wall surface (E_{ah}) = 0.5*(21.23 + 2.61)*2.9 = 34.57 kN. The inclination of potential failure plane is given by

$$\vartheta = \phi + \operatorname{arccot} \left[\tan(\phi + \alpha) + \frac{1}{\cos(\phi + \alpha)} \sqrt{\frac{\sin(\phi + \delta) \cdot \cos(\alpha + \beta)}{\sin(\phi - \beta) \cdot \cos(\delta - \alpha)}} \right] = 42.17^\circ.$$

According to the monolithic theory, the external forces acting on the crib wall will be transferred to the ground like gravity walls. The calculation is carried out by taking the average density (γ_w) of composite structure. The unit weight of bamboo varies from 3.7 to 8.5 kN/m³. Here an average unit weight of 5.0 kN/m³ is assumed.

The total volume of bamboo per unit length of wall = πd²/4 * total length of crib elements = π * (0.1²/4)*(2*10*3*1 + 2*11*1.5) = 0.73 m³. The total volume of wall = 2.9*1.5*1 = 4.35 m³. Then the average density of wall (γ_w) = (0.73*5.0 + 3.62*18)/4.35 = 15.81 kN/m³.

By considering the centre of gravity of wall and all acting forces, the point of action of the resultant earth pressure has been calculated as 1.07 m above the wall base. Vertical component of earth pressure (E_{av}) = E_{ah}*tan|δ-α| = 34.57*tan|20°-20°| = 0 kN. Normal force due to self-weight of soil and crib wall (N) = 1.5*2.9*15.81 + 0.5*1.5*1.5 tan23.49°*18 = 77.57 kN.

Resultant (R) = {(77.57)² + (34.57)²}^{0.5} = 84.92 kN.

Angle of inclination of the resultant (δ_s) = tan⁻¹(34.57/77.57) = 24.02° to the vertical, 65.98° to the horizontal. Taking moment at the centre of the wall base of all acting forces, neglecting the passive pressure from the front side, we have:

$\Sigma M_o = 77.57 \cdot (0.75 - 0.22) - 34.57 \cdot 1.07 = 4.12$ kN-m clockwise.

The eccentricity (e) = $\Sigma M_o / \Sigma V = 4.12 / 77.57 = 0.053$ m.

$$e_{Per} \leq \frac{b'}{4} = 1.4/4 = 0.35 \text{ m, } e < 0.35 \text{ m ok.}$$

The resultant of forces passes through the point with a distance of 0.053 m inwards from the centre of the wall base, i.e. it lies at inner half of the base. Then the maximum and minimum pressures at bottom soil are given by $\sigma_{1,2} = \frac{N}{A} \cdot \left(1 \pm \frac{6e}{b'}\right)$.

Where A is the total area of the wall base, b' the effective width and e is the eccentricity. $\sigma_1 = 77.57 / 1.5 (1 - 6 \cdot 0.053 / 1.4) = 39.93$ kN/m² and $\sigma_2 = 77.57 / 1.5 (1 + 6 \cdot 0.053 / 1.4) = 63.49$ kN/m². There will be a higher pressure at the inner part of the base of the wall.

In case of displaced horizontal joints situation the maximum pressure will be given by $\sigma_1 = \frac{2N}{c}$, where c = distance of resultant from outer edge of the wall. $\sigma_{1max} = 2 \cdot 77.57 / (1.5 - 0.22) = 121.20$ kN/m².

Permissible stress for unconsolidated mixed soil = 200 kN/m² (assumed), which is greater than σ_{max} . It is safe for design.

Safety against bearing capacity failure:

Safety factor = $200 / 63.49 = 3.15 > 1.5$ ok.

The safety factor for overturning is given by

$$F_O = \frac{M_{Passive}}{M_{Active}} = \frac{77.57 \cdot (0.75 - 0.22)}{34.57 \cdot 1.07} = 1.11 < 1.5 \text{ not ok.}$$

Since the wall does not fulfill the safety criteria, either the height of wall should be reduced or the slope angle should be reduced or the fill material having high density and high friction angle should be used.

For the safety against sliding, the following criteria shall be fulfilled. $H \leq V \cdot \tan \delta_1$ and $F_s = \frac{V \tan \delta_1}{H} \geq 1.5$

$$\text{Then } F_s = (77.57 \cdot \tan 20^\circ) / 34.57 = 0.81 < 1.5.$$

Since the wall is not safe against sliding, the base layer of the crib should be secured against sliding by providing wooden or steel pegs hammered into the ground or making an inclined foundation base to increase the frictional resistance. Therefore in this case it is suggested to use wooden pegs of lengths 1–1.5 m at 1.5 m centres. Moreover, if the base of foundation is inclined at an angle of 15°, then the factor of safety against sliding will be increased to about 1.57 which will be then safe.

7 Internal stability analysis

a. Safety against the lifting up of crib elements or joint displacements:

1. From horizontal and vertical forces (refer Fig. 6):

For the given wall, $z = z_{max} = 2.9$ m and $b^* = 1.30$ m. The weight of crib elements (G_3) = $0.73 \cdot 5 = 3.65$ kN/m and the self-weights (\bar{A}, \bar{B}) of crib elements and soil inside the cell will be calculated as follows:

$$\begin{aligned} \bar{B} &= \frac{3.65}{2} + 10 \cdot 0.1 \cdot 0.2 \cdot 1.3 \cdot 18 \\ &+ \frac{2.9}{2 \cdot 1.3} \cdot \frac{\tan 20^\circ}{\cos 20^\circ} \\ (3.65 + 2 \cdot 10 \cdot 0.1 \cdot 0.2 \cdot 1.3 \cdot 18) &= 10.54 \text{ kN} \end{aligned}$$

$$\begin{aligned} \bar{A} &= \frac{3.65}{2} + 10 \cdot 0.1 \cdot 0.2 \cdot 1.3 \cdot 18 \\ &- \frac{2.9}{2 \cdot 1.3} \cdot \frac{\tan 20^\circ}{\cos 20^\circ} \\ (3.65 + 2 \cdot 10 \cdot 0.1 \cdot 0.2 \cdot 1.3 \cdot 18) &= 2.46 \text{ kN.} \end{aligned}$$

The joint pressure at the front side of walls will be less compared to earth side and therefore for extreme condition, only earth side joint pressure is considered. Vertical pressure at joints from other loads will be given by:

$$B = f(k \cdot j \cdot G_1, E_v, G_3, N_A^*, N_B^*, \alpha, b^*, z, h_E)$$

Total vertical pressure at earth side joints (B) = $10.54 + (1.3 \cdot 2.9 \cdot 18) / 2 = 44.47$ kN.

Tensile force at the earth side of crib wall because of horizontal component of earth pressure will be given by $B' = \frac{E_h}{b^* \cdot \cos \alpha} \cdot (h_E - b^* \cdot \sin \alpha) = 17.69$ kN. Then the safety factor against the lifting up of crib elements at earth side will be given by $F_{LC} = \frac{B}{B'} = 2.51 > 1.5$. It is ok.

2. From the forces parallel to wall inclination (refer Fig. 6):

In this case the forces are resolved in parallel and normal direction to the wall inclination and the forces at the joint are calculated.

For inside joints: $N_B = k_B \cdot j \cdot G_{1N} + G_{3B,N} + E_{VN} = 19.37$ kN and the safety against uplifting is given by $F_{LC} = \frac{N_B}{N'_B}$ where $N'_B = E_h \cdot \cos(90 - \alpha) \cdot \frac{h_E}{\cos \alpha} \cdot \frac{1}{b^*} = 10.35$ kN.

Then $F_{LC} = 19.37/10.35 = 1.87 > 1.5$, which is ok.

b. Safety against the breaking of joints

According to monolithic theory, the maximum compression at the outer side of crossing point is given by $N_A = a' \cdot N \cdot (0.5 \pm \frac{e}{b'})$.

Using the equation above, from the force diagram (Fig. 6), the forces acting on the front and back joints (N_A, N_B) can be calculated as

$$N_A = 1.3 \cdot 77.57 \cdot (0.5 - 0.053/1.4) = 46.60 \text{ kN}$$

$$N_B = 1.3 \cdot 77.57 \cdot (0.5 + 0.053/1.4) = 54.23 \text{ kN}$$

These two forces are compressive forces acting on the respective joints on stretcher members, which will be transferred to the ground in form of bending and compression. Note that these forces act over the stretcher member on the surface area equal to the cross sectional area of the stretcher member, then the compressive stress at the joints will be

$$\sigma_{c,A} = 46.60 / (\pi \cdot 0.1 \cdot 0.1/4) = 5933.29 \text{ kN/m}^2 = 5.93 \text{ N/mm}^2$$

$$\sigma_{c,B} = 54.23 / (\pi \cdot 0.1 \cdot 0.1/4) = 6904.76 \text{ kN/m}^2 = 6.90 \text{ N/mm}^2$$

The allowable compressive strength for bamboos = $60/1.5 = 40 \text{ N/mm}^2 > \sigma_{c,A}$ or $\sigma_{c,B}$, which is safe.

Then the safety factor will be given by

$$F_{CJ,A} = \frac{N_{A,Break}}{N_{A,Available}} \geq 1.5$$

$$F_{CJ,A} = 40/5.93 = 6.74 > 1.5 \text{ ok.}$$

$$F_{CJ,B} = 40/6.90 = 5.79 > 1.5 \text{ ok.}$$

According to silo theory, the compressions at the joints are calculated as follows:

The model tests and site measurements showed that the joint pressures at the earth side of the wall are normally higher than at the outer side. The pressure at the inside and outside joints are calculated by $N_B = \frac{1}{2} \cdot G_3 + N_B^* + \frac{1}{2} \cdot G_1 + E_v$ and $N_A = \frac{1}{2} \cdot G_3 + N_A^* + \frac{1}{2} \cdot G_1$. Where G_1 is given by $G_1 = a \cdot b \cdot \gamma_v \cdot [z - z_0 \cdot (1 - e^{-\frac{z}{z_0}})] = a \cdot b \cdot (\gamma_v \cdot z - p_{vz}) = 140.68 \text{ kN}$.

$N_B = 0.5 \cdot 3.65 + 4.68 + 0.5 \cdot 140.68 + 0 = 76.84 \text{ kN}$. Since the vertical component of the earth pressure is zero in the present calculation. The joint pressure $N_B = N_A = 76.84 \text{ kN}$. Consider that these forces are acting over the stretcher member on the surface area equal to the cross sectional area of the stretcher member, then the compressive stress at the joints will be $\sigma_{c,A} = \sigma_{c,B} = 76.84 / (\pi \cdot 0.1 \cdot 0.1/4) = 9784.20 \text{ kN/m}^2 = 9.78 \text{ N/mm}^2$. If we assume

a compressive strength of bamboo = 40 N/mm^2 , Then the safety factor against the breaking of joints will be:

$$F_{CJ,B} = \frac{N_{B,Break}}{N_{B,Available}} = \frac{40}{9.78} = 4.08 \geq 1.5 \text{ which is ok.}$$

c. Safety against the shear failure through crib wall

The crib wall is assumed as filled slope construction with different material having different weights and shear strengths (fill materials and crib elements separately) and the stability analysis was carried out using Bishop’s method. The fill slope is assumed as reinforced earth slope with bamboo elements. In the analysis the same bamboo crib wall and same soil parameters are used. The result of the analysis is presented in Fig. 9 which showed a factor of safety of 1.4 and is safe.

8 Conclusion and Recommendation

Vegetated log crib walls can be designed as composite gravity walls similar to the design of concrete crib walls. Compare to concrete crib walls, vegetative log crib walls are generally made $20^\circ\text{--}30^\circ$ inclined from vertical to ease the plant growth. In addition, the wall base is also kept inclined ($5^\circ\text{--}15^\circ$) to increase the sliding resistance. In crib walls, concentration of stresses will take place at the crossing points between long and short elements; therefore care should be taken while selecting the size of crib elements so that the compressive and bending stresses at each element will not exceed the permissible limits.

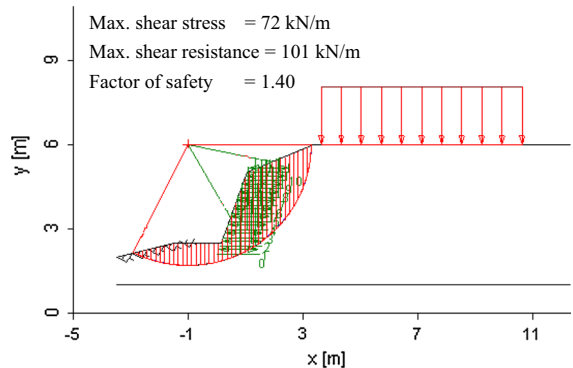


Fig. 9 Results of stability analysis for the designed slope using Bishop’s method

For external stability analysis, the same procedure for concrete crib with some additional considerations in mobilisation of friction and earth pressure can be followed. But for the internal stability analysis a contribution from vegetative cuttings can also be taken into consideration, if there exists a substantial amount of vegetative cuttings with relatively longer embedment length and good fill material with good compaction. However, if the contribution of the vegetative parts (cuttings) is very small compared to the strength of crib structure it can be neglected in the initial analysis and design.

Experiences on different soil bioengineering works carried out in stabilising road side slopes in Nepal (after monitoring the slope for more than 5 years) showed that, if there are no significant slope movements or erosions processes observed, it can be assumed that the slope has been stabilised fully by regaining its strength with the help of plants through root reinforcement and increase in cohesion. After 5 years the slope will be stabilised further by natural processes of revitalisation of vegetation or improvement in other slope parameters. In this situation, no further safety check is required. However, at this stage also there is a possibility to calculate a factor of safety to verify the desired level of safety.

Based on the results of past constructions and the past experiences, a consideration of the future situation of the planned log crib wall should also be made in the design so that the slope would remain stable with the help of grown plants after the decay of wooden crib elements. To ensure the serviceability of the retaining structure, the log crib wall should be well monitored and regular maintenance of vegetation at site should be carried out. When there is lack of past experience, the designed retaining structure should be monitored on a regular basis and appropriate safety measures shall be taken.

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