Letter to the Editor

An error correction about TACIT MODELS and INFINITY

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Dear Editor,

I am thinking that the expression “students questioned whether 0.333...is equal to 1/3 or tends to 1/3, answer usually that: 0.333...tends to 1/3 which is not mathematically correct” in the second page of the article titled “TACIT MODELS and INFINITY” in vol. 48, pp. 309–329, of Educational Studies in Mathematics. It is possible that some of the realities were not noticed in the editing process of the article after the death of Professor Fischbein. After the explanation below, I hope you can understand that the true version should be “0.333...tends to 1/3.”

In mathematics, the sum of infinite multiplicity cannot be equal to a number which is finite. For example, consider that a father divides an apple into two equal parts and gives one half to his son. Then, he again divides the one half into two equal parts and gives one quarter to his son, again. Then, this process goes to the infinite times. At the end, the son cannot get a whole apple. However, the amount of apple that the father has will be ignorable after a specific number of this process. This process brings us the concept of the sum of infinite series and that of limit. The amount of apple that the son has can be presented as $(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots)$, and this amount can be calculated as 1 (a whole apple) with the help of geometric series. However, it is obvious that infinitesimal amount of apple will be on the father's hand. However, the amount is ignorable. Therefore, the limit is zero. However, a zero limit cannot be considered as absolute zero.

The sum of the series $\sum_{i=1}^{\infty} \alpha_i$ never becomes a finite number. In this sum, the intended argument is that the limit is for $n \rightarrow \infty$ in the partial sums of the sequence $(S_n) = \sum_{i=1}^n \alpha_i$. If the value of $\lim_{n \rightarrow \infty} (S_n)$ is a finite number, then the sum of the series is considered as this number.

Therefore, if $0.33333 \dots = (\frac{3}{10} + \frac{3}{100} + \frac{3}{1,000} + \dots) = \sum_{i=1}^{\infty} (\frac{3}{10^i})$, then, the expression of “0.333...tends to 1/3” will be mathematically true.

I hope God rests Professor Fischbein's soul. And, I want to correct this mathematically incorrect situation.

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