Letter to the Editor

Backward Reaction Force in a Firehose

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Because the ejected water jet in a firehose resembles the ejected gases that propel rockets, many people believe that the ejected water jet produces a backward reaction force on the firehose, a force that must be counteracted by firefighters. In a previous experimental work published in this journal [1] we showed that the reaction force produced at the nozzle does not exist and that the force counteracted by firefighters exists but is a consequence of the interaction of water with the inner walls of a bent hose. In a recent paper in this journal entitled Firefighter Nozzle Reaction, Selena K. Chin, Peter B. Sunderland, and Grunde Jomaas [2], concluded that the water jet momentum ejected from a firehose is produced at the nozzle and that the backward nozzle reaction in a firehose is always equal to the jet momentum flow rate. Although this backward force on the firefighters apparently agrees in magnitude and direction with firefighting experience, it contradicts the experiment reported in our previous work [1]. Here, we argue that Chin et al. results are the consequence of using an equation for the tension of the hose that is difficult to reconcile with a conceptual understanding of the force on the nozzle and with calculations found in two Fluid Mechanics textbooks cited in their paper [3, 4].

In this letter we use some simple fluid flow examples and calculations using Newton's second law in the control volume approach, to give support to our conceptual explanation for the backward force on a firehose. We also show how our explanation in [1] differs from Chin et al. results [2]. The explanation for the backward force on the firefighters given in our previous work [1] is that the force is produced in a bend behind the firefighters and it is transmitted by the hose, producing a force that is interpreted by firefighters as being produced at the nozzle. In that work we also showed that the conceptual error of a backwards reaction force with a magnitude equal to the jet momentum flow rate is of widespread use in several physics textbooks [5–7].

The issue has its origins in what we deem as an incorrect use of Newton third law to calculate the backward force using the formula for the thrust that is, for example, commonly used to explain rocket propulsion. In a firehose, the ejected water jet is not propelled by the nozzle, it is propelled by the fluid pressure along

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the hose and therefore the water jet can not push backwards on the nozzle. Thus, the issue is a consequence of using Newton third law for a macroscopic system (that states that there is a reaction force somewhere) without paying attention to the microscopic interaction forces (that allow to understand where the reaction force is really exerted).

In what follows, we use the control volume form of Newton's second law (see for example the chapter Basic Equations in Integral Form for a Control Volume in Fox and McDonald's Book) [8]:

$$\mathbf{F} + \int_{CS} -p d\mathbf{A} + \int_{CV} \rho \mathbf{g} dV = \frac{\partial}{\partial t} \int_{CV} \mathbf{v} \rho dV + \int_{CS} \mathbf{v} \rho \mathbf{v} \cdot d\mathbf{A} \quad (1)$$

In this formula, CV is the control volume, CS is the surface of the CV , $\mathbf{v}$ is the velocity of the fluid, $\mathbf{F}$ is the external or anchor force applied to the system with F_x pointing to the right and F_y pointing upwards, $d\mathbf{A}$ is a vector pointing out of the control volume, $\rho \mathbf{g} dV$ is the force of gravity, the first term of the right side is the rate of change of momentum within the control volume and the last term of the right side is the net rate at which momentum is leaving the control volume through the control surface.

An appropriate choice of the control volume can simplify some calculations [8], but it can also hide the nature of the interactions between the fluid and the walls containing the fluid flow. For example, let us consider a simplified model for a rocket that is anchored to a launching platform. Although the rocket is expelling gases, the platform (or another external agent) exerts anchor forces F_x , F_y to keep the rocket motionless. To understand the microscopic mechanism that propels rockets we will use two different control volumes to analyze this problem.

Figure 1 shows a rocket anchored to the launching platform, the segmented lines at the left identify the control volume CV_1 that includes the rocket and the combustion chamber, the segmented lines at the right identify the control volume CV_2 that includes the rocket but excludes the combustion chamber. Using the control volume CV_1 , the external vertical force on the rocket F_y in Eq. (1) obeys

$$F_y - Mg = -v dM/dt, \quad (2)$$

where M is the mass of the rocket. Therefore the magnitude of the force that propels rockets can be identified with the typical formula for the ejected momentum rate $+v dM/dt$. Using the control volume CV_2 shown in Fig. 1 right, the force on the rocket F_y in Eq. (1) obeys

$$F_y + (p - p_0)A - Mg = 0. \quad (3)$$

Thus, the propulsion of rockets can be calculated using the rate of ejected momentum $v dM/dt$ of Eq. (2), but the physical mechanism for rocket propulsion is produced by the higher pressure zone $(p - p_0)$ that appears as a consequence of high temperatures in the combustion chamber, that is included in Eq. (3). In other

Figure 1. A simplified version of a rocket anchored to the launching platform. Left: the control volume CV_1 includes the combustion chamber. Right: the control volume CV_2 excludes the combustion chamber.

words, the forces propelling rockets are microscopic forces produced by molecular collisions acting at the upper part of the combustion chamber and not where the gases are expelled.

Another interesting example is a straight hose of inner area A emptying a container (see Fig. 2). In this experiment a pressure gradient must appear to connect in a continuous way the values for the pressures at the bottom of the container and the atmospheric pressure at the end of the hose. In the absence of friction forces between the fluid and the inner walls of the hose (the inviscid fluid approximation) the pressure along the hose should be constant in contradiction with any simple demonstration of this experiment. Using the control volume CV shown in Fig. 2, the external force F_x obeys

$$F_x + p_2 A - p_3 A = v_3 \frac{dM}{dt} - v_2 \frac{dM}{dt}. \quad (4)$$

$p_2 > p_3$ because of fluid friction, and because water enters and leaves the hose at the same speed, the right hand side of this equation equals to zero and therefore the net external force on this CV is given by

$$F_x = -(p_2 - p_3)A \quad (5)$$

and is directed against the direction of the flow. The non zero force necessary to hold this segment and the pressure gradient that appears along the hose, are direct consequences of friction forces. In this situation, any anchor or a firefighter holding this CV should be pulled forwards and not backwards as suggested by Chin et al. main result. In this example friction forces can not be neglected because they produce the pressure gradient observed experimentally. However, when a nozzle is included at the end of the hose a large pressure gradient will appear at the position of the nozzle and therefore a large fluid speed will be produced inside the nozzle. The smaller speed at the hose justifies the usual approximation of considering an almost constant pressure along the hose.

Figure 2. A straight hose emptying a container. The control volume CV includes the end of the hose and the water flowing inside it.

The main purpose of Chin et al. paper was to include the tension of a flexible hose in the calculation of the external force on a nozzle to maintain it static, assuming a steady inviscid flow and a bent flexible hose in frictionless contact with the ground. The net external force on the nozzle was separated as a force produced by the tension of the hose on the nozzle and the force of a firefighter or other anchor on the nozzle. After including their formula $T = p_g A + \rho A v^2$ for the tension along a hose, they found a backwards nozzle reaction force that is always equal to the momentum flow rate emerging from the nozzle.

Figure 3 shows a firehose forming a loop and the external tension forces necessary to maintain the loop static. Here we will use the small section of the loop enclosed by CV_1 and the tensions $T_1 \hat{t}_1$ and $T_2 \hat{t}_2$ that maintain this small segment static to derive the formula for the tension obtained in Chin et al. paper. Applying Eq. (1) for the control volume CV_1 , neglecting the effects of gravity and regrouping terms, we find

$$(T_1 - p_{g1}A - v dM/dt) \hat{t}_1 + (T_2 - p_{g2}A - v dM/dt) \hat{t}_2 = 0, \quad (6)$$

Figure 3. Tension forces on a small segment of a hose making a circular loop. The flow enters at the left end of the hose with speed v . The figure shows the control volume CV_1 used to calculate the magnitude of the tensions acting on this segment. The figure also shows a control volume CV_2 at the end of the hose.

$p_g = p - p_0$ are gauge pressures. Because the unitary vectors $\hat{t}_1$ and $\hat{t}_2$ are independent each term in parenthesis must vanish independently and the magnitude of the tensions T_1 and T_2 required to maintain any segment of the hose static are given by

$$T_1 = p_{g1}A + \rho Av^2 \quad (7)$$

and

$$T_2 = p_{g2}A + \rho Av^2. \quad (8)$$

These equations give the value for the magnitude of the tension T at any point of the loop when the pressure (and therefore the tension) varies along the hose. If there is no friction, then $p_{g1} = p_{g2} = p$ and $T_1 = T_2 = T$, and the expressions for T_1 and T_2 reduce to formula (5) of Chin et al. paper: $T = p_g A + \rho A v^2$. The first term corresponds to the hydrostatic part of the force that is present even when the fluid is motionless. The second term is the force originated at the inner parts of the hose by the fluid momentum.

Chin et al. procedure can not be applied to find the hose tension for the case of a straight hose. In the limit of a straight hose segment, both terms in parenthesis in Eq. (6) are not independent but can be used to obtain the net external force on a segment of the hose. Because the hose segment is supported only by the tensions $\mathbf{T}_1$ and $\mathbf{T}_2$, the net force $\mathbf{F}$ acting on it is the vectorial sum of these tensions. In the limit of a straight hose the unitary vectors $\hat{t}_1$ and $\hat{t}_2$ lie on the same straight line and satisfy $\hat{t}_1 = -\hat{t}_2$. If we choose the direction of $\hat{t}_2$ and the x-axis along the hose, and using the standard notation $\hat{i}$ for the unitary vector that points in the positive direction of the x-axis, we have $\hat{t}_2 = \hat{i}$ and $\hat{t}_1 = -\hat{i}$. Thus, the x-component of the net force F_x is given by $T_2 - T_1$ and using the expressions for T_1 and T_2 we obtain

$$F_x = -(p_1 - p_2)A. \quad (9)$$

As mentioned previously, $(p_1 - p_2)A$ corresponds to the friction force and is a positive quantity. Thus, F_x points towards the negative side of the x-axis to sustain a straight hose segment that is pushed by the fluid in the direction of the fluid flow.

For a straight hose anchored at its ends, the magnitude of the tension varies along the hose as a consequence of friction forces and it also depends on the magnitude of the applied external forces. However, in the absence of friction forces, the magnitude of the tension equals the magnitude of the external forces that maintain the hose straight. Similarly, when the end segment of the straight hose is constrained to move only along the x-axis, the tension $\mathbf{T}$ on this segment points against the fluid flow with a magnitude equal to the magnitude of the friction force and is zero for an inviscid fluid.

Although we just showed that the net force on the end segment of a straight hose vanishes in the absence of friction forces, here we will explore the consequences of following Chin et al. procedure to find the force applied by a firefighter $\mathbf{F}_f$ on a straight segment at the end of a hose (without a nozzle). Using the control volume shown in Fig. 2 that encloses this segment and assuming a pressure $p_2 \neq p_3$ near the end of the hose, we obtain

$$\mathbf{F}_f + \mathbf{T} + p_2 A \hat{i} - p_3 A \hat{i} = 0 \quad (10)$$

Using $p_3 = p_0$, $\mathbf{F}_f = F_{fx} \hat{i}$, $\mathbf{T} = -T \hat{i}$, $p_g = p_2 - p_0$, and

$$T = p_g A + \rho A v^2, \quad (11)$$

Chin's procedure gives

$$F_{fx} = (p_g A + \rho A v^2) - p_g A = \rho A v^2. \quad (12)$$

This equation implies that a firefighter must exert a force on the end segment of a straight hose pointing in the direction of the flow, with a magnitude equal to the ejected rate of momentum $\rho A v^2$. In fact, it follows from Chin et al. paper that for any bending of the hose, no matter how small, and for any nozzle design (which includes the no nozzle case), there is always a reaction force $\mathbf{R}$ of this magnitude in the backwards direction. This result is a consequence of using a constant pressure in the formula $T = p_g A + \rho A v^2$ for the tension at the end region of a hose. Notice however that friction forces can produce a pressure gradient along the hose (as was previously discussed in the example of a horizontal straight hose emptying a container) invalidating this result. It should be also noted that in real firefighting practice there are large friction forces exerted by the ground on the hose and firefighters use their own weight to increase the normal force to increase the maximum value of the static friction force. When these forces are present it is not possible to use the value of the hose tension given by Chin et al. equation at the nozzle.

Finally, we will use Chin et al. procedure to examine the simple case of a converging nozzle with $A_1 > A_2$ attached at the end of the hose at the position of the control volume CV_2 for the loop shown in Fig. 3. The pressure p at the initial part of the nozzle is larger than the atmospheric pressure, and following Chin et al. assumption of an inviscid flow, the pressure at the loop would have this same magnitude. In this case,

$$F_{fx} - T + p_g A_1 = \rho A_2 v_2^2 - \rho A_1 v_1^2, \quad (13)$$

and using

$$T = p_g A_1 + \rho A_1 v_1^2, \quad (14)$$

the force applied by a firefighter would be given by

$$F_{fx} = \rho A_2 v_2^2, \quad (15)$$

which implies a backward reaction force with a magnitude equal to the rate of ejected momentum.

The net external force F_x that needs to be applied on this nozzle to maintain it static is calculated in textbooks of fluid mechanics using the control volume approach. For this example, the books by Nazarenko [4] and Lautrup [3] find:

$$F_x = -\frac{1}{2} \rho v_2^2 \frac{(A_1 - A_2)^2}{A_1}. \quad (16)$$

The negative sign of F_x means that it is necessary to exert a net external force against the fluid flow to maintain the nozzle static. In this example the net external force can be decomposed as $F_x = F_{fx} - T$ and the pressure drop at the nozzle might be large enough to produce a nearly constant pressure along the hose. Therefore, the pressure at the left side of the nozzle could be used to find the tension at the loop. However, friction forces with the ground make it unrealistic to use the value of the tension T (originated at the loop) at the nozzle, to find the value for the anchor force F_{fx} .

Comparing the previous results for the net force on the end segment of a straight hose given by Eq. (5) and the net force to maintain static a nozzle at the end of a hose given by Eq. (16), it is possible to see that these net forces point against the fluid flow. Although the tension force produced by the hose on the nozzle points against the fluid flow, it is not necessarily true that this tension will produce a net force on the firefighters in the backwards direction. In other words, the firefighters must counteract the combined effects of the tension and the force exerted by the fluid on the nozzle. A simple example where the force of the nozzle on a firefighter would be directed along the direction of the flow is the experiment reported in our previous work [1]. In this experiment it is evident that when the pressure at the inlet of a straight hose is increased the force on the nozzle increases along the direction of the flow. Although straight firehoses are not used in fire-fighting practice, our straight hose experiment was designed to show in a simple way that the backward reaction force is not produced at the nozzle. However, understanding the origins of the backward reaction force allowed us to propose a new possibility to combat a fire: the end of the hose can be connected to an anchored flexible ball joint attached to a straight rigid hose segment to combat a fire without backward forces on the firefighters.

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