Erratum to: Modeling clusters of extreme values

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The erratum concerns Theorems 2, 3 and Lemma 1. The equation numbers of the original paper are preserved.

The partition (9)–(10) has to be replaced by the following one. Let

$$1 = k_{n,0}^* \leq k_{n,1}^* \leq k_{n,2}^* \leq k_{n,3}^* \leq k_{n,4}^* \leq k_{n,5}^* = j \quad (9)$$

be a partition of the interval $[1, j]$ and positive integers $\{k_{n,i}^*\}$ be such that

$$\{k_{n,i-1}^* = o(k_{n,i}^*), i \in \{1, 2, \dots, 5\}\}. \quad (10)$$

The next theorem is a refinement and correction of Theorem 2.

Theorem 2 *Let $\{R_n\}_{n \geq 1}$ be a stationary process with the extremal index θ . Let $\{x_{\rho_n}\}$ be a sequence of quantiles of R_1 of the levels $\{1 - \rho_n\}$, that satisfies the conditions (1) and (2) if u_n is replaced by x_{ρ_n} and, $q_n = 1 - \rho_n$. Let positive integers $\{k_{n,i}^*\}$, $i = \overline{0, 5}$, be as in Eqs. 9 and 10, respectively, $\Delta_{n,i} = k_{n,i}^* - k_{n,i-1}^*$, $q_{n,i}^* = o(\Delta_{n,i})$,*

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$i \in \{1, 2, \dots, 5\}$, be such that for each $\varepsilon > 0$ there exist n_ε and $j_0 = j_0(n_\varepsilon)$ such that for all $n > n_\varepsilon$ and $j > j_0(n_\varepsilon)$

$$\alpha_n^*(x_{\rho_n}) = \max \left\{ \alpha_{k_{n,4}^*, q_{n,1}^*}; \alpha_{k_{n,3}^*, q_{n,2}^*}; \alpha_{\Delta_{n,3}, q_{n,3}^*}; \alpha_{j+1-k_{n,2}^*, q_{n,4}^*}; \right. \\ \left. \alpha_{j+1-k_{n,1}^*, q_{n,5}^*} \right\} < \varepsilon \quad (11)$$

and

$$\alpha_{j+1, k_{n,4}^* - k_{n,1}^*} / \rho_n < \varepsilon \quad (12)$$

hold, where $\alpha_{n,q} = \alpha_{n,q}(x_{\rho_n})$ is determined by (7). Then for the same n and j it holds

$$\left| P\{T_1(x_{\rho_n}) = j\} / \left(\theta^2 \rho_n (1 - \rho_n)^{(j-1)\theta} \right) - 1 \right| < \varepsilon, \quad (13)$$

$$\left| P\{T_2(x_{\rho_n}) = j\} / \left(\theta^2 q_n (1 - q_n)^{(j-1)\theta} \right) - 1 \right| < \varepsilon. \quad (14)$$

The partition (9) is modified by means of Eq. 10 in such a way that each subinterval $\Delta_{n,i}$ is infinite as $n \rightarrow \infty$. This allows to apply mixing conditions in the proof. In the new formulation, former $\rho_n^* = (1 - q_n^\theta)^{1/\theta}$ and $q_n^* = 1 - \rho_n^*$ are replaced by ρ_n and q_n , respectively, because $n\rho_n^*$ does not tend to the same τ as $n\rho_n$ for $\theta \neq 1$. A new assumption (12) is added. It specifies formula (42) in Markovich (2014) and provides

$$\left| \frac{P\{R_{j+1} > x_{\rho_n} | R_1 > x_{\rho_n}\}}{\rho_n} - 1 \right| < \varepsilon \quad (15)$$

for all n and j sufficiently large. We can apply

$$|P\{R_{j+1} > x_{\rho_n} | R_1 > x_{\rho_n}\} - P\{R_{j+1} > x_{\rho_n}\}| \leq \alpha_{j+1, k_{n,4}^* - k_{n,1}^*}(x_{\rho_n}) < \varepsilon \rho_n$$

and obtain (15). These improvements do not change the proof of Theorem 2 (Markovich 2014) apart of one item. In formula (43) (Markovich 2014) one can use

$$P\{M_{1, k_{n,1}^*} \leq x_{\rho_n} | R_1 > x_{\rho_n}\} \rightarrow \theta, \quad P\{M_{k_{n,4}^*, j} \leq x_{\rho_n} | R_{j+1} > x_{\rho_n}\} \rightarrow \theta$$

instead of $P\{M_{1, k_{n,1}^*} \leq x_{\rho_n} | R_1 > x_{\rho_n}\} \rightarrow 1$, $P\{M_{k_{n,4}^*, j} \leq x_{\rho_n} | R_{j+1} > x_{\rho_n}\} \rightarrow 1$ as $n \rightarrow \infty$. This follows from the representation by O'Brien (1987)

$$\lim_{n \rightarrow \infty} P\{M_{1, p_n} \leq u_n | R_1 > u_n\} = \theta,$$

where $\{p_n\}$ is an increasing sequence of positive integers, $p_n = o(n)$ as $n \rightarrow \infty$. Hence, we can replace ' $\rho_n(1 - \rho_n)^{(j-1)\theta}$ ', and ' $q_n(1 - q_n)^{(j-1)\theta}$ ', in Theorem 2 (Markovich 2014) by ' $\theta^2 \rho_n(1 - \rho_n)^{(j-1)\theta}$ ', and ' $\theta^2 q_n(1 - q_n)^{(j-1)\theta}$ ', respectively, without the requirement of the mixing condition $D''(x_{\rho_n})$ as in Markovich (2014).

For all n and j sufficiently large one can rewrite (14) in a geometric form as

$$\left| \frac{d_n P\{T_2(x_{\rho_n}) = j\}}{\chi_n (1 - \chi_n)^{j-1}} - 1 \right| < \varepsilon, \quad (16)$$

where $d_n = \chi_n / (\theta^2 (1 - (1 - \chi_n)^{1/\theta}))$, $0 < \chi_n < 1$, using the replacement $(1 - q_n)^\theta = 1 - \chi_n$.

Lemma 1 in Markovich (2014) is reformulated and its proof is simplified.

Lemma 1 *Let the conditions of Theorem 2 be satisfied. Then for the same n and j as in Theorem 2 it follows*

$$|E(T_2(x_{\rho_n}))/\Lambda_n - 1| < \varepsilon,$$

where

$$\Lambda_n = \frac{\theta^2 q_n}{(1 - (1 - q_n)^\theta)^2} (1 - q_n)^{\theta j_0(n_\varepsilon)} (j_0(n_\varepsilon) (1 - (1 - q_n)^\theta) + 1),$$

$$\text{and } E(T_2(x_{\rho_n})) = \sum_{j=j_0(n_\varepsilon)+1}^{\infty} j P\{T_2(x_{\rho_n}) = j\}.$$

The condition (18) in Lemma 1 (Markovich 2014) is not required since the series $\sum_{j=j_0(n_\varepsilon)+1}^{\infty} j P\{T_2(x_{\rho_n}) = j\}$ converges uniformly by all $n > n_\varepsilon$ by Weierstrass' theorem. From Eq. 16 $E(T_2(x_{\rho_n}))$ is upper bounded by

$$\begin{aligned} \frac{1 + \varepsilon}{d_n} \sum_{j=j_0(n_\varepsilon)+1}^{\infty} j \chi_n (1 - \chi_n)^{j-1} &= \frac{(1 + \varepsilon) \theta^2 q_n}{(1 - (1 - q_n)^\theta)^2} (1 - q_n)^{\theta j_0(n_\varepsilon)} \\ &\times (j_0(n_\varepsilon) (1 - (1 - q_n)^\theta) + 1) = (1 + \varepsilon) \Lambda_n \end{aligned}$$

and $\Lambda_n \rightarrow 0$ since $q_n \sim 1 - \tau/n$ as $n \rightarrow \infty$.

The formulation regarding $S_{T_2(x_{\rho_n})}$ in Theorem 3 (Markovich 2014) can be improved and its condition (36) is not required. We assume the same Conditions 1–3 as in Markovich (2014).

Theorem 3 *Let the conditions of Theorem 2 be fulfilled. Let $\{X_n\}_{n \in \mathbb{N}}$ be a strictly stationary sequence of nonnegative rvs, jointly regularly varying with index $\alpha \in (0, 2)$, and whose tail process $^1 (Y_n)_{n \in \mathbb{Z}}$ almost surely has no two values of opposite sign, $\{a_n\}$ be a sequence of positive real numbers such that $\lim_{n \rightarrow \infty} n P\{|X_1| > a_n\} = 1$ holds. Conditions 1 and 2 are supposed if $\alpha \in (0, 2)$ holds. If $\alpha \in [1, 2)$ holds Condition 3 is also required. Then taking $j_0(n_\varepsilon) = [nt]$, we get for all t in a dense subset of $[0, 1]$ including 0 and 1 and any $x > 0$*

$$\lim_{n \rightarrow \infty} \frac{P\{S_{T_2(x_{\rho_n})} - d > x a_n\} - P\{V(t) + b > x\}}{\psi(n)} \leq 1$$

holds, where for $\alpha \in (0, 1)$ $d = 0$, $t = 1$ and $b = (p - r)\alpha/(1 - \alpha)$, and for $\alpha \in [1, 2)$ $d = [nt]E(X_1 \mathbf{1}\{|X_1| \leq a_n\})$ and $b = 0$ and,

$$\psi(n) = \theta^2 q_n (1 - q_n)^{\theta [nt]} / (1 - (1 - q_n)^\theta).$$

¹The existence of the process $(Y_n)_{n \in \mathbb{Z}}$ with $P\{|Y_0| > y\} = y^{-\alpha}$ for $y \geq 1$ such that as $x \rightarrow \infty$, $((x^{-1} X_n)_{n \in \mathbb{Z}} | |X_0| > x) \rightarrow^{fidi} (Y_n)_{n \in \mathbb{Z}}$, where 'fidi' denotes convergence of finite-dimensional distributions, is equivalent with the property of regular variation with tail index α for the sequence (X_n) , see Theorem 2.1 in Basrak and Segers (2009).

Since $1 - q_n \sim \tau/n$ the statement of Theorem 3 provides the rate of convergence $n^{-\theta[nt]}$. The condition (36) in Theorem 3 (Markovich 2014) is not required since from Eq. 16 the series $\sum_{j=j_0(n_\varepsilon)+1}^{\infty} P\{T_2(x_{\rho_n}) = j\}$ converges uniformly by all $n > n_\varepsilon$ by Weierstrass' theorem.

Proof The proof is the same as in Theorem 3 (Markovich 2014) apart of the evaluation of $P\{T_2(x_{\rho_n}) > [nt]\}$ which follows from the changing in Theorem 2. From Eq. 16, we get

$$\begin{aligned} P\{T_2(x_{\rho_n}) > [nt]\} &= \sum_{j=j_0(n_\varepsilon)+1}^{\infty} P\{T_2(x_{\rho_n}) = j\} \\ &< \frac{1+\varepsilon}{d_n} \sum_{j=j_0(n_\varepsilon)+1}^{\infty} \chi_n (1-\chi_n)^{j-1} = (1+\varepsilon)\psi(n), \\ P\{T_2(x_{\rho_n}) > [nt]\} &> (1-\varepsilon)\psi(n), \end{aligned}$$

where $\psi(n)$ is determined as in the statement of the theorem.

Since $V(1)$ is stable distributed (see Theorem 3.1 in Davis and Hsing 1995) we can rewrite (61) in Markovich (2014) replacing t by 1. Hence, the result of the theorem follows. $\square$

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