Preface

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The present Notes are a re-elaboration of the lectures delivered at the First Sperlonga Summer School on Mechanics and Engineering Sciences, organized by the *Fondazione Tullio Levi-Civita* of Cisterna di Latina, and held in Sperlonga, not far from Rome, in September 2011. In the year and a half elapsed after the Course, the material has been revised and enriched with new and partially unpublished results. Significant additions have been introduced in the occasion of the course *The variational approach to fracture and other inelastic phenomena*, delivered at SISSA, Trieste, in March 2013.

The Notes reflect a research line carried on by the writer over the years, addressed to a comprehensive description of the many aspects of the phenomenon of fracture, and to its relations with other phenomena, such as the formation of microstructure and the changes in the material's strength induced by plasticity and damage. Roughly, the first four Lectures summarize the contents of the paper [4], and Lectures 5 and 6 reflect the subject treated in the paper [5] and successive developments.

A unifying element of these Notes is the concept of cohesive energy. This idea is already explicit in Galilei's book [7] who, in 1638, gave birth to the new science of the strength and rupture of solids. In the following century, this concept was dominant in the mathematical theories of the structure of matter based on interatomic interactions [2], and in Coulomb's theory of rupture [3]. In the second half of the 20th century, the cohesive energy of Barenblatt [1] and Dugdale [6] provided a fruitful regularization of the pre-existing theory of Griffith [8] for brittle fracture.

After a presentation of Griffith's theory in Lecture 1, in Lecture 2 the Barenblatt-Dugdale regularization is discussed. In Lecture 3 it is shown that the concept of cohesive energy can be successfully used to describe the formation of microstructure. A description of the phenomenon of elastic unloading, based on the assumption of dissipativity of the cohesive energy, is given in Lecture 4. A sharp change comes in Lecture 5, in which the cohesive energy, instead of being defined on surfaces as usual, is supposed to be diffused over the

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volume. With this assumption, a description of the process zones which usually accompany the crack growth is included into the model. The result is almost identical to the classical plasticity theory based on Drucker's material stability postulate. In Lecture 6 a description of the strain-softening phenomenon, not included in the preceding model, is obtained by adding to the cohesive energy a non-local term of the gradient type.

The whole analysis is carried on using a single mathematical tool, the incremental energy minimization. This explains the uniform structure of all Lectures: the definitions of equilibrium and stability based on the signs of the first and second variation of the energy, followed by the study of the quasi-static evolution and by the analysis of the response curves. In each Lecture, reference and comments are collected in a final Section.

The bibliography is far from complete, and the writer apologizes for unintentional omissions. The criterion followed for citations was to privilege two types of papers: those marking the origin of an idea or of a solution technique, and those providing a, possibly recent, overview on a specific subject.

An evident limit for these Notes is that the whole treatment is one-dimensional. This choice has been made to enlarge the field of the theory as much as possible, keeping away from the complex mathematical apparatus involving, for instance, non-regular fracture sets, non-convex elastic energies, and technically complicated function spaces. An extension to higher dimension is far from trivial. It could be a very challenging perspective for future work.

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