Advanced Fresnel X-ray telescopes for spectroscopic imaging

Christoph Braig · Peter Predehl

Received: 27 March 2008 / Accepted: 20 October 2009 / Published online: 19 November 2009
© The Author(s) 2009. This article is published with open access at Springerlink.com

Abstract We present an effective achromatic imaging technique for diffraction-limited X-ray telescopes. For a common focal length, independent optical sections of the aperture are dedicated to two or more spectral bands and optimized with respect to their efficiency. For this purpose, we introduce the “achromatic gain” of an X-ray hybrid lens. Large-scale segmented and nested apertures provide a promising implementation of that scheme. An optimized numerical example and astrophysical simulations prove its capabilities for an energy range between 6 and 16 keV.

Keywords Fresnel lenses · X-ray optics · Instrumentation · High resolution imaging techniques

1 Introduction

Fresnel lenses and their derivatives have been suggested for next generation X-ray telescopes [1, 2]. Being inexpensive and lightweight, diffraction limited optics with an angular resolution down to 10^{-3} or even 10^{-6} arcsec may be manufactured from low-Z lens materials like beryllium (Be) or even ordinary plastics. Regarding Abbe’s law, aperture diameters of a few decimeters would

C. Braig (✉)
Friedrich-Schiller-Universität, 07745 Jena, Germany
e-mail: christoph.braig@uni-jena.de

P. Predehl
Max-Planck-Institut für Extraterrestrische Physik, 85748 Garching, Germany

Table 1 Total transmission $\mathcal{T}_1(s)$ of achromatic hybrid lenses for $1 \leq s \leq 7$

s	1	2	3	4	5	6	7
$\mathcal{T}_1(s)$	78.7%	63.2%	51.8%	43.2%	36.7%	31.7%	27.7%

The absorption of the diffractive component is neglected

be sufficient beyond photon energies of about 1 keV. Preliminary calculations [1, 3] were followed up by novel schemes [4] for hard-X and Gamma-ray applications. Recent papers analyzed specific aspects [5] and proposed an optimized design for simple transmissive X-ray telescopes [6].

However, little success has been made in the improvement of the dispersion limited spectral bandwidth. Even the bandwidth of achromatic doublets is restricted to few 10^2 eV below 20 keV. For scientific purposes, multi-band telescopes that cover two or more energy intervals simultaneously are of great interest.

In this paper a possible improvement is discussed. We review fundamentals of diffractive-refractive X-ray lenses and outline principles of segmented large-scale apertures. Such optics provide all conditions for the development of nested two-band achromatic telescopes. We introduce the “achromatic gain” as the central concept of such common-focus instruments. An optimized configuration is designed, briefly analyzed with respect to material insufficiencies and applied to an astrophysical target. The simulation, performed on the central region of a galaxy, demonstrates the usability of our scheme.

Due to the large quantity of symbols within this work, we overview the most important definitions in Table 1 at the end of Section 2.

2 Hybrid Fresnel lenses

Diffractive lenses may be regarded as circular diffraction gratings. For an aperture radius R and a photon energy $E = hc/\lambda$, the focal length F_Z is given as:

$$F_Z = R^2 N^{-1} E (hc)^{-1}, \quad (1)$$

where h and c denote Planck’s constant and the vacuum velocity of light. The geometrical zone number $N \gg 1$ counts the number of phase shifts $\Delta\varphi = \pi$ under which the aperture is seen from the focal point. Binary zone plates (ZP) [7] and even phase ZPs [8] distribute main fractions of the incoming flux to diffraction orders different from the first. Only 10% in case of binary ZPs or 41% for phase ZPs can be used for imaging. Fresnel lenses, i.e. blazed circular diffraction gratings, provide an efficiency up to 100%, with the exception of absorption losses.

2.1 Fresnel diffraction in the X-ray regime

The field amplitude $\mathcal{A}(E)$ on the optical axis in a distance z from the Fresnel lens plane can be obtained from the incoming amplitude, which is transformed by the Fresnel lens to $\mathcal{A}_0(r)$ and directed to the focal plane [9]:

$$\mathcal{A}(E) = i \frac{k}{z} \int_0^R \mathcal{A}_0(r) \exp\left(-i \frac{k}{2z} r^2\right) r dr. \quad (2)$$

In (2), the wave number is given as $k \equiv 2\pi/\lambda$ and the transformed amplitude can be written as $\mathcal{A}_0(r) = \exp(-ik(n-1)t_Z(r))$ for an index of refraction $n = 1 - \delta - i\beta$. We neglect absorption losses for pure diffractive lenses, i.e. $\beta = 0$. The stepwise Fresnel profile function for a blaze energy E_c reduces the lens thickness by integer multiples of the 2π -thickness every 2nd Fresnel zone:

$$t_Z(r) = \frac{r^2}{2F_Z\delta(E_c)} \bmod t_{2\pi}, \quad t_{2\pi} \equiv \frac{hc}{E_c\delta(E_c)}. \quad (3)$$

The real contribution δ to the refractive index follows a normal dispersion $\propto E^{-2}$. Generalized to normalized energies $\psi \equiv E/E_c$, the numerical value $\mathcal{A}(\psi)$ of the axial field amplitude is obtained from the phase shift across all Fresnel rings. The radius r is transformed via $r \equiv \tau R$ with $0 \leq \tau \leq 1$:

$$\mathcal{A}(\psi) = \mathcal{C} \sum_{n=1}^{N/2} \exp\left(2\pi i \frac{1-n}{\psi}\right) \int_{\tau_n}^{\tau_{n+1}} \mathcal{F}_0(\psi, \tau) \tau d\tau. \quad (4)$$

The function $\mathcal{F}_0(\psi, \tau) \equiv \exp(-i\pi N(m - \psi^{-1})\tau^2)$ contains the diffraction order m , which is introduced via

$$F_Z^{(m)}(\psi) = m^{-1} \psi F_Z(E_c), \quad (5)$$

according to (1). The proportionality constant in (4) is defined as $\mathcal{C} \equiv 2\pi i m N$. Since one Fresnel ring contains two (binary) Fresnel zones, the phase shift must be integrated for $\tau_n = \sqrt{2(n-1)/N}$, with $1 \leq n \leq N/2$. The total amplitude would be given as the coherent superposition from the contributions of all individual Fresnel rings. Wherever the condition $m = \psi^{-1}$ is fulfilled, incoming X-rays are regularly focused to an image point z on the optical axis at a normalized distance $\zeta \equiv z/F$, where $\zeta > 0$ and $F = F_Z^{(1)}(1)$ denotes the focus for $m = 1$ and an energy $\psi = 1$ (see Fig. 1). From (4), we obtain the efficiency $P_m(\psi)$,

$$P_m(\psi) = \left| \frac{\mathcal{A}(\psi)}{\pi m N} \right|^2 \quad \text{with} \quad \sum_{m=-\infty}^{\infty} P_m(\psi) = 1, \quad (6)$$

Fig. 1 The phase condition for blazed Fresnel X-ray lenses with normal dispersion $\delta \propto E^{-2}$. The deflected wave fronts are shown for one Fresnel ring and the first three diffraction orders (exaggerated drawing)

since the point spread function (PSF) in the plane ζ always shows an Airy shape whose peak intensity scales with $(\pi m N)^2$ whereas the spot size $\varnothing_{PSF}$ decreases with $(\pi m N)^{-1}$. So we get the fraction $P_m(\psi)$ of the incoming flux

Fig. 2 Focusing efficiency $P_m(\psi)$ of diffractive X-ray lenses without absorption. For blazed Fresnel lenses the first three diffraction orders are shown according to (7)

which is found in the diffraction order m from (6). For all diffraction orders $m \geq 1$, this focusing efficiency $P_m(\psi)$ follows the “sinc” function [5]:

$$P_m(\psi) = \left(\frac{\sin(\pi \mathcal{H}_m(\psi))}{\pi \mathcal{H}_m(\psi)} \right)^2, \quad \mathcal{H}_m(\psi) \equiv m - \frac{1}{\psi}. \quad (7)$$

Aside from integer fractions with $\psi = m^{-1}$, the blaze condition is not fulfilled for the Fresnel grooves, and the X-rays are distributed to several diffraction orders, as shown in Fig. 2.

2.2 Achromatic hybrid optics

Dispersion corrected achromatic X-ray systems are made of closely spaced diffractive (Z) and divergent refractive (L) devices whose focal lengths are related as $F_L = -2F_Z$ [2]. This setup is sketched in Fig. 3. Within the thin lens approximation, the focal lengths can be added via $F^{-1} = F_Z^{-1} + F_L^{-1}$ and we obtain:

$$F_m(\psi) = 2F_Z (2m\psi^{-1} - \psi^{-2})^{-1} \quad (8)$$

for the diffraction order m . In Fig. 4, this function is compared to the linear dispersion of a pure diffractive Fresnel lens. Within the diffraction limited focal depth of field (DOF) [5, 7], the spectral width is enlarged from $\Delta E = E/N$ for pure diffractive optics to $\Delta E = 2N^{-1/2}E$ for achromatic lenses. Since massive lens components cause absorption—absorption losses of the very thin diffractive Fresnel lens component are neglected within this work—the geometrical zone number N of the (diffractive) lens should be compared to an intrinsic parameter of the material, the “critical zone number” N_0 . As defined

Fig. 3 Achromatic hybrid X-ray lens, as suggested by Skinner [2] and Wang et al. [10]. An additional support structure will be neglected within this work. The focal plane of our telescope contains several 10^2 resolution elements in diameter

Fig. 4 Diffractive (dotted) and achromatic (solid) normalized focal length dispersion ζ as a function of ψ for the first three diffraction orders. For blazed Fresnel lenses, the X-ray light is completely concentrated to only one of these orders $1 \leq m \leq 3$ wherever $\mathcal{H}_m(\psi) = 0$

by Yang [9], we set $N_0 \equiv \delta / (2\pi\beta)$, for an imaginary part β of the refractive index n . The total photon transmission may be calculated by integration of the punctual transmission over the lens radius,

$$\mathcal{T}(s) = \frac{2}{R^2} \int_0^R \exp\left(-\frac{4\pi}{\lambda} \beta t_L(r)\right) r dr. \quad (9)$$

In (9), the radial profile of the convex spherical correction lens is given as $t_L(r) = (R^2 - r^2) / (2\xi)$, where we used the lens curvature $\xi \equiv F \delta(E_c)$. As for the diffractive (Fresnel) component we assume an ordinary dispersion $\delta(E) \propto E^{-2}$ for the refractive device, too. With the definition of $s \equiv N/N_0$, (9) yields

$$\mathcal{T}(s) = 2 \int_0^1 \exp\left(-\frac{1}{2} \psi^{-1} s (1 - \tau^2)\right) \tau d\tau. \quad (10)$$

Since the achromatic hybrid lens produces properly focused images only for energies $\psi^{-1} = m$, the result may be expressed in parametrical dependence on the diffraction order m . Apart from that, the total transmission is an exclusive function of the zone ratio s :

$$\mathcal{T}_m(s) = 2m^{-1}s^{-1}(1 - \exp(-ms/2)). \quad (11)$$

The product $ms \propto mN$ can be interpreted with respect to the Fresnel lens: In case of higher orders with $m \geq 1$, the zone number N must be replaced by mN , which counts the effective number of phase shifts $\Delta\phi = \pi$ over the lens radius (see Fig. 1). Table 2 provides data on the hybrid lens transmission for the first diffraction order.

3 Segmented apertures

In case of large apertures with diameters beyond ~ 2 m, it is useful to replace common compact lenses by segmented optics. In the following, we review and extend the detailed considerations of [5, 6].

Table 2 Global symbols and definitions used within this work

Symbol	Description	Usage
R	Outer lens radius	
r_{obs}	Radius of central obstruction	$r_{\text{obs}} \leq R$
a	Fractional lens obstruction	$a \equiv r_{\text{obs}}/R$
T_N	#(segment rings)	
k	Index of segment rings	$1 \leq k \leq T_N$
η_N	Segmented conversion	$\eta_N(a) \lesssim 3$
E_c	Fresnel blaze energy	
ψ	Relative energy	$\psi \equiv E/E_c$
ΔE	Absolute spectral width	$[\Delta E] = \text{eV}$
F	Focal length at energy E_c	
ζ	Fractional focal distance	$\zeta \equiv z/F$
m	Diffraction order	$m \geq 0$
N	#(geometrical Fresnel zones)	$R^2 = N\lambda F$
$N_\star$	#(Fresnel zones) per segment	$N = T_N N_\star$
N_0	Critical zone number	$N_0 = \delta/(2\pi\beta)$
s	Zone ratio	$s \equiv N_\star/N_0$
E_0	Basic multi-band energy	
ψ_0	Relative multi-band energy	$\psi_0 \equiv E/E_0$
$E_{p,q}$	Multi-band blaze energies	$E_{p,q} \propto E_0$
$r_{\text{lens}}^{(i,o)}$	Inner, outer partial lens radius	$r_{\text{lens}}^{(i)} < r_{\text{lens}}^{(o)}$
$\varnothing_{\text{FOV}}$	Detector diameter	$\varnothing_{\text{FOV}} = 2r_{\text{FOV}}$
n_{FOV}	#(resol. elements) in FOV	$n_{\text{FOV}} \propto \varnothing_{\text{FOV}}$
$\varnothing_{\text{PSF}}$	Focal spot size (HEW)	$[\varnothing_{\text{PSF}}] = \text{mm}$
$\Delta\epsilon$	Angular resolution	$\Delta\epsilon = \varnothing_{\text{PSF}}/F$
w	#(coh. steps) within segment	
$\mathcal{T}$	Mean lens transmission	$\mathcal{T} = \mathcal{T}_{(w)}(s)$
$\mathcal{G}$	Achromatic gain	$\mathcal{G} = \mathcal{G}(N, N_0)$
A_{eff}	Effective lens area	$A_{\text{eff}} = \pi R^2 \mathcal{T}(s)$

Their usage is given for most common applications. The notation [...] describes the dimension unit of a symbol

3.1 Conventional segmentation

Convenient segmented apertures are divided into T_N rings of constant zone numbers $N_\star = N/T_N$, where N denotes the total Fresnel zone number. Each of those rings, counted by $1 \leq k \leq T_N$, contains an increasing number of segments, so that their shapes approximate squares for sufficiently large $k \gg 1$ (see upper left image in Fig. 5). In order to minimize absorption losses, the radial thickness profile of an ordinary, convex refractive lens component should be reduced periodically with respect to subsequent segment rings. The resulting radial cross section of such conventional segmented apertures is sketched below the lens shape in Fig. 5. Our suggested “sandwich design” contains the diffractive lens within two symmetric refractive lens components. The maximum thickness of the lens is reduced as follows:

$$\Delta t_{\text{max}} = \frac{N_\star}{4} \frac{hc}{E_c \delta(E_c)}, \quad (12)$$

and the overall transmission is still given by (11) for the modified parameter $s \equiv N_\star/N_0$.

Fig. 5 Geometrical structure and spectral properties of an obstructed, segmented aperture for conventional (left) and coherently stepped versions (right). This example assumes $T_N = 7$ segment rings, from which $k_{obs} = 3$ are obstructed. The coherently stepped design is shown for $w = 3$ steps within one segment. Axial symmetry is presumed, $0 \leq \varphi < 2\pi$

As it was calculated elsewhere [6], the detectable spectral bandwidth within the DOF is given as

$$\Delta E = 2N_{\star}^{-1/2} E, \quad (13)$$

in analogy to compact achromatic hybrid lenses. The spectral band from (13) is also illustrated on the left in Fig. 5. Within this energy range, the PSF of the segmented lens may be recorded with an accuracy near the diffraction-limit which is constrained by the segment size and the non-uniform gray-wedge transmission due to the refractive lens profile.

3.2 Segmentation with coherent stepping

For some instrumental applications, e.g. large apertures or short focal lengths, large zone numbers N are desirable whereas reasonable segment sizes should still ensure a sufficient spatial and angular resolution. As a consequence, high s -ratios, again defined as $N_{\star}/N_0$, with radial transmission gradients close to 100% within individual segments are required. Wherever such technical constraints cause significant absorption losses for $s \gg 1$, an *additional* “coherent stepping” of the refractive component within each segment may conserve the focal spot size and an acceptable transmission as well. This advanced scheme is compared to the conventional case on the right in Fig. 5. We denote the number of steps or “teeth” within one segment by an integer w . Each of these

coherent steps contains N_*/w Fresnel zones and the maximum thickness is reduced to:

$$\frac{\Delta t_{\max}}{w} = \frac{N_*}{4w} \frac{hc}{E_c \delta(E_c)}, \quad (14)$$

with respect to the conventional segmentation scheme. In extension of (11), the total transmission should be written as:

$$T_m^{(w)}(s) = 2m^{-1}ws^{-1} \left(1 - \exp(-mw^{-1}s/2)\right). \quad (15)$$

In other words, the s -ratio N_*/N_0 is reduced by the factor w to an “effective” value s/w .

Since all refractive steps within one segment now act coherently, interference effects lead to an oscillating spectral response in the focal plane [2]. The diffraction-limited PSF is only found for certain energies:

$$\psi_n = 1 \pm 4n \frac{w}{N_*} \quad \text{with} \quad n \geq 0, \quad (16)$$

where $w \geq 2$ determines the number of spectral spikes and their constant spacing $4w/N_*$. The resulting comb of energies spreads over a total bandwidth which is enlarged by a factor of $\sqrt{w}$ with respect to the conventional version ($w = 1$). The *net* bandwidth which effectively contributes to the luminous power is therefore given as:

$$\Delta E = \sqrt{w} \frac{1}{2w} \frac{2E}{\sqrt{N_*}} = \frac{E}{\sqrt{wN_*}}. \quad (17)$$

An example is shown on the bottom in Fig. 5. Such comb structures may be properly detected with a spectral resolution $\Delta E_{\text{comb}} = 2E/N_*$. Special detection techniques beyond conventional CCDs are required for that purpose. Promising candidates include transition edge sensors (TES) and superconducting tunnel junction (STJ) devices and will be discussed elsewhere (Braig and Predehl, unpublished manuscript).

3.3 Angular resolution of segmented lenses

The spatial and angular resolution provided by an incoherently assembled aperture—with or without coherent stepping—is governed by the “mean”

Table 3 Conversion for segmented apertures

a	0	0.1	0.2	0.4	0.6	0.8	1.0
η_N	2.27	2.30	2.35	2.55	2.86	3.19	3.55

The geometrical correction η_N depends on the central obstruction a

Fig. 6 Fourier transformation (right) of almost squared shaped and rotated segments (left). That example assumes $s = 30$, $w = 3$

segment size, in contrast to common lenses, whose diffraction-limited angular resolution scales with the reciprocal lens radius R^{-1} : Since all single segments act independently, an optical coherence is exclusively maintained therein.

The actual angular resolution depends not only on the—slightly varying—size of the lens segments but also on the radial transmission gradient therein. The contribution of such an achromatic prism-like device to the focal point spread function coincides with the squared Fourier transform of its amplitude transmission. The latter depends on the radial coordinate r within the lens according to Fig. 5. As it was shown in detail elsewhere [5, 6], the diffraction pattern in the focal plane can be approximated assuming almost squared shaped segments for ring numbers $k \gtrsim 3$, averaged along $0 \leq \varphi < 2\pi$.

In summary, the focal spot of conventional as well as coherently stepped segmented apertures is a function of the effective geometrical segment size and the non-uniform absorption due to the refractive lens profile. Since both features affect the PSF independently, their contributions can be factorized.

In a first approach, we neglect any absorption and restrict to the geometrical contribution caused by segmentation. Previous investigations [5] proved a linearity of the incoherent angular resolution $\Delta\epsilon_{\text{inc}}$ in the ring number T_N , modified by an empirical correction $\eta_N(a)$. This parameter stands for radial decreasing segment sizes and obstructions $0 \leq a \leq 1$, as listed in Table 3. With respect to the well known Abbe limit $\Delta\epsilon_{\text{coh}}$ of compact apertures with the same radius, we obtain

$$\Delta\epsilon_{\text{inc}} \approx \eta_N(a) T_N \Delta\epsilon_{\text{coh}}, \quad (18)$$

where $\Delta\epsilon_{\text{coh}} = \alpha \lambda R^{-1}$ describes the angular resolution of the coherent analogue. The HEW is associated with $\alpha = 0.535$.

Table 4 Relative angular resolution for segmented hybrid optics without reduction ($w = 1$) for various s -ratios

s	1	2	3	4	5	6	7	8	9
Q_1	1.00	1.01	1.03	1.06	1.09	1.12	1.17	1.21	1.27

Table 5 Relative angular resolution for coherently stepped segments, based on absorption

s	5	10	15	20	25	30	35	40	45	50
Q_5	1.00	1.02	1.04	1.06	1.10	1.15	1.21	1.28	1.36	1.48
Q_{10}	1.00	1.00	1.01	1.02	1.03	1.04	1.05	1.07	1.08	1.10

Deviations $Q_w(s)$ less than 2×10^{-2} are neglected

Next we take the absorption of the refractive lens component into account. In general, the HEW will suffer from the non-uniform, i.e. periodical gray-wedge transmission. The relative spatial resolution $Q_w(s)$ of an arbitrary, squared-shaped segment should be calculated using a two-dimensional Fourier transformation of the amplitude transmission function [6]. The diffraction pattern is obtained from rotated and superposed segments, as shown in Fig. 6. The factor $Q_w(s)$ describes how much the focal spot size (HEW) is enlarged by absorption with respect to a fully transparent segment of the same size and must be calculated numerically for all relevant s -ratios and w -numbers. Some data are listed in Table 4 for segments without reduction ($w = 1$) and Table 5 for coherently stepped versions ($w \geq 5$). Obviously, coherent stepping becomes useful in particular for s -ratios beyond ~ 10 ; strong reductions $w \gtrsim 5$ conserve the resolution of the ideal transparent segment to a far extent. Using $Q_w(s)$, the focal spot size of arbitrary segmented hybrid lenses can be estimated. Finally, we write the resulting equation in the form:

$$\Delta\epsilon_{\text{inc}} \approx \eta_N(a) T_N Q_w(s) \Delta\epsilon_{\text{coh}}. \quad (19)$$

Compared to an extremely time-consuming accurate numerical calculation, which means the direct Fourier transform of the complete segmented aperture, the factorized approximation from (19) works well within an error range of 10^{-2} .

It should be noted that “segmentation” does not necessarily mean an *incoherent* superposition of the focal spots provided by all lens segments—if accurately aligned, the phase condition would be preserved over the whole aperture and the Abbe limit $\Delta\epsilon_{\text{coh}}$ is still valid. However, this fine adjustment on a scale of microns will cause substantial problems in space and is therefore considered to be practically impossible.

4 Dual band apertures

Many astronomical investigations benefit from multi-wavelength observations. For instance, insights into conversion mechanisms of AGN or X-ray binaries may arise from hardness ratios. Simple achromatic two-band telescopes for such applications use the common-focus concept mentioned above. Its theory is based on the one developed for purely diffractive apertures [5]. We will repeat the main results in short.

A Fresnel lens blazed on an energy E_p focuses mistuned X-rays with an energy $E_q \neq E_p$ to other, often numerous, diffraction orders, according to (7). The fractional achromatic focal length dispersion for $\zeta_{\pm} \equiv z/F$, where z denotes the focus of the scattered light,

$$\zeta_{\pm} = (2m\psi^{-1} - \psi^{-2})^{-1}, \quad (20)$$

distinguishes “red” ($\zeta_- < 1$) from “blue” ($\zeta_+ > 1$) focal positions, as it follows from (8). The geometry is shown in Fig. 7. In the focal plane, those defocused X-rays evoke ring-like halos around the detector. In order to protect the active detector field from any scattered X-rays, the aperture must be divided into partial ring lenses, each of them dedicated to one of the desired energy bands. We consider an obstructed lens with an outer radius $r_{\text{lens}}^{(o)}$:

$$r_{\text{lens}}^{(i)} = ar_{\text{lens}}^{(o)}, \quad \text{with } 0 \leq a \leq 1 \quad (21)$$

defines the inner radius of that lens partition. Simple geometric formula, in particular the theorem on intersecting lines, prove the condition:

$$ar_{\text{lens}}^{(o)} \geq \mathcal{S}(\zeta_{\pm})r_{\text{FOV}} \quad , \quad 2r_{\text{FOV}} = n_{\text{FOV}}\varnothing_{\text{PSF}} \quad (22)$$

for n_{FOV} resolution elements that may be detected across an “unsoiled” field of view, measured in units of the circular detector size $2r_{\text{FOV}}$. The scaling factor:

$$\mathcal{S}(\zeta_{\pm}) \equiv \frac{r_{\pm}}{r_{\text{FOV}}} = \pm 2(1 - \zeta_{\pm}^{-1})^{-1} \quad (23)$$

depends on the fractional focal length $\zeta \equiv z/F$, where the signs $\pm$ are used as above. The lens radii $r_{\pm}$ correspond to $r_{\text{lens}}^{(i)}$ from (21).

Compared to simple diffractive Fresnel objectives, the stray light behavior of achromatic hybrid lenses is more complicated, as it follows from their focal length dispersion in (20). For a basic energy E_0 , we obtain:

$$\mathcal{S}_m(\psi_0) = \pm 2(1 - \psi_0^{-1}(2m - \psi_0^{-1}))^{-1}, \quad (24)$$

with $\psi_0 \equiv E/E_0$ and the diffraction order $m \in \mathbb{Z}$.

Fig. 7 Geometry and notation for detuned X-rays in dual-band telescopes. In general, different detector diameters are assumed for the soft and hard band, respectively

5 The achromatic gain

In Fig. 4 the normalized focal lengths of Fresnel lenses are shown for the first three diffraction orders.

5.1 Definition for hybrid lenses

However, this linear dispersion limits the diffractive spectral bandwidth $\Delta E = E/N$, which is related to the DOF [5, 7]. Accordingly, the—photon limited—luminous power, defined as the product of the effective area and the spectral width:

$$(A_{\text{eff}} \times \Delta E)_{\text{dif}} = P_m(\psi) \pi h c F_Z^{(m)} \quad (25)$$

depends on the focal length F_Z .

The luminous power of an achromatic lens suffers from the serious absorption in the refractive lens device. In particular, we have:

$$(A_{\text{eff}} \times \Delta E)_{\text{hyb}} = \pi R^2 P_m(\psi) \mathcal{T}_m(s) \frac{2E}{\sqrt{N_{(\star)}}}, \quad (26)$$

where the radius is given as $R^2 = (N/2)\lambda F$, since the achromatic focal length is twice as large as the diffractive one. Within this section, we use the index $(\star)$ for the zone number $N_{(\star)}$ wherever the formula is valid for compact and segmented hybrid lenses as well. Using (25) and (26), we define the achromatic gain $\mathcal{G}(N_{(\star)}, N_0)$ that relates the luminous power of the hybrid lens to the one of the solely diffractive lens:

$$\mathcal{G}(N_{(\star)}, N_0) \equiv \frac{(A_{\text{eff}} \times \Delta E)_{\text{hyb}}}{(A_{\text{eff}} \times \Delta E)_{\text{dif}}} = 2\sqrt{N_{(\star)}} \mathcal{T}_m(s). \quad (27)$$

In (27), we introduced the quantity $\mathcal{G}(N_{(\star)}, N_0)$ in order to compare the luminous power of two transmissive X-ray lenses which are based on the *same diffractive component*. An alternative definition would relate the luminous powers of achromatic hybrid lenses to their diffractive analogues with *equal focal lengths*. Though both options share advantages and drawbacks, we chose an experimental or technical approach: *Existing*, i.e. already produced Fresnel lenses can be easily corrected for chromatic focal length dispersion by *additional* refractive optics. In contrast, the detailed groove profile of real Fresnel lenses would always get the fingerprint of the difficult production process – and different Fresnel components would always be involved in case of an alternative definition based on equal focal lengths.

Fig. 8 Achromatic gain for different critical zone numbers $400 \leq N_0 \leq 1400$. The straight dotted line indicates the optimized arrangement with $N_{(*)} \approx 2.5N_0$

5.2 Optimization of common-focus instruments

The achromatic gain as it was introduced above is clearly independent of the Fresnel lens efficiency $P_m(\psi)$. We may use (27) and write the luminous power of the hybrid lens with $m = \psi^{-1}$ as:

$$(A_{\text{eff}} \times \Delta E)_{\text{hyb}} = \pi \frac{hc}{2} \mathcal{G}(N_{(*)}, N_0) F. \quad (28)$$

The subscript “hyb” will be neglected from now on. Interestingly, the luminous power depends on two quantities or “degrees of freedom” alone, namely $\mathcal{G}(N_{(*)}, N_0)$ and the focal length F . The first one describes an internal feature of each individual hybrid lens. The second one is in common with other partial telescopes.

In (27) the spectral bandwidth and the absorption counteract each other as a function of $N_{(*)}$, and an optimized achromatic gain is obtained for the condition $\partial_{N_{(*)}} \mathcal{G}(N_{(*)}, N_0) = 0$. The result of the partial differentiation depends on N_0 :

$$\mathcal{G}_{\text{opt}}(N_{(*)}, N_0) \approx 1.8\sqrt{N_0} \quad \text{at} \quad N_{(*)} \approx 2.5N_0. \quad (29)$$

In Fig. 8 the achromatic gains and its optimal values are illustrated for some typical values of N_0 . Since the optical performance of low-Z materials like Li and Be peaks between 10 and 20 keV with $N_0 \sim 10^3$, the luminous power

Table 6 Zone numbers of Li and Be optics for a given focal length

E(keV)	4	6	8	10	12	14	16	18	20
$N_{\text{opt}}^{(\text{Li})}$	690	1582	2430	2898	3010	2888	2688	2480	2278
$N_{\text{opt}}^{(\text{Be})}$	218	516	912	1326	1668	1884	1982	1994	1946

The data optimize $A_{\text{eff}} \times \Delta E$

Fig. 9 Luminous intensity of achromatic X-ray lenses in the diffraction-limited operation mode for various gains $\mathcal{G}$ ($N_{(*)}$, N_0). The line for $\mathcal{G} = 16$ is a guide to the eye on the logarithmic scale. The lower limit with $\mathcal{G}_{\min} = 4$ refers to an acceptable minimum (see text)

of an achromatic lens may be more than 50 times higher than for the solely diffractive one. For a given energy, the material with the largest N_0 should be chosen. The following list overviews the maximum values $\mathcal{G}_{\max}$ for the lightest elements:

H_2 @ 2.90	keV : $\mathcal{G}_{\max} = 135$	for $N_0 = 5615$
He @ 6.90	keV : $\mathcal{G}_{\max} = 84.0$	for $N_0 = 2164$
Li @ 11.7	keV : $\mathcal{G}_{\max} = 62.5$	for $N_0 = 1199$
Be @ 17.3	keV : $\mathcal{G}_{\max} = 50.9$	for $N_0 = 795$

In the soft X-ray range of a few keV, liquid elements like H_2 and He provide the best performance but may hardly be exploited, at least in astronomical applications. Realistic X-ray optics might profit from solid-state materials like Li and Be, at least if surface and bulk impurities are handled properly [6].

Within this work we assume crystals without impurities [6] and, as far as possible, zone numbers according to (29) as listed in Table 6. Telescopes with these optimized zone numbers would obtain maximum achromatic gain. Special applications might need minimized focal spots, however. A lower bound for the achromatic gain may be set around $\mathcal{G} = 4$ because the corresponding Fresnel lens with the half focal length $F_Z = F/2$ would reduce the focal spot size and the luminous power $A_{\text{eff}} \times \Delta E$ by the same factor, i.e. 50%, according to (25). Thus one could also take the solely diffractive analogue instead of an achromatic one. Luminous intensities for various gains are shown in Fig. 9.

6 An optimized design

Based on (22), several possible solutions for the lens radii within the nested aperture can be found. In general, compared to purely diffractive systems [5], the obstruction for the outer lens is enlarged by 50–100 % in case of an achromatic scaling factor. Following (24), the small ring lens for E_2 must be

Fig. 10 Optimized obstruction of nested two-band telescopes. The obstructions for the two lowest diffraction orders ($m = 1$ and $m = 2$) coincide for the condition $S_{m\pm}(\psi_0) = 2\psi_0$ (straight dotted line) at $\psi_0 \approx 2.62$

encircled by the large partial lens for the soft band at E_1 . So it is desirable to find an optimized arrangement that favors the soft X-ray band.

6.1 Lens parameters

We ask for the *smallest* radius $r_{\pm}$ that fulfills the condition (22) for all orders m , in order to avoid any stray light on the detector. The inner radius of the partial lens for E_0 will be minimized for:

$$S_{m\pm}(\psi_0) = 2\psi_0 \quad \rightarrow \quad \psi_0 \approx 2.62 \quad \text{for} \quad \begin{bmatrix} m_+ = 1 \\ m_- = 2 \end{bmatrix},$$

and we obtain $r_{\min}(E_0) \approx 5.24 r_{\text{FOV}}$. The condition $S_{m\pm}(\psi_0) = 2\psi_0$ is equivalent to the relation $\zeta_{\pm}/|1 - \zeta_{\pm}| = \psi_0$, which represents an equal focal ratio for “blue” ($\zeta_+ > 1$) and “red” ($\zeta_- < 1$) detuned X-rays with an energy $E_2 = 2.62 E_0$, scattered by the outer ring lens for $E_1 = E_0$. An illustration of this semi-analytical investigation is given in Fig. 10.

The obstruction of the hard band at E_2 defines the central hole of the aperture and is conservatively set to $1.0 r_{\text{FOV}}$, somewhat more than what the scaling factor $S_m(\psi_0)$ requires. We choose a set of energies between 1 and 20 keV. However, X-rays below 5 keV would be mainly absorbed. As a compromise, we take $E_1 = 6.0$ keV and $E_2 = 15.7$ keV and calculate the lens performance for the case of massive hybrid lens segments (on the left of Fig. 5) in the hard band and coherently stepped versions (Fig. 5, right) for E_1 . The technical data are listed in Table 7. Compared to large diffractive instruments [5], the achromatic gain allows for compact lens radii. In particular, the huge

Table 7 Technical details of the optimized nested two band telescope

Band	Energy	w	a	$N_{\star}$	$\mathcal{G}$	Mat.	T_N	$r_{\text{lens}}^{(o)}$
E_1	6.0 keV	4	0.57	4480	22.2	Li	15	1.39 m
E_2	15.7 keV	–	0.19	2750	49.5	Be	20	0.79 m

The model refers to Fig. 11. See Table 8 for performance data

Table 8 Performance parameters of the optimized two-band telescope design, according to Fig. 11

Parameter	Symb.	E_1 (6.00 keV)	E_2 (15.7 keV)
Resolution	$\Delta\epsilon$	0.67 mas	0.54 mas
Spot size	$\varnothing_{\text{PSF}}$	0.94 mm	0.75 mm
Field of view	$\varnothing_{\text{FOV}}$	39 cm	30 cm
Focal length	F	287 km	
$A_{\text{eff}} \times \Delta E$		1212 cm ² keV	5331 cm ² keV

geometric area of pure Fresnel objectives is compensated as shown in Table 7, with an almost optimized gain in the hard band.

Apart from the limited FOV, those achromatic dual-band arrangements also compete well with single band telescopes [6] with respect to all other parameters such as resolution, sensitivity and geometrical dimensions. However, the large aspect ratios up to ~ 2 must be regarded in particular since properly shaped refractive lens profiles are crucial for an imaging performance free from aberrations. Table 8 overviews the most important variables. Figure 11 sketches the general design of the telescope. The radial dimensions are compared, whereas the focal length is not to scale.

In order to obtain an optimized image quality in E_1 , the comb structure should be detected with a spectral resolution within:

$$4.4 \times 10^{-4} \leq \left(\frac{\Delta E_1}{E_1} \right)_{\text{det}} \leq 6.7 \times 10^{-4}. \quad (30)$$

Albeit currently developed micro calorimeters might match those requirements in the middle-term future for energies around $E_1 \sim 6$ keV, their application to hard X-rays beyond 10 keV seems questionable so far.

6.2 Stray light halos

The lateral position of a detector spacecraft should be fixed with respect to the optical axis in order to exclude all scattered X-rays. As it follows from

Fig. 11 Schematic view of the achromatic dual band telescope. Both ring lenses are segmented (simplified drawing). The outer halo in the focal plane is only shown in part

Fig. 12 Secure stray light protection of the detector plate. On the *left side*, the on-axis point source yields an axially symmetric halo. The halo of extremely displaced point sources on the edge of the detector field just touches but does not contaminate the detector (*right picture*)

the telescope design based on Fig. 7 and (22), the circular detector field for the energy band E_p is protected from any stray light photons in this case: The inner radius of the halo consisting of photons with an energy E_p measures at least twice the corresponding detector radius. Point sources which are imaged to an edge pixel of the circular detector field evoke an excentric but still not detected halo (Fig. 12). We estimate the size and intensity of the halos in the focal plane. The inner and outer radii $r_{\text{scat}}^{(i,o)}(E_q)$ of the halo (Fig. 12) are given as:

$$r_{\text{scat}}^{(i,o)}(E_q) = |1 - \zeta_{\pm}^{-1}| r_{\text{lens}}^{(i,o)}(E_p), \quad (31)$$

where $r_{\text{lens}}^{(i,o)}(E_p)$ describes the inner and outer radius of the partial hybrid lens for an energy E_p . In Fig. 13 the dimensions are illustrated in comparison to the detector plate. However, a misaligned detector spacecraft would also detect X-rays from the surrounding halos. The scattered diffraction efficiencies P_{scat} result from absorption losses in segments dedicated to $E_p \neq E_q$ and the focal dispersion. For an equal spectral width as in the regular detection, the luminous power in the halos is found as:

$$(A_{\text{eff}} \times \Delta E)(E_q) = \pi \Delta r_{\text{geo}}^2(E_p) P_{\text{scat}} \Delta E_q. \quad (32)$$

Fig. 13 Stray light halos for the two-band telescope design according to Fig. 11. The *left picture* illustrates photons with an energy around E_1 in the focal plane accidentally “imaged” by the inner partial lens for E_2 . On the right, photons with an energy around E_2 , scattered by the outer partial lens for E_1 , surround the detector field

Table 9 Data of the stray light halos from Fig. 13

Band	Energy	m	ζ_-	ζ_+	P_{scat}	$(A_{\text{eff}} \times \Delta E)_{\text{scat}}$
E_1	6 keV	2	0.28	–	1.31%	$1.1 \times 10^1 \text{ cm}^2 \text{ keV}$
E_2	16 keV	2,1	0.72	1.62	23.9%	$5.8 \times 10^3 \text{ cm}^2 \text{ keV}$

The scattering fraction P_{scat} refers to the scattered imaging efficiency in the corresponding energy band

The abbreviation $\Delta r_{\text{geo}}^2(E_p)$ denotes the squared radial distance $[r_{\text{lens}}^{(o)}(E_p)]^2 - [r_{\text{lens}}^{(i)}(E_p)]^2$ for the partial lens dedicated to E_p . In (32), the scattering efficiency P_{scat} can be calculated using (7), multiplied with the refractive lens transmission:

$$P_{\text{scat}}(E_q) = P_m(E_q/E_p) \times T_m(s). \quad (33)$$

In this special case, the “mistuned” s -ratio is given as

$$s = w_p^{-1}(N_*(E_p)/N_0(E_q)) E_p E_q^{-1}. \quad (34)$$

The effective “net” energy width ΔE_q is calculated for the regular spectral band around E_q :

$$\Delta E_1 = \frac{2E_1}{\sqrt{N_*(E_2)}} \quad \text{and} \quad \Delta E_2 = \frac{E_2}{2\sqrt{N_*(E_1)}}. \quad (35)$$

Table 9 summarizes the scattering data. Obviously, the soft band lens operates quite efficiently, since only $\sim 1\%$ of the incident flux is lost in the 2nd diffraction order. Significant but still acceptable fractions of the hard X-ray component are useless scattered on the other hand by the outer ring lens for E_1 .

7 Material issues

Though only very few empirical studies exist on the effect of elementary and other impurities on (X-ray) photons propagating through the lens medium, theoretical predictions clearly describe how photons are scattered by such inhomogeneities [11]. In this work we restrict to predominant absorption losses caused by atomic substitutions in the regular crystal lattice consisting of Li or Be, respectively.

For N types of atoms with concentrations c_n , $1 \leq n \leq N$ and scattering factors f_n , the averaged scattering factor $\bar{F}$ is defined as

$$\bar{F} \equiv \sum_{n=1}^N c_n f_n \quad \text{with} \quad \sum_{n=1}^N c_n = 1. \quad (36)$$

Since all substitutional atoms have their origin in an imperfect manufacturing process, we can assume randomized positions within the crystal lattice. In agreement with an incoherent superposition of the scattered elementary waves, the impurity atoms would enhance the background in the focal plane over $\text{Re}(\bar{\mathcal{F}})$ and reduce the lens transmission by absorption via $\text{Im}(\bar{\mathcal{F}})$ as well.

We use the data for a commercially available ultra-high purity sample [12] of Li around 6 keV [13]:

Li	:	$f = 3.00226 + 0.000651 i$
Na @	1.0×10^{-5}	: $f = 11.2067 + 0.217395 i$
Ca @	5.0×10^{-5}	: $f = 20.1061 + 2.165730 i$
Si @	5.0×10^{-6}	: $f = 14.3470 + 0.566843 i$
Fe @	5.0×10^{-6}	: $f = 24.3204 + 0.642017 i$
Al @	5.0×10^{-6}	: $f = 13.2986 + 0.420026 i$
Ni @	5.0×10^{-6}	: $f = 26.8777 + 0.883844 i$

In a similar way, the imperfect Beryllium crystal [14] may be analyzed for an energy of 15.7 keV [13]:

Be	:	$f = 4.00065 + 0.000329 i$
Cu @	5.0×10^{-6}	: $f = 29.2963 + 1.538240 i$
Pb @	3.0×10^{-6}	: $f = 75.0876 + 10.30430 i$
Cd @	2.0×10^{-6}	: $f = 47.3821 + 1.464520 i$
Zn @	2.5×10^{-5}	: $f = 30.2366 + 1.746820 i$
Ni @	7.5×10^{-6}	: $f = 28.3358 + 1.344800 i$
Co @	2.0×10^{-6}	: $f = 27.3538 + 1.156970 i$
Mn @	2.0×10^{-6}	: $f = 25.3674 + 0.880966 i$
Fe @	1.7×10^{-5}	: $f = 26.3687 + 1.019410 i$

The averaged scattering factors, which are slightly larger than the pure values, are found as

$$\bar{\mathcal{F}}_{\text{Li}} = 3.00362 + 7.74 \times 10^{-4} i, \quad (37)$$

$$\bar{\mathcal{F}}_{\text{Be}} = 4.00240 + 4.46 \times 10^{-4} i. \quad (38)$$

The real parts differ by less than 0.1 % from those of pure crystals, whereas deviations beyond 10 % in $\text{Im}(\bar{\mathcal{F}})$ indicate that the contamination with other atoms will predominantly affect the transmission by enhanced absorption rather than the PSF by scattering. Using (36), the parameter η , an often used measure for the optical quality of the material, can be written as

$$\eta(E) = \frac{\sum_j m_j f_j^{(2)}(E)}{\sum_j m_j f_j^{(1)}(E)} = \frac{\text{Im}(\bar{\mathcal{F}})}{\text{Re}(\bar{\mathcal{F}})}, \quad (39)$$

where $f_j^{(1,2)}$ refer to the energy dependent atomic scattering factor $f_j = f_j^{(1)} + if_j^{(2)}$. The statistical weight of an atomic type j is denoted by m_j . We find

$$\eta(6.00 \text{ keV}) = 2.58 \times 10^{-4} \quad \text{for Li} \quad (+18.85\%),$$

$$\eta(15.7 \text{ keV}) = 1.12 \times 10^{-4} \quad \text{for Be} \quad (+35.61\%).$$

Even for contaminations on the ppm level, a significant but still acceptable decline of the optical performance is observed. The relative data, compared to the values of perfect crystals, are listed in (+%). The usage of ultra-high purity crystals will thus be indispensable for practical implementations of such diffractive-refractive X-ray telescopes in the future.

8 Simulations

We demonstrate the astronomical capability of the presented single-focus dual band instrument. As mentioned at the beginning of Section 4, dual band telescopes might be applied to investigations of hardness ratios, probably the simplest form of imaging spectroscopy. It is convenient to assume an ordinary power law whose photon index may vary within $1.2 \lesssim \gamma \lesssim 1.8$.

8.1 X-ray emission from NGC 4594

In the following, we consider a typical example of AGN spectra, the “sombbrero” galaxy NGC 4594 (http://heawww.gsfc.nasa.gov/users/.../liner_llagn.html). As it was measured by ASCA, the spectral slope towards hard X-rays can be described by $\gamma \approx 1.70$ for core emission. Clearly, any synchrotron contributions from potential jets are included in the spectrum shown in Fig. 14. Beyond 10 keV, the power law may be extrapolated up to 20 keV.

Fig. 14 ASCA X-ray spectrum of the sombrero galaxy NGC 4594. The straight lines describe a power law with the slope $\gamma \approx 1.70$. Figure adopted from the GSFC website (http://heawww.gsfc.nasa.gov/users/.../liner_llagn.html)

The second soft component in Fig. 14 will not be considered within this work. For an energy-dependent luminous power $(A_{\text{eff}} \times \Delta E)(E_p)$, the partial photon count rate $\Phi(E_p)$ for an energy E_p can be written as:

$$\Phi(E_p) \approx \Phi_0 \left(\frac{E_0}{E_p} \right)^\gamma (A_{\text{eff}} \times \Delta E)(E_p), \quad (40)$$

where Φ_0 describes the flux density for the basic energy $E_0 = 6$ keV from Fig. 14. Obviously, the count rate in each energy band scales proportional to the effective area and spectral bandwidth of the corresponding ring lens for E_1 and E_2 , respectively. These luminous power data $(A_{\text{eff}} \times \Delta E)(E_p)$ are given in Table 8.

The flux density at 15.7 keV amounts to about 20% of the corresponding one at 6 keV. Since the telescope provides an almost inverse sensitivity, we expect similar count rates in both energy bands. Integrated to an exposure time Δt_{obs} of 10^6 s, photon numbers greater than 2×10^4 in the soft and hard band may be expected. In general, the target photons will be distributed among the number of pixels corresponding to the resolution of the X-ray source. However, in the case of NGC 4594, the central black hole has been estimated to $10^9 M_\odot$ [15]. The corresponding gravitational radius R_s of 2.9×10^{12} m implies an overall diameter of the accretion disk of at least 10^{15} – 10^{16} m, depending on the theoretical model. The angular resolution of $\lesssim 10^{-3}$ arcsec yields an estimated length scale of the same order, about 10^{15} m. Though the detailed structure of the accretion disk will hardly be resolved by our telescope, an optimal signal-to-noise ratio (SNR) may be expected for signal photons distributed over a few pixels. In particular, an assumed accretion disk which is extended to $10^3 R_s$ would correspond to an angular diameter of 4.2 mas.

Radio observations [15] with an angular resolution of 5×10^{-3} arcsec unearthed no extended X-ray emission aside from the central nucleus. Other investigations speculated on the existence of some directed outflows, e.g. jets [16]. If present on length scales in the order of 10^{-3} arcsec, they could be resolved to several pixels. Assuming dimensions of $A_{\text{pix}} = (395 \times 395) \mu\text{m}^2$, we expect an image similar to that presented in Fig. 15.

Fig. 15 Central region of NGC 4594, as it might be seen by the two-band telescope. The soft and hard bands are shown on the left and right, respectively

8.2 Signal-to-noise ratio

We estimate the SNR in a pixel numbered by (i, j) :

$$SNR_{ij}(E_p) = \frac{S_{ij}(E_p)}{\sqrt{S_{ij}(E_p) + B_{ij}(E_p)}} \quad (41)$$

for signal counts S_{ij} and background events B_{ij} . The total number of signal counts, integrated from (40) for an observation time of 10^6 s, amounts to 2.87×10^4 and 2.46×10^4 , respectively, summed up from both disk and jet contributions. We presume a hardness ratio of 2.5 for disk-to-jet conversion between the soft and the hard band emission. Thus we obtain the total number of signal counts in the disk (S_{disk}) and jets (S_{jets}), respectively (Table 10). In order to estimate the diffuse XRB, an overall power law of the form $n_b(E) = n_0 E^{-\Gamma}$, with $n_0 = (9.8 \pm 0.3) \text{ s}^{-1} \text{ cm}^{-2} \text{ keV}^{-1} \text{ sr}^{-1}$ and $\Gamma = 1.42 \pm 0.02$ was taken from Revnivtsev et al. [17]. The number of background counts is roughly given as

$$n_b(E_p) = \frac{1}{5} \pi n_0 E_p^{-\Gamma} \Delta t_{\text{obs}} A_{\text{pix}} \Delta E_p, \quad (42)$$

if the radiation is collimated in front of the detector to a solid angle of 10% of 2π . Following (17) and Table 7, the net achromatic bandwidth ΔE_p amounts to 90 eV at E_1 and 600 eV around E_2 . We find about 68 natural background counts per pixel within the assumed exposure time of 10^6 s for the soft band and 115 disturbing photons around 15.7 keV.

The SNR may be thus as good as 8.3 for 6 keV and 10 in the hard band, as it is confirmed by numerical simulations: For each pixel within the FOV, the background level is superimposed by the presumed original intensity distribution of the target. The result is convoluted with the discrete PSF and “randomized” by an artificial Poisson noise (see Fig. 15).

Obviously, almost nothing is known about the actual length and brightness of the synchrotron jets and the accretion disk might be 10 times smaller than assumed above. However, our simulations prove the feasibility of fruitful observations using two-band achromatic telescopes.

Table 10 Photon counts and covered number of pixels $\#(\text{pix})$ for the modeled disk-jet system in the core of NGC 4594

Target region	E_1 (6.0 keV)	E_2 (15.7 keV)
$S_{\text{disk}} / \#(\text{pix})$	$8.32 \times 10^3 / 105$	$1.75 \times 10^4 / 105$
$S_{\text{jets}} / \#(\text{pix})$	$2.04 \times 10^4 / 185$	$7.13 \times 10^3 / 185$

Following the data from Table 8, each pixel covers $(0.28 \times 0.28) \text{ mas}^2$ in the focal plane

9 Conclusion

An extension to single-band hybrid Fresnel X-ray telescopes for spectroscopic applications is developed. The detectable energy range may be doubled at least, using several partial instruments with a common focal length. An efficient design relies on segmented, nested ring lenses for the simultaneous imaging of two energy bands. The “achromatic gain” is introduced as an essential concept for optimized assemblies of that type. Central detector units minimize the natural as well as artificial background noise. The observational sensitivities of the soft and hard band are very suitable for measurements of hardness ratios for power-law emitting sources like AGN or X-ray binaries.

Further theoretical steps should enhance the luminous power and the number of energy bands. From a technological point of view, dedicated detectors for the spectral structure of stepped hybrid profiles are of great interest. Other challenges point to investigations of low- Z lens materials and an accurate control system for the positioning of lens and detector in space—problems that will be attended to forthcoming publications.

Acknowledgements This work was financially supported in part by the Heidenhain foundation, Traunreut (Germany). The authors gratefully recognize encouraging discussions with Joachim Trümper. Special thanks are dedicated to Karsten Jessen who carefully read the manuscript.

Open Access This article is distributed under the terms of the Creative Commons Attribution Noncommercial License which permits any noncommercial use, distribution, and reproduction in any medium, provided the original author(s) and source are credited.

References

1. Skinner, G.K., et al.: Fresnel lenses for X-ray and Gamma-ray astronomy. *Proc. SPIE* **5168**, 459 (2004)
2. Skinner, G.K., et al.: Diffractive-refractive optics for high energy astronomy—II. Variations on the theme. *Astron. Astrophys.* **383**, 352 (2002)
3. Gorenstein, P.: Role of diffractive and refractive optics in X-ray astronomy. *Proc. SPIE* **5168**, 411 (2004)
4. Skinner, G.K.: Design and imaging performance of achromatic diffractive-refractive X-ray and gamma-ray Fresnel lenses. *Appl. Opt.* **43**, 4845 (2004)
5. Braig, C., Predehl, P.: Large-scale diffractive X-ray telescopes. *Exp. Astron.* **21**, 101 (2006)
6. Braig, C., Predehl, P.: Efficient Fresnel X-ray optics made simple. *Appl. Opt.* **46**, 2586 (2007)
7. Michette, A.G.: *Optical Systems for Soft X-rays*. Plenum, New York (1986)
8. Kirz, J.: Phase zone plates for X-rays and the extreme UV. *JOSA* **64**(3), 301 (1974)
9. Yang, B.X.: Fresnel and refractive lenses for X-rays. *Nucl. Instrum. Methods A* **328**, 578 (1993)
10. Wang, Y., et al.: Achromatic Fresnel optics for wideband extreme-UV and X-ray imaging. *Nature* **424**, 50 (2003)
11. Guinier, A.: *X-ray Diffraction in Crystals, Imperfect Crystals and Amorphous Bodies*. Dover, New York (1994)
12. American Elements: American Elements homepage. Los Angeles, Calif. <http://www.americanelements.com> (2006)
13. Henke, B.L., et al.: X-ray interactions: photoabsorption, scattering, transmission, and reflection at $E = 50 - 30000$ eV, $Z = 1 - 92$. *At. Data Nucl. Data Tables* **54**, 181 (1993)

14. Goode, G.C., et al.: The determination of some impurities in high purity Beryllium by differential cathode-ray polarography. *Analyst* **91**, 719 (1966)
15. Pellegrini, S., et al.: An XMM-Newton and Chandra investigation of the nuclear accretion in the sombrero galaxy (NGC 4594). *Astrophys. J.* **597**, 175 (2003)
16. Di Matteo, T., et al.: Limits on the accretion rates onto massive black holes in nearby galaxies. *Astrophys. J.* **547**, 731 (2001)
17. Revnivtsev, M., Gilfanov, M., Sunyaev, R., Jahoda, K., Markwardt, C.: The spectrum of the cosmic X-ray background observed by RXTE/PCA. *Astron. Astrophys.* **411**, 329–334 (2003)