Numbers and Propositions Versus Nominalists: Yellow Cards for Salmon & Soames

Rafal Urbaniak

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Abstract Salmon and Soames argue against nominalism about numbers and sentence types. They employ (respectively) higher-order and first-order logic to model certain natural language inferences and claim that the natural language conclusions carry commitment to abstract objects, partially because their renderings in those formal systems seem to do that. I argue that this strategy fails because the nominalist can accept those natural language consequences, provide them with plausible and non-committing truth conditions and account for the inferences made without committing themselves to abstract objects. I sketch a modal account of higher-order quantification, on which instead of ranging over sets, higher order quantifiers are used to make (logical) possibility claims about which predicate tokens can be introduced. This approach provides an alternative account of truth conditions for natural language sentences which seem to employ higher-order quantification, thus allowing the nominalist to evade Salmon's argument. I also show how the nominalist can account for the occurrence of apparently singular abstract terms in certain true statements. I argue that the nominalist can achieve this by, first, dividing singular terms into real singular terms (referring to concrete objects) and only apparent singular terms (called onomatoids), introduced for the sake of brevity and simplicity, and then providing an account of nominalistically acceptable truth conditions of sentences containing onomatoids. I develop such an account in terms of modally interpreted abstraction principles and argue that applying this strategy to Soames's argument allows the nominalists to defend themselves.

R. Urbaniak (✉)
Chair of Logic, Methodology and Philosophy of Science, University of Gdansk,
Gdansk, Poland
e-mail: rfl.urbaniak@gmail.com

R. Urbaniak
Centre for Logic and Philosophy of Science, Ghent University, Ghent, Belgium

One would hope and perhaps conjecture that the whole general set theory, however beautiful it is, will in the future disappear. With the higher types Platonism begins. The tendencies of Chwistek and others ('Nominalism') of speaking only of what can be named are healthy. [Alfred Tarski]¹

1 Introduction

Salmon in his argument against nominalism about numbers (Salmon 2008) and Soames in his argument for the existence of sentence types (Simons 1987; Soames 1999) employ (respectively) higher-order and first-order logic to model certain natural language inferences. In both cases, the conclusions of those natural language inferences are supposed to carry commitment to abstract objects, partially because their renderings in those formal systems seem to do that.

I argue that this is not a compelling strategy. The nominalist can accept those natural language consequences as long as he provides them with sensible and non-committing truth conditions and is able to account for the validity of the inferences made without conjuring abstract objects.

In particular, I sketch a modal account of higher-order quantification, on which instead of ranging over sets, higher order quantifiers are used to make possibility claims about which predicate tokens can be introduced. This approach provides an alternative account of truth conditions for natural language sentences which seem to employ higher-order quantification. I also explain how this approach allows the nominalist to undermine Salmon's argument.

Another phenomenon that needs to be accounted for is the occurrence of apparently singular abstract terms in certain true statements. I argue that the nominalist can explain this fact by, first, dividing singular terms into real singular terms (referring to concrete objects) and only apparent singular terms (called onomatoids), introduced for the sake of brevity and simplicity. Then I provide an account of nominalistically acceptable truth conditions of sentences containing onomatoids. I do so in terms of modally interpreted abstraction principles and argue that applying this strategy to Soames's argument allows the nominalists to defend themselves.

For most of the paper I focus on the argument about numbers. As I proceed, it becomes clear that the two anti-nominalistic arguments are similar enough for the considerations to apply, *mutatis mutandis*, to both of them.

I first describe Salmon's argument for the existence of numbers and Soames's argument for the existence of sentence types. Then, I discuss in general terms what I consider the weak point of both arguments and sketch a nominalistic way to use this weakness. In Sect. 6 I describe the nominalist's response to Salmon's argument. In Sect. 7 I provide more details of a nominalistic strategy of handling both arguments. In Sect. 8 I explain how it can be used against Soames's argument.

¹ As cited by Carnap, folder RC 090-16-09 from the Carnap archive. Translated by Mancosu (2005, 334).

2 Exhibit One: Salmon on the Existence of Numbers

Salmon (2008) argues that nominalists cannot plausibly deny the inference from [A] to [B]:

- [A] There are exactly two Martian moons.
 [B] Something is such that it is number two and there are exactly that many Martian moons.

He insists that [B] commits one to the existence of numbers, and thus the nominalist is committed to the existence of numbers as soon as they accept [A].

As Salmon observes, the standard nominalist response to the claim that [B] follows from [A] is that existential generalization introducing a variable replacing ‘two’ in [A] is illegitimate, since this word, as it stands in [A], is an adjective, not a noun.

To circumvent this response, Salmon concedes that ‘two’ doesn’t occur in [A] as a singular term, and yet argues that [B] still can be derived from [A]. The argument starts from a symbolic, arguably nominalistically acceptable, formulation of [A]:

$$[A'] \quad \exists x \exists y (Mx \wedge My \wedge x \neq y \wedge \forall z (Mz \rightarrow x = z \vee y = z))$$

Formula [A'] says that exactly two objects are M . On the assumption that higher-order logic is available, we can apply second-order λ -expansion to [A'] and obtain a formula which says that M has the property of being a property F such that there are exactly two objects falling under F :

$$[1] \quad \lambda F [\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge \forall z (Fz \rightarrow x = z \vee y = z))] (M)$$

If 3-rd order quantification is available, [1] entails the claim that there is some n which is identical with the property of being a property F such that there are exactly two objects falling under F , and that M has this n :

$$[B'] \quad \exists n [n = \lambda F [\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge \forall z (Fz \rightarrow x = z \vee y = z))] \wedge n(M)]$$

Salmon comments:

Since nothing other than a single premiss, a definition, and logic is employed [...[B']] is exactly as it seems: a genuine existential-generalization consequence of [[A']]. The nominalist therefore cannot consistently accept [[A']] while rejecting [[B']]. Score one goal for the numbers against their opponents. [...] since] that there are such things as the Frege-Russell numbers is provable in higher-order logic alone [...] [n]umber deniers unintentionally commit themselves, through inconsistency, to there being things of every sort whatsoever, even round squares. Score another goal for the numbers. (Salmon 2008, 180-1)

Salmon agrees that the nominalist may simply reject higher-order logic for ontological reasons (he cites Quine as having done so). Still, he insists, it's enough that the argument sounds convincing for neutral agnostics about *abstracta* who have no bias “concerning the validity of natural language arguments” [181].

Keeping this argument in mind, let's take a look at another anti-nominalistic argument. Both arguments rely on taking certain natural language utterances at face value, and both arguments can be dealt with in a similar manner.

3 Exhibit Two: Soames on the Existence of Sentence Types

Soames (1999; Simons 1987) starts his argument for the existence of propositions by first considering an argument for distinguishing sentence types and tokens.² Imagine that two people, x and y , utter the same sentence. Now, x 's utterance (U_x) is composed of different sounds than y 's utterance (U_y). Thus we get:

$$[P1] \quad U_x \neq U_y$$

On the other hand, we already assumed that the sentence uttered by x (S_x) is the same as the sentence uttered by y (S_y):

$$[P2] \quad S_x = S_y$$

This entails that it is not the case that both x 's utterance is identical to the sentence x uttered and y 's utterance is identical to the sentence y uttered:

$$[C1] \quad \neg(U_x = S_x \wedge U_y = S_y)$$

which is equivalent to:

$$[C1a] \quad U_x \neq S_x \vee U_y \neq S_y$$

To obtain the more general claim that none of the sentences involved is identical to an utterance, Soames adds another premise:³

$$[P3] \quad \forall u, s[uRs \rightarrow \forall u'(s = u' \rightarrow s = u)]$$

[P3] says that for any u and s , if u is an utterance of s , then if s is identical with any utterance, it is also identical with u .⁴

[P1], [P2] and [P3], Soames claims, jointly entail that none of the utterances is identical to the sentence whose utterance it is:

$$[C2] \quad U_x \neq S_x \wedge U_y \neq S_y$$

² The argument isn't of course extremely new, but I'm not interested in identifying the first person who formulated it. Suffice it to say, Soames's formulation is commendable for its clarity.

³ He is not explicit about how R should be read, but the context indicates that ' xRy ' means ' x is an utterance of y '.

⁴ Soames's formulation is semi-formalized, I'm giving a slightly more formalized version. Also, it is worth emphasizing that the variables involved have to be interpreted as two-sorted, s ranging over sentences and u ranging over utterances. Otherwise, [P3] on its own would entail that any sentence is identical with all its utterances. For suppose uRs . By [P3] we get: $\forall u'(s = u' \rightarrow s = u)$. If we are allowed to "mix" variables, by universal instantiation this yield $s = s \rightarrow s = u$. But the antecedent is trivially satisfied, so $s = u$. Rather, the premise says that a sentence is identical to all its utterances *if* it is identical to any utterance at all.

Strictly speaking, what is needed is not only the addition of (P3), but also assuming that U_xRS_x and U_yRS_y . For only then (P3) will allow us detach $\forall u'(S_x = u' \rightarrow S_x = U_x)$ and $\forall u'(S_y = u' \rightarrow S_y = U_y)$. Then, proof by cases from (C1a) allows to apply *modus tollens* and prove that either $\neg\exists u'S_x = u'$ or $\neg\exists u'S_y = u'$. Then, one more step is needed in which one says that if one of the sentences is not an utterance, neither is the other one.

Soames claims that this strategy can be generalized to distinguish propositions from utterances, propositions from sentence types, and propositions expressed by sentences from the meanings of those sentences.⁵ Although, perhaps, some of the steps in such generalizations can be plausibly challenged by a nominalist, I will put this issue aside. Instead, I will argue that the initial argument about sentence types and tokens fails. If Soames's initial argument successfully generalizes, so does my criticism.⁶

4 Natural Language, Formalization and Ontological Commitment

There is a difference between natural language phenomena and mathematical theories meant to capture them. It is one thing to accept a certain inference as it is formulated in natural language, it is another to claim that its rendering in a particular formal system preserves ontological commitment and that validity in a formal system of our choice corresponds to the validity of natural language arguments.

These are two separate issues: the formalization might capture valid natural language arguments *in extenso* without getting the ontological commitment right. Even if we assume that Salmon's or Soames's formal apparatus achieves the former (which I grant), it by no means follows that it also does the latter.

There is a difference between accepting the natural language inference from [A] to [B], thinking that the formal framework employed by Salmon or Soames captures *in extenso* the valid arguments⁷ and assuming that [A'] and [B'] are in *all* respects adequate renderings of [A] and [B]. Salmon, without any further argument, moves from the claim that the nominalist should make the first two moves to the claim that they also should make the last one.

⁵ "The reason for considering this argument in some detail is that it generalizes. For example, to distinguish statements (propositions) from utterances, imagine a case in which x and y assert the same proposition. Since their utterances are different but their statements are the same, either x 's utterance is not identical with what x stated or y 's utterance is not identical with what y stated or both. To complete the argument, one may note the implausibility of claiming that the statement asserted by one of the speakers was a sequence of sounds (marks) that the speaker did not produce. [...] The same arguments can be made for distinguishing statements from sentences. Just as the same statement can be made by different utterances, so utterances of different sentences can result in assertion of the same statement [...] Although there is a close relationship between the meanings of sentences and the propositions they express, the two cannot always be identified. To see why, we need to consider two different kinds of cases: those in which utterances of sentences with different meanings express the same proposition and those in which utterances of sentences with the same meaning express different propositions." (Soames 1999).

⁶ Perhaps a nominalist can be happy with reconstructing sentence types as equivalence classes of sentence tokens. In what follows, however, I will not assume that a nominalist is happy with classes, whether they are equivalence classes or not.

⁷ That is, it correctly diagnoses valid natural language arguments modulo representation in the formal system.

If the nominalist can account for the validity of the natural language argument (which involves what appears to be higher-order quantification) without relying on Salmon's rendering, they won't have to be committed to whatever [B'] seems committed to. The nominalist, however, needs to tell a sensible story about the natural language argument from [A] to [B], and one of my main goals is to sketch such a story.

This point applies to Soames as well. He also makes his formalization do the heavy-lifting. Once you interpret sentence-terms as real singular terms and the sameness predicate as applicable only to genuine singular terms, you've fixed the outcome: accepting the claim that x and y made different utterances but uttered literally the same sentence will, in this rendering, force you to take sentence types to be *objects* distinct from the tokens. But if the nominalist can say a sensible story about the behavior of what *prima facie* seems like singular terms without taking them to be referring singular terms, they don't have to be committed to whatever Soames's formalization is committed to. Another goal of this paper is to sketch such a story.

The anti-nominalist might try to prevent this move by claiming that Salmon's and Soames's renderings are default interpretations of the relevant natural language sentences, and the *onus probandi* of showing their inadequacy belongs to the nominalist. One problem is that this strategy will lead nowhere. The anti-nominalist will insist on and the nominalist will deny such claims. There seems to be no clear procedure for deciding what the default formalization is, and there are no obvious reasons to think that what's default is right.⁸ Another issue is that even among logicians and philosophers of language there is no univocally accepted approach to abstract terms and the platonist interpretation of abstract terms in natural language is not commonly accepted.⁹ For this reason, I will not devote any more attention to this

⁸ Suppose even that the objectual reading is the default one. Still, it might have become default in virtue of its capturing the arguments *in extenso* and its simplicity, despite its ignored ontological inadequacy.

⁹ Let's skip a rather long history of nominalism before the XXth century and ignore the fact that from a larger perspective first-order and higher-order logics were developed quite recently and so by no means are tools that naturally come to mind. Even among prominent representatives of modern logic naive trust in the reference of apparently abstract terms hasn't been that common.

Just to give a few examples: Frege didn't settle with Hume's Principle, didn't take natural numbers to be objects *sui generis*, but tried to define them as composed objects whose nature was by no means immediately obvious to generations of mathematicians rather familiar with arithmetic. Dedekind and structuralists also put forward a rather elaborate theory of how the language of arithmetic works, rather avoiding simplistic objectual interpretations of natural number terms. Russell decided not to take class terms to refer to anything and developed his no-class theory (Whitehead and Russell 1910, vol. I, Introduction, ch. III, section (2), p. 75). Tadeusz Kotarbiński, Tarski's teacher, had an even more general view about singular terms. For him, the only genuine singular terms were those singular terms which referred to spatio-temporal objects and the truth conditions of sentences containing other apparent singular terms were to be reduced to sentences containing only genuine singular terms (Kotarbiński 1929). More recently, Boolos's (1998) approach to plural quantification suggests that plural quantification in natural language does not commit one to the existence of sets just because standard-second order translations of certain true sentences do. (Boolos himself was rather concerned with plural quantifiers in set theory, so he assumed the existence of sets; still, he didn't think that it is plural quantification that carries ontological commitment.)

In general, various nominalistic approaches are being recently developed (Chihara 1990; Hellman 1989; Lewis 1991; Goodman 1966; Goodman and Leonard 1940; Simons 1987; Martin 1988, 1992) and to claim that the platonist reading of mathematical statements is the default reading that almost everyone accepts (even only among logicians, philosophers, and perhaps mathematicians) is to ignore evidence.

issue and move on to sketching philosophical stories that nominalists can develop to account for the validity of the relevant natural language arguments.

5 Choice of Strategy

Let's grant that the nominalist should accept the natural language inference from [A] to [B]. This is not only because an unbiased observer might still buy into Salmon's argument, but also because arguments whose higher-order formal renderings are valid are abundant in mathematics, and a nominalist who would deny their correctness would set themselves against a legion of working mathematical theories (Lewis 1991). Putting the issue of making nominalistic sense of all mathematical theories, I will focus on sketching a way the nominalist can accept the natural language inference discussed by Salmon without ontological guilt.

The first thing to observe is that going merely substitutional about higher-order quantification doesn't seem viable for the nominalist. If they think that the admissible substituents, predicate tokens, are finite sequences over a finite alphabet and only actually existing things can be quantified over, they run out of tokens when they try to emulate higher-order quantification over infinite domains.¹⁰

Another stab would be to go inferentialist about higher-order quantification in the vein of (Rossberg and Cohnitz 2009). On this approach, the meaning of logical constants and the logic itself are specified by means of purely syntactic rules of inference. All there is to second-order logic is the notion of second-order derivability and standard rules for handling Boolean connectives, identity and first- and second-order quantifiers. While Rossberg and Cohnitz convincingly argue that the resources used on this approach are nominalistically acceptable, I choose not to employ this strategy.

A wider discussion of inferentialism is beyond the scope of this paper, but my dissatisfaction with the inferentialist approach in this case stems roughly from the following intuition. Natural language claims containing *prima facie* higher-order quantification not only occur in inferences, but also can be true or false. The task of the nominalist is not only to explain how such claims can be successfully used in inferences, but also to account for this ability. Some account of how such *prima facie* higher-order contingent claims are related to the world is needed. Perhaps the inferentialist could insist that such contingent claims are true in virtue of being derivable from some first-order contingent claims. I'm not sure how successful this strategy would be, especially in light of independence phenomena. I cannot pursue

¹⁰ Rossberg and Cohnitz (2009) describe a certain interesting strategy of avoiding this problem while still going substitutional. Roughly, the idea is that all the needed substituents are there, in the world. They exist as proof- or token-shaped configurations of, say, particles, yet to be discovered or pointed at. My worry about this strategy is that, slight eccentricity of this proposal aside, application to infinite domains still requires assuming that there are uncountably many such configurations. This suggests going modal and speaking about possible configurations or the possibility of world containing more matter than it currently does; but if that's the case, going modal without the additional eccentricity of the proposal might be preferred.

these issues in this paper and I find a more direct account of such a relation between sentences and the world preferable to the one which proceeds *via* derivation.

One way out of this quandary for the nominalist is to go modal. In this setting, the second order ‘For some F , ϕ ’ reads ‘it is possible to introduce a predicate token F such that ϕ ’ rather than ‘there is a set F such that ϕ ’. Thus, the general strategy is to take natural language claims about which predicate tokens it is possible to introduce as intuitively clear, to accept the modality involved as primitive. Then, the truth-conditions of natural language claims containing *prima facie* higher-order quantification are to be given in terms of such “nameability” claims, and the validity of *prima facie* higher-order natural language arguments is to be explained in terms of truth preservation *modulo* such translation.

This deals with the problem faced by going merely substitutional. Given a predicate token P and a set A say that P captures A iff P is true of all and only those objects which are in A . While there might be not enough predicate tokens to mimic quantification over all subsets of an infinite domain (it is not true that for each subset there exists a predicate token which captures it), there are sufficiently many ways predicates *could* be to do the job.

One important intuition about nameability, which will allow us to avoid paradoxes, is that the approach is typed. The initial domain of extralinguistic objects (which is the domain of first-order quantification) doesn’t change with the introduction of new predicate tokens. Predicate tokens are at specific levels and can apply to objects one level below only. (Typing can be weakened to obtain something more resembling a cumulative hierarchy. I will briefly discuss this move at some point.)

A crucial move here is the assumption that the possibility involved is sufficiently clear and can be taken as a primitive. One can challenge it by requesting a more formal truth theory. This is a tricky challenge, because once the nominalist tries to accommodate this request, the truth theory is bound to employ some set-theoretic apparatus, and the nominalist is asked to explain why this apparatus should be available to them. But the claim that a formal truth theory is needed should not be accepted without argument. For the sake of brevity and simplicity, in this paper I will simply assume that a formal semantics is not needed and the nameability-speak is clear enough. However, a more detailed formal approach is developed in Urbaniak (2010). Notice that even if the philosophical viability of the nominalist approach described here needs further defence, sketching the approach suffices as a challenge to Salmon or Soames: as long as they do not provide further reasons to prefer their frameworks over the nominalist reading, their initial arguments are not satisfactory. Let’s now see in more detail how this approach allows the nominalist to handle the arguments in question.

6 The Nominalist Story About Moving from [A] to [B]

The symbols of the formal language are: standard brackets, Boolean connectives ($\neg, \wedge, \vee, \rightarrow, \equiv$), first-order individual variables ($x, y, x_1, \dots$), second-order predicate

variables ($F, G, \dots$), third-order predicate variables ($F^3, G^3, \dots$), first-, second- and third- order identity ($=, =_2, =_3$), lambda operator (λ), first-order quantifiers ($\exists, \forall$), modal second-order and third-order existential and universal quantifiers ($\Sigma, \Pi, \Sigma^3, \Pi^3$). The formation rules are standard.¹¹

The language is devised to speak of extra-linguistic objects and possible predicate tokens. First-order identity and Boolean connectives are understood in a standard way. Higher-order identity is coextensiveness (e.g. $P =_2 Q$ is true just in case predicate tokens associated with P and Q apply to the same things). The first-order quantifiers have their standard meaning: they quantify over the initial domain of extra-linguistic objects. The second-order quantifier ' ΣF ' is read as 'it is possible to introduce a first-order predicate token such that' (the universal ' ΠF ' is understood analogously and so are third-order quantifiers). The lambda operator constructs a third-order predicate: ' $\lambda F[\dots F \dots](K)$ ' is interpreted as "the predicate token K could *salva veritate* serve the role of F in ' $\dots F \dots$ '", which boils down to the deflated reading of ' $\dots K \dots$ '. ' $\lambda F[\dots F \dots] =_3 K^3$ ' reads 'for any $H, \dots H \dots$ just in case $K^3(H)$ '.

Now, if the natural language higher-order existential quantifier is interpreted as Σ , the nominalists can safely accept the second-order inference from [A] to [B]. They will accept A' (which they will interpret in a standard manner and take to be a correct reading of [A]) and (1).

$$\begin{aligned} [A'] \quad & \exists x \exists y (Mx \wedge My \wedge x \neq y \wedge \forall z (Mz \rightarrow x = z \vee y = z)) \\ [1] \quad & \lambda F [\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge \forall z (Fz \rightarrow x = z \vee y = z))] (M) \end{aligned}$$

Their interpretation of the lambda operator in the latter, however, will be different, for [1] will have the interpretation on which it is quite easily deflated back to [A']. But instead of [B'] (which, they will insist, doesn't follow from [1] for it embraces literal commitment to sets, which is not present in [A]) they will rather accept:

$$\begin{aligned} [B^N] \quad & \Sigma^3 G^3 [G^3 =_3 \lambda F [\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge \\ & \wedge \forall z (Fz \rightarrow x = z \vee y = z))] \wedge G^3 (M)] \end{aligned}$$

Let's see what [B^N] says in the nominalistic reading described above.

- To start with, ' $\Sigma^3 G^3$ ' says that it is possible to introduce a predicate token G^3 .
- The first conjunct in the scope of the quantifier

$$G^3 =_3 \lambda F [\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge \forall z (Fz \rightarrow x = z \vee y = z))]$$

boils down to saying that the third-order predicates on both sides of " $=_3$ " apply to the same lower-order predicates:

$$\forall H [G^3(H) \equiv \lambda F [\exists x \exists y (Fx \wedge Fy \wedge x \neq y \wedge \forall z (Fz \rightarrow x = z \vee y = z))](H)]$$

¹¹ While I mentioned that the approach is rather cumulative than typed, I here follow the standard first- and second-order typing, mostly for the sake of simplicity and because this yields a system most similar to the one used in the original argument.

- For any H , $\lambda F[\exists x\exists y(Fx \wedge Fy \wedge x \neq y \wedge \forall z(Fz \rightarrow x = z \vee y = z))](H)$ boils down to $\exists x\exists y(Hx \wedge Hy \wedge x \neq y \wedge \forall z(Hz \rightarrow x = z \vee y = z))$ which simply says that H applies to exactly two objects.
- The last conjunct says that G^3 applies to M .

Thus, the formula reads:

It is possible to introduce a third-order predicate token G^3 , such that G^3 would apply to the same predicate tokens as the lambda expression which applies to a predicate just in case this predicate applies to exactly two objects, and such G^3 would apply to M .

As far as commitment goes, this is just a long-winded way of saying:

It is possible to introduce a predicate token G^3 , which would apply to all and only those lower-order predicate tokens which apply to exactly two objects, and such G^3 would apply to M .

Which seems like an elaborate way of saying:

M applies to exactly two objects.

Thus, *prima facie*, $[B^N]$ is non-committing, for it only says that it is possible to introduce a certain predicate-token satisfying a certain condition, rather than saying that a certain set exists.

What bearing does all this have on whether $[B]$ follows from $[A]$? Well, on this nominalist reading $[B]$, if it is taken to be committing and interpreted as $[B']$, doesn't follow from $[A]$. Yet, the nominalist will insist, our usual willingness to accept $[B]$ is justified because $[A']$ is a correct rendering of $[A]$ and entails $[B^N]$ which translated back to natural language yields $[B]$. Salmon argues that since $[B]$ follows from $[A]$ and $[B]$ is ontologically committing, so is $[A]$. Our nominalist rather suggests that since $[B]$ follows from $[A]$ and $[A]$ is non-committing, nor is $[B]$.

Perhaps things are not as they seem and there are reasons to think the approach sketched in this section is not nominalistically acceptable.¹² But even if this is the case, Salmon's original argument needs to be supplemented with an argument against the nominalistic acceptability of alternative accounts of higher-order quantification.^{13,14}

¹² A more detailed discussion of the nominalistic acceptability of the theoretical devices employed in this strategy lies beyond the scope of this paper.

¹³ One might feel tempted to think that introduction of predicates itself is not nominalistically acceptable, because each predicate has to have a set at its denotation. Thinking that natural language predicates are best construed as referring to sets is just a bias. Equally well one can think of predicates as applying directly to (perhaps multiple) objects (and third-order predicate tokens to apply to multiple second-order predicate tokens).

¹⁴ The approach, so far, is similar to that of (Chihara 1990) who takes existential quantifiers to be constructibility quantifiers (thus, existential higher-order quantifier is read 'it is possible to construct a formula'). Some differences arise when we look at more formal details (Urbaniak 2010), especially when cumulative approach is chosen over the typed one. More important differences arise when abstraction principles are brought in. These are the subject of sections to follow.

7 Numerical “Singular” Terms and Abstraction

The nominalist should be able to explain how numerical terms can function in true sentences not only as adjectives, but seemingly also as singular terms. One way to start is to say that since there are no abstract objects (or at least, since it is preferable to account for the truth of mathematical statements without appealing to abstract objects), apparently singular abstract terms don’t refer: terms that pretend to refer to such objects are introduced for brevity and simplicity of our discourse. On philosophical grounds, such nominalists divide *prima facie* singular terms into those that really refer to objects and those which only pretend to do that and syntactically behave like singular terms, but in fact don’t refer to anything (let’s call the latter *onomatoids*).¹⁵ Thus, such a nominalist says that most of those sentences in which no abstract terms occur have literal truth-conditions, whereas truth-conditions of sentences containing abstract terms are more elaborate and don’t require realism about the reference of such terms.

One way to handle onomatoids is to say that their introduction and the truth-conditions of sentences containing them are determined by appropriate abstraction principles interpreted linguistically. For instance Hume’s Principle, which says:

[HP] The number of *F*s is the same as the number of *G*s if and only if some *R* establishes the equinumerosity of *F*s and *G*s.

when introduced above a certain fixed domain tells us (in this nominalistic interpretation) that for each possible predicate token σ over our fixed domain we can introduce an onomatoid ‘ $N(\sigma)$ ’ (‘the number of σ ’). Moreover, it tells us that insofar as truth-conditions of sentences containing such terms are concerned, we are supposed to ignore all differences between onomatoids $N(\sigma)$ and $N(\tau)$ if σ and τ are equinumerous (that is, if it is possible to introduce a binary predicate token which satisfies the obvious conditions).¹⁶

For heuristic purposes and to avoid lengthy informal description, here’s a slightly more formal account of how this approach can be developed. First, since the nominalist doesn’t believe in concepts, instead of using Fregean comprehension with concepts, they rather speak of *possible predicates*, along the lines outlined above.

Definition 1 $\langle D, P, \Delta \rangle$ is an infinite naming structure (INS), if D is an infinite domain of objects, $\Delta \subseteq P \times D$, and for any $A \subseteq D$, there is a $\sigma \in P$ such that $\forall x \in D [x \in A \equiv \Delta(\sigma, x)]$.¹⁷ $\square$

¹⁵ This approach is inspired by certain ideas of Kotarbiński (1929), a Polish logician and Tarski’s teacher, who held this view and dubbed such apparent singular terms *onomatoids*.

¹⁶ Thus, to introduce equinumerosity we have to talk also about possible binary second-order predicate tokens which apply to two objects in a certain order. Then, given a fixed domain, F and G are equinumerous iff it is possible to introduce a binary predicate token R such that (1) for any x with Fx there is a y with Rxy and Gy , (2) for any y with Gy there is an x with Fx and Rxy , (3) if Fx , Fz , Gy , Rxy and Rzy , then $x = z$ and (4) if Gx , Gz , Fy , Ryx and Ryz , then $x = z$.

¹⁷ A more nominalistically kosher description would go: Consider a situation in which there are infinitely many objects and for any selection of those a predicate token can be introduced which applies exactly to them.

We are putting the issue of actual infinity aside and don't worry about the assumption that the domain of objects is infinite.¹⁸ P is the intended assembly of possible predicate tokens over D satisfying comprehension and Δ is the 'satisfaction' or 'true predication' relation.¹⁹

If an account of how second-order quantification is understood over such structures is needed, here's a sketch of one. A valuation function v would map individual variables into D and predicate variables to possible predicate tokens P . First-order quantification is just objectual quantification over D . The only two interesting satisfaction clauses are:²⁰

- [2] $\langle D, P, \Delta, v \rangle \models Fx$ iff $\Delta(v(F), v(x))$
- [3] $\langle D, P, \Delta, v \rangle \models \Sigma F\phi$ iff $\langle D, P, \Delta, v' \rangle$ where v' is an F -variant of v .

On a more informal and nominalistic description, this reads:

- [2] Fx is satisfied (given an association of predicate variables with predicate tokens and of individual variables with individuals) iff the predicate token (associated with) F applies to the object associated with x .
- [3] $\Sigma F\phi$ is satisfied (given an association of predicate variables with predicate tokens and of individual variables with individuals) iff it is possible to associate F with a possible predicate token (leaving the association of other variables unchanged) so that ϕ comes out satisfied.

So the satisfaction of a quantified phrase depends on whether certain possible predicate tokens are available.

Now (since we are heading to a nominalist interpretation of the language of arithmetic) a slightly more formal description of what [HP] does "on top of" INS:

Definition 2 An *HP-model* is an INS $\langle D, P, \Delta \rangle$ extended with a set $N_M = \{N(\sigma) \mid \sigma \in P\}$ and an equivalence relation $\approx$ on N_M such that for all σ, τ in P , $[N(\sigma) \approx N(\tau) \equiv \sigma \sim \tau]$ where $\sim$ is the equinumerosity relation between possible predicate tokens. $\square$

N can be seen as belonging to the model and being a function from P onto N_M . Equally well one can simply speak of P and N_M being just sets such that the conditions built into the definition are satisfied. The equivalence relation is the "identification" relation between apparent singular terms, and the definition above can be easily seen as embracing Hume's Principle. We also say that $N(\tau)$ is the

¹⁸ For a few strategies the nominalist can employ to avoid that see (Urbaniak 2010).

¹⁹ One worry about this approach is whether a nominalist can legitimately use semantics formulated in set-theoretic terms. Again, this issue lies beyond the scope of current considerations, but the main moves described in this paper can be described without the set-theoretic apparatus, in the spirit of footnote 17. It is also worth mentioning that even Quine didn't think that using first-order logic commits one to sets just because the semantics of first-order logic is set-theoretic.

²⁰ An F -variant of an assignment is an assignment which differs from the original at most in what it assigns to F .

successor of $N(\sigma)$ (i.e. $S(\sigma, \tau)$) iff τ is equinumerous to a possible predicate that applies to all those objects which σ applies to, and one more object).²¹

Here's one way to interpret the language of first-order Peano Arithmetic (PA_1) within this setting. First, we define the range of first-order quantification in the language of PA_1 :

- If $\sigma \in P$, $\neg(\exists x \in D)\Delta(\sigma, x)$, then $N(\sigma)$ is a natural numerical term (NNT) and all elements of $N_M \cup N_A \approx$ -related to it are NNTs.
- If σ is an NNT and $S(\sigma, \tau)$, τ is an NNT.
- Nothing else is an NNT.

The first clause ensures that any numerical term corresponding to an unsatisfied predicate is an NNT. The second one postulates that every successor term of an NNT is NNT. The third one requires that nothing else is.

The language of PA_1 contains individual constants: $0, S(0), S(S(0)), \dots$, individual variables $x, y, \dots$, Boolean connectives, brackets and quantifiers binding individual variables. Formation rules are standard. To interpret such a language over a minimal HP-model we have to:

- Associate individual constants with elements of N_M . This is straightforward: '0' is associated with $N(\tau)$ for some unsatisfied possible predicate token τ .²²
- Associate St for any term τ with the successor of the NNT associated with t .
- Interpret identity in the language of PA_1 as $\approx$, the "fake identity" relation.
- Understand the Boolean connectives, addition and multiplication in the standard manner.²³
- Interpret quantification in PA_1 as ranging over N_M .

On this approach, numerical constants do not really refer: they correspond with natural numerical terms which also do not refer. However, they can occur in true (or false) "fake" identity statements, because their systematic behavior is governed by Hume's Principle. Quantification is understood modally: ' $\exists x$ ' is read as 'it is possible to introduce a natural numerical term such that'. A few examples of translations of arithmetical statements into the modal nominalist language:²⁴

- [$SS0 + SS0 = S0 + SSS0$] The possible NNT associated with ' $SS0+SS0$ ' bears $\approx$ to the possible NNT associated with ' $S0+SSS0$ '.
- [$\exists x 0 + x = SSS0$] It is possible to introduce an NNT σ such that the NNT associated with ' 0 ' added-within-model to σ yields a possible NNT which bears $\approx$ to the NNT associated with ' $SSS0$ '.

²¹ Minimal HP-models allow us to introduce numerical terms of a very specific form: 'the number of σ '. However, nothing prohibits us from introducing other numerical terms; they don't have to be all of the form ' $N(\sigma)$ '. Such an extension can be easily accounted for by a slight modification of the notion of an HP model. For the sake of simplicity, we skip it.

²² That such predicate token is possible follows from the comprehension principle built into the notion of INS.

²³ Given that NNT are an ω -structure ordered by the (transitive closure of) successor relation, this is pretty straightforward.

²⁴ 'Added within the model'—this just refers to the function on N_M which does the job of addition in the ω sequence generated by N_M and the successor relation.

On the account described above, identities between numerical terms (which syntactically behave like singular terms) can be true, but are not grounded in the existence of numbers. We could consider a language in which also mixed identities (identities whose one argument is a real singular term and another argument is an onomatoid) are well-formed. Such mixed identities would always come out false, because no real object is the referent of an abstract term on this view.

How does this relate to Salmon's argument? Recall the problematic sentence together with its two readings:

[B] Something is such that it is number two and there are exactly that many Martian moons.

[B'] $\exists n[n = \lambda F[\exists x\exists y(Fx \wedge Fy \wedge x \neq y \wedge \wedge \forall z(Fz \rightarrow x = z \vee y = z))] \wedge n(M)]$

[B^N] $\Sigma^3 G^3[G^3 =_3 \lambda F[\exists x\exists y(Fx \wedge Fy \wedge x \neq y \wedge \wedge \forall z(Fz \rightarrow x = z \vee y = z))] \wedge G^3(M)]$

Framework sketched in this section (a modal interpretation of second-order quantification and of apparently singular numerical terms) allows for something I consider a more natural reading:²⁵

[B^S] $\exists x[x = SS(0) \wedge N(\text{Martian Moon}) = x]$

How to decipher its nominalistic reading? Quantification is the modal quantification over possible NNT. So we start:

It is possible to introduce a natural numerical term...

Then we have an identity between numerical terms, so we continue:

... which bears $\approx$ (the fake identity introduced by [HP]) to the natural numerical term corresponding to a predicate which applies to exactly two objects...

(This easily boils down to saying: 'It is possible to introduce a natural numerical term which corresponds to predicate which applies to exactly two objects'). The last bit of [B^S] requires that this possible numerical term should bear $\approx$ to the numerical term corresponding to 'is a Martian moon'. So we continue:

... and this newly introduced NNT bears $\approx$ also to the NNT 'N(Martian Moon)'.

After a rather straightforward simplification this says:

It is possible to introduce a natural numerical term corresponding to predicates satisfied by exactly two objects, and such numerical term would also correspond to the predicate 'is a Martian moon'.

²⁵ 'S' in 'B^S' stands for 'simple'.

which, I submit, commitment-wise, says nothing more than ‘there are exactly two Martian moons’.

One worry about this approach might be that it is typed, and (if the system is extended to other higher-orders) there are separate numbers at each level, so that there is no unique number two etc. While this is true for the typed approach (which I followed for the sake of simplicity), the issue does not arise if a cumulative hierarchy is used instead. On this approach, introducing predicate tokens proceeds in stages, pretty much mimicking the (initial steps of) the iterative hierarchy of sets. First there are extra-linguistic individuals and predicate tokens can be introduced which apply to those individuals. Then, predicate tokens can be introduced which apply to such first-order predicate tokens or extra-linguistic individuals, and so on. (It is important that the hierarchy is rather cumulative than typed: predicates can have “mixed” reference.) After each level of predicates the numerical onomatoids are expanded and associated with equinumerous predicates from below (even if those predicates belong to different levels). On this approach, the number of Martian moons is the same as the number of ‘ $x = 0 \vee x = 1$ ’, and so on. More details on the cumulative approach are available in Urbaniak (2010).

8 Nominalistic Abstraction and Sentence Types

The strategy employed to account for the behavior of numerical singular terms can be generalized. In particular, the nominalist can take terms referring to utterances to be real singular terms but interpret “sentence type” terms as onomatoids introduced by means of modally interpreted abstraction principles.

When we look at Soames’s argument from this perspective, [P1] which says that x ’s utterance is not identical to y ’s utterance, is understood literally and true. ‘ U_x ’ and ‘ U_y ’, unlike ‘ S_x ’ and ‘ S_y ’, are genuine singular terms. Sentence-type onomatoids, on the other hand, are introduced by means of an abstraction principle, perhaps of the following form:

[ST] The sentence uttered by x is the same as the sentence uttered by y if and only if the utterances made by x and y are sufficiently similar not to make a linguistic difference.

(If you find the notion of sufficient similarity problematic, say because you think it isn’t transitive, replace it with any nominalistically acceptable appropriate equivalence relation on sentence tokens.)

This abstraction principle, on the nominalistic reading, tells us that given that x and y make two utterances, onomatoids ‘ S_x ’ and ‘ S_y ’ can be introduced. [ST] gives us also truth-conditions for formulas in which sentence-terms occur: mixed identities are false by default and the well-formed claim ‘ $S_x = S_y$ ’ is true just in case U_x and U_y remain in an appropriate equivalence relation.

Thus, also Soames’s [P2], which says that $S_x = S_y$, comes out true (given the assumption that the relevant conditions on U_x and U_y are satisfied). Since mixed identities are false by default, the nominalist can also safely accept Soames’s conclusion [C2]:

$$U_x \neq S_x \wedge U_y \neq S_y$$

There is an important difference between Soames and the nominalist. Soames seems to think that it is enough to establish [C2] to accept the existence of sentence types. Our nominalist doesn't think the truth of [C2] carries commitment to such objects. Soames would accept, as a consequence of [P2], ' $\exists xx = S_y$ '. The nominalist, on the other hand, would accept ' $\Sigma xx = S_y$ ' instead, where Σx means 'it is possible to introduce a sentence-type term x such that' and identity is understood as in [ST]. For the nominalist, [P2] is true not in virtue of ' S_x ' and ' S_y ' referring to one and the same object, but rather because other, non-committing truth conditions of [P2] (determined by the right-hand side of [ST]) are satisfied. And just because [C2] tells us that sentences are not to be identified with utterances it doesn't follow that sentence terms are genuine singular terms.

Perhaps, the relevant notion of sufficient similarity employed in [ST] requires more explication and perhaps its nominalistic acceptability needs to be additionally defended. On the other hand, claiming that shape and sound similarity recognition, available even to not very well developed animals, clearly requires the existence of abstract objects is a bit of a stretch.²⁶ Whatever your view on the nominalistic acceptability of [ST] is, issues like these have to be seriously considered before taking sentence-type terms to be genuine singular terms.

9 Final Remarks

Salmon's and Soames's arguments rely on specific formal interpretations of certain natural language sentences. They take those interpretations to be a guide to the ontological commitment of natural language sentences in question. Even if their formal interpretation captures the natural language arguments adequately, it does not follow that it also gets the ontological commitments right. I argued that the nominalist can provide an alternative account of the validity of the arguments in question, on which the conclusions do not have the commitments Salmon and Soames would like them to have. The conclusions are true, but for quite different reasons.

The differences between philosophers sympathetic to Salmon's and Soames's arguments and the kind of nominalist we were talking about are deeper than it might initially seem and often result from certain meta-theoretic convictions. The former are inclined to think that most of philosophical questions about existence are trivial and can be answered just by looking at what natural language existence claims people accept. The latter tend to believe that apart from a weak interpretation of quantifiers (or a weak notion of an object) on which mathematical existential claims

²⁶ Would we also require that for any bunch of sufficiently similar sounds or shapes there is an abstract objects (abstract coconut-look, abstract dog-bark, etc.) in virtue of which they are similar? If yes, the doctrine seems to be neither parsimonious nor helpful. Parsimony considerations aside, no one doing research on how certain animals recognize, say, the sound of a dog, seriously tries to solve the problem by proposing that there is an abstract object, the ultimate bark in whose barkhood all barks participate and that it is in virtue of accessing this abstract object and comparing the sounds they hear against it, squirrels know that a dog is nearby.

come out true (and on which, presumably, existence claims about correctly introduced onomatoids come out true), a more robust quantifier or notion of an object is available, and answering what exists in this robust sense is not easy. Arguing for the plausibility of one of these meta-theoretic approaches or finding a sensible alternative to both of them is a challenge I will not address in this paper.²⁷

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²⁷ Chalmers et al. (2009) is a great collection of papers on this meta-theoretic problem and related issues.