Growth Overfishing: The Race to Fish Extends to the Dimension of Size

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Abstract The gravity of growth overfishing is increasingly recognized. The size-distribution of fish stocks is often severely truncated, even when the overall biomass is reasonably well managed. In a first part of this article, I show how the "race to fish" extends to the dimension of size: Akin to the classical Bertrand competition in prices, each agent has an incentive to target fish at a smaller size. In fact, for perfect selectivity, competition between two agents is sufficient to dissipate all rents. In a second part of this article, I explore the implications of size-differentiated harvesting for ITQ regulation. I show that quotas specified in terms of numbers are far superior to those specified in terms of weight or value.

1 Introduction

Fish stocks could play a significant role in providing food security for the world's growing population—if they are properly managed (Smith et al. 2010). Yet, there is probably no other area of environmental economics where the gap between potential and actual performance is as large (Heal 2007). Even in fisheries whose aggregate biomass is reasonably well managed, growth overfishing is increasingly seen as a serious problem (Beamish et al. 2006; Hsieh et al. 2006; Ottersen 2008). Due to the size-selective properties of harvesting gears, large fish are over-proportionally removed from the stock. Since older and larger fish are better able to buffer adverse environmental conditions (Ottersen et al. 2006), this truncation

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of the fish stock's size structure can lead to magnified fluctuations of abundance (Anderson et al. 2008). Moreover, if harvesting has evolutionary effects (Guttormsen et al. 2008), the increased variability may be irreversible (Stenseth and Rouyer 2008).

Growth overfishing is also a substantial economic problem. In fact, it could be much more important than recruitment overfishing.[1] Weight and value of an individual fish grows considerably over time in most commercially harvested species. Moreover, recruitment is often influenced by random environmental conditions. It is therefore crucial to fully account for the natural growth potential of fish stocks. For example, Diekert et al. (2010) pointed out that the resource rent in the Barents Sea cod fishery could be more than doubled simply by changing the mesh size of trawlers.

However, under non-cooperative exploitation the ability to conditionally harvest on size does not lead to a situation where the young and growing fish are spared in order to catch more fish of optimal size. To the contrary: Growth overfishing is the result of a situation where every agent has an incentive to target fish at a smaller size than his opponent. The race to fish extends to the dimension of size.

The argument is developed for two stylized forms of gear selectivity that could be thought of as extreme points of the range of real-world applications: First, *perfect selectivity*, where agents can condition their fishing intensity on size and only incur costs when a fish is actually harvested. Second, *knife-edge selectivity*, where agents incur costs for the overall fishing intensity and only choose a first-size-at-capture.

I show that two players playing a non-cooperative game choosing gear with perfect selectivity is sufficient to dissipate all rents in the fishery. While this result is formally similar to a well known theorem from Clark (1980, 1990), Clark's result is cast in terms of time and biomass.[2] I moreover show that the same mechanism is at work under *knife-edge selectivity*, though in a less pronounced form. Here, some rents can be retained since part of the externality is internalized due to the imperfect harvesting technology.

Fishing is a process which removes individuals from a population, but it is the value of these individuals that generates economic profits. Restricting the number of harvestable fish with an individual transferable quota (ITQ) is therefore superior to an ITQ in terms of biomass. The former will increase in value at the same rate as the individual fish, while the latter increases in value only at the rate at which a larger fish is able to fetch a higher price per kg (if this is at all the case). However, as long as it is less costly to harvest fish when they are more abundant, no indirect regulation will be sufficient to achieve the first best.

The article proceeds as follows: The next section looks at the related literature. The model is presented in Sect. 3. Subsequently, the optimal harvesting pattern and the feedback Nash equilibrium are derived for both *perfect selectivity* (Sect. 4.1) and *knife-edge selectivity* (Sect. 4.2). Section 5 then analyzes the implications of growth overfishing for ITQ regulation. The results are discussed in Sect. 6. Section 7 concludes.

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[1] Recruitment overfishing is defined as harvesting too many fish before they have matured, so that the replenishing potential is restricted; Growth overfishing can be defined as harvesting fish before they have reached their socially optimal size. Whereas it is often argued that recruitment overfishing is the less likely but far more severe form of overfishing, it is important to note that growth overfishing, via the link described above, could also increase the risk of stock collapse.

[2] Clark analyzed an aggregate biomass model whose growth function developed autonomously over time but not age. As the growth function in my model is non-autonomous and develops over age but not time, his result does not immediately apply to my case. The essence of the proof is however the same and does not depend on the growth function.

2 Related Literature

The problem of growth overfishing is long known to practitioners and scientists. Already in 1893, Petersen discusses growth overfishing as one of the main causes of declining yields in the Danish flounder fisheries. The importance of accounting for the size-selectivity has clearly been recognized in the literature of the 1950s and 1960s (Allen 1953; Beverton and Holt 1957; Turvey 1964; Smith 1969), but it has received surprisingly little attention thereafter. Today, it is mainly treated from a technical perspective (see for example Bethke 2006; Kronbak et al. 2009; Kvamme and Frøysa 2004). The gear regulations of most fisheries remain largely ad hoc (Froese et al. 2008).

In general, there is a growing interest in using size-differentiated and stage-based models in empirical resource economics (Grafton et al. 2007; Massey et al. 2006; Pintassilgo and Duarte 2002; Smith et al. 2008, 2009). In an early, but largely neglected contribution, Stollery (1984) has compared optimal and open-access harvesting in a fishery which develops continually both over age/size and over time. Recently, Tahvonen (2009a,b) has provided a solution in a discrete setting, in spite of the previous pessimism regarding analytical solutions (Wilen 1985; Clark 1990). In addition, there exists a large mathematical literature on optimal harvesting (although with little or no economic content; see e.g. Brokate 1985; Getz and Haight 1989; Murphy and Smith 1990) and a related literature in forestry economics (see Xabadia and Goetz 2010, and the references therein). Skonhoft et al. (2011) analyze selectivity in a three-stage model and find that only few straightforward results appear. Quaas et al. (2010) look at incentives for optimal management of structured fish populations in a model with two catchable cohorts. I model growth as a continuous process to circumvent some of the difficulties implied by a discrete analysis, allowing me to present sharp results.

The harvest technology of *perfect selectivity* was first applied in the seminal analysis of Clark et al. (1973), who follow a single cohort over time. The present model is mathematically parallel to theirs, but models the entire size-distribution of the stock in equilibrium. I show how two players are sufficient to dissipate all rents, akin to the classical Bertrand competition in prices.[3] Perfect selectivity is among others employed by Stollery (1984) and Quaas et al. (2010). Clark (1990, Chap. 9.3) uses perfect selectivity when discussing the optimal harvesting of a single cohort. For his discussion of ITQ management (in terms of biomass) under presence of multiple cohorts (Clark 1990, Chap. 9.8) however, he uses *knife-edge selectivity*. This description of the harvesting technology is also employed in the classical exposition from Beverton and Holt (1957) as well as in Tahvonen (2009a,b) and in much of the empirical literature (e.g. Bjørndal and Brasão 2006; Kulmala et al. 2007; Kjærsgaard and Frost 2008).

There is a long standing debate on the efficiency and effectiveness of ITQ management. Their inability to restore optimality in face of resource heterogeneity has been discussed i.a. by Boyce (1992), Townsend (1995), Gavaris (1996), and Costello and Deacon (2007). I contribute to this literature by pointing to the effect of different quota specifications and by highlighting the superiority of number-quotas.[4] This confirms the finding of Quaas et al.

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[3] On this note that I use the Bertrand game as a metaphor explaining the mechanism of undercutting the size-at-first-capture of the respective competitors. A differential game of Bertrand pricing in a fishery has been analyzed by Beard (2008).

[4] Turner (1997) finds that value-quotas are preferable over weight-quotas (number-quotas are not discussed) with respect to quota induced discarding under imperfect selectivity. In contrast to Turner, I assume that it is possible to harvest one class of fish without also harvesting something from another class. Therefore, my model does not yield the potential advantages of value-quotas. For a further discussion of value-quotas under imperfect selectivity see Singh and Weninger (2009).
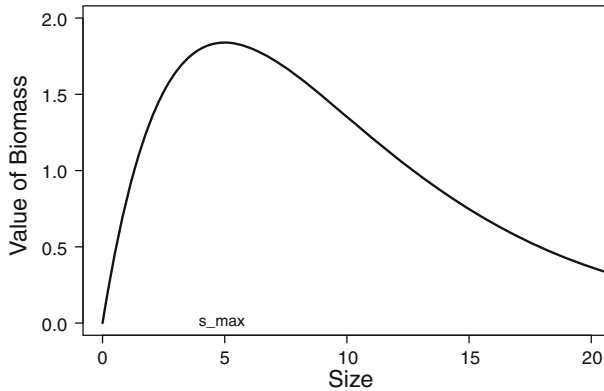
Fig. 1 Development of biomass value in absence of fishing

(2010), but I show that for more general cases no undifferentiated ITQ scheme is sufficient to prevent growth overfishing.

3 Model

In a fully differentiated fishery model, the dynamics of the fish stock unfold both over size and time. To describe the mechanisms clearly and to focus on growth overfishing, I abstract from the time variable (i.e. I look at the situation in equilibrium) and assume that recruitment is exogenous. Then the development of one cohort over time is the same as the distribution of cohorts over size (Beverton and Holt 1957). The other necessary features for my model to work is to describe natural mortality as constant[5] and growth as a non-decreasing process. Finally, I presume precise targeting in the sense that unwanted fish are not harvested.

Let $x(s)$ be the value of the biomass of a fish stock (biovalue for short) as a function of size s. The biovalue is $x(s) = n(s) \cdot v(s)$, where $n(s)$ is the number of fish of size s and $v(s)$ is their individual value. The value of a fish is $v(s) = p(s) \cdot w(s)$, where $w(s)$ is its weight in kg and $p(s)$ is the price per kg. Denote the relative individual gain in value by $\varphi(s) = \frac{\partial v(s)/\partial s}{v(s)}$. The instantaneous development of the biovalue over size is then given by:

$$\frac{\partial x(s)}{\partial s} = [\varphi(s) - m - f(s)] \, x(s). \tag{1}$$

It consists of two parts: First, the relative individual gain in value: $\varphi(s)$ Second, the decline in the number of individuals due to the instantaneous natural mortality rate m and the instantaneous fishing mortality rate $f(s)$: $\frac{\partial n(s)}{\partial s} = -[m + f(s)] \cdot n(s)$. The development of natural biovalue over size in absence of fishing is exemplified in Fig. 1. For future reference, the biovalue in absence of fishing is denoted by $x_0(s)$ and can be written as $x_0(s) = v(s) \cdot Re^{-ms}$ (where recruitment is exogenous $n(0) = R$ and the solution of $\frac{\partial n(s)}{\partial s} = -mn(s)$ is Re^{-ms}). Denote the size where the unharvested biovalue reaches its maximum by s_{max}. That is, $\varphi(s_{max}) = m$.

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[5] Constant natural mortality is a strong assumption, which—similar to the assumption of exogenous recruitment—enables closed-form solutions when also fishing mortality is constant. It is customary to assume a constant natural mortality in many stock-assessments (see e.g. ICES 2010). Recent contributions (Aanes et al. 2007; Brinch et al. 2011) have shown that—for North-East Arctic cod at least—natural mortality is indeed rather stable across different age-classes.

The control variable that enters the state equation (1) is the applied fishing mortality f. From an economic perspective, the cost of fishing are related to a set of inputs—conventionally subsumed as "effort"—which then translates into fishing mortality. Without loss of generality, it is assumed that one unit of effort equals one unit of fishing mortality,[6] but I distinguish two technologies: One where fishing mortality is perfectly flexible, and one where it is the same for all targeted size-classes. Let costs be denoted by $c(f) = c \cdot f$ and at this highly simplifying level of aggregation, take them to be proportional to the level of f. Note that even though the cost per unit of effort is constant, the cost per unit of harvest depends on the abundance of fish, in particular when effort can be conditioned on size.

If, on the one hand, costs are mainly associated with taking fish out of the water and precise targeting of individual fish is possible, the appropriate formulation of the profit function would be given by (P), where costs appear within the integral and effort is targeted at each size. This technology is called *perfect selectivity*.

$$\pi(f(s)) = \int_0^\infty [x(s) - c] f(s) \, ds \tag{P}$$

If, on the other hand, a large part of the cost is incurred regardless of whether a fish is actually caught (e.g. by steaming to and from the fishing grounds), and the fishing process can be divided into two separate control dimensions, gear selectivity (the first-size-at-capture s_0), and the application of effort (translating into a uniform fishing mortality f), then the appropriate formulation of the profit function would be given by (K). This technology is called *knife-edge selectivity*.

$$\pi(s_0, f) = \int_{s_0}^\infty f \cdot x(s) \, ds - c(f) \tag{K}$$

My aim is to direct attention to the importance of non-cooperative interactions that play out on the qualitative properties of the stock. While I believe that this approach offers a new and useful perspective, I am aware that it can be but a first step. In order to isolate the mechanism of growth overfishing and its effect on ITQ regulation, I have stripped the model to its essentials. This necessitated some simplifications which deserve to be discussed explicitly:

First, assuming that things do not change over time and that recruitment is exogenous implies that recruitment is constant, whereas in reality stochastic environmental conditions have a large impact. The assumption further implies that recruitment is continuous. For many species, spawning is seasonal, so that the size distribution is characterized by several peaks rather than a uniform distribution. Moreover, maturity is likely to depend strongly on size and fertility may be growing more than linearly with size. These effects are not taken into account here.

Second, there is no discounting as the time variable is not present in the model. In so far as the model is the steady state of an originally time- and size-differentiated problem, I implicitly assume a zero discount rate. Intuitively, the effect of discounting on the realized steady state is that it becomes less worthwhile to wait for the fish to grow to a larger size.

Third, I assume growth in value to be non-decreasing. Fish do not shrink, but there might be cases where the price decreases with size (e.g. due to quality deterioration). Strictly speaking,

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[6] In the following, the terms effort, fishing mortality, and fishing intensity are used interchangeably.
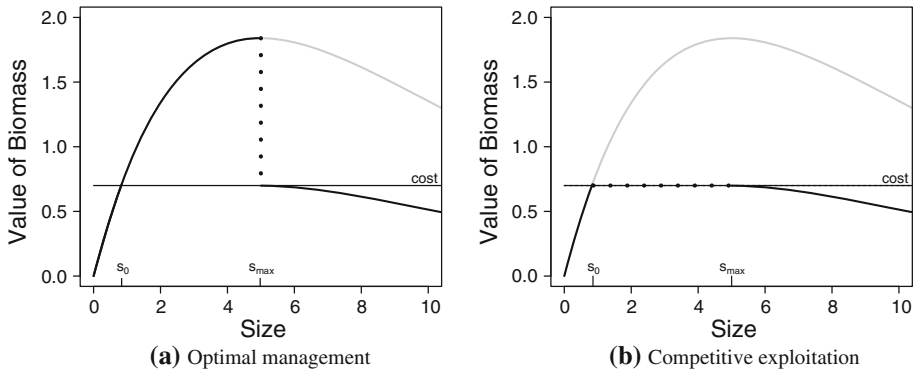
Fig. 2 Equilibrium harvesting patterns under perfect selectivity: the *thin line* indicates the level of costs c, the *black line* shows the development of biovalue over size $x(s)$. Under optimal management with unbounded f, fishing is an impulse control at s_{max} (*dotted line*). Under competitive exploitation, harvesting occurs over an interval (*dotted line*) which begins when $x(s)$ reaches c at s_0 and ends when the biovalue declines below c even in absence of fishing. For comparison, the *grey line* indicates the development of biovalue in absence of fishing

what is needed for my model to work is that the biovalue is a one-peaked concave function. This is satisfied when value-growth is non-decreasing, but it might also be satisfied when the value first increases and then decreases. More variable cases with interchanging intervals of increasing and decreasing prices would need special attention.

4 Optimal and Non-Cooperative Harvesting

4.1 Perfect Selectivity

Perfect selectivity could be a realistic description for a coastal fishery or a (recreational) fishery with individual handling (such as e.g. lobster fisheries).

4.1.1 Optimal Harvesting Under Perfect Selectivity

The solution to the problem of a sole-owner maximizing sustainable profits given technology (P) and the fish stock development (1) is parallel to the problem of harvesting a single cohort over time as it has been described i.a. by Clark (1990, Chaps. 9.3 and 9.4). Intuitively, when the fishing intensity is unbounded, the sole-owner would only target fish of the highest value at s_{max} and at that size extract the entire available surplus $x_0(s_{max}) - c$. This harvesting pattern is illustrated in Fig. 2a. When the maximum fishing intensity is bounded ($f_{max} < \infty$), such an exploitation path is not feasible. Harvesting would occur over the shortest interval $[s_0^*, S]$ symmetric around s_{max}, so that the extracted surplus biovalue is maximized: Fish that are too small are spared because they would still grow in value in the water, and fish that are too large are not targeted because there are so few of them that harvesting them becomes exceedingly costly. When marginal costs are increasing with increasing effort, harvesting will follow a more gradual path (Wilen 1985), but fishing intensity will still be highest at the size of maximum biovalue (to get, loosely speaking, most bang per buck).

4.1.2 Competitive Harvesting Under Perfect Selectivity

How does the harvesting pattern change when we move from the sole-owner solution to the competitive situation of many agents? Let there be N symmetric agents and denote the fishing mortality from agent i by f^i, where $f^i \in [0, f^i_{max}]$. The development of the biovalue then reads:

$$\frac{\partial x(s)}{\partial s} = \left[\varphi(s) - m - \sum_i f^i(s) \right] x(s). \tag{1'}$$

and profits for player i are:

$$\pi(f^i(s)) = \int_0^\infty [x(s) - c] f^i(s) \, ds$$

Proposition 1 (Growth overfishing under perfect selectivity) *Provided that $\sum_{i \neq j} f^i_{max} \geq \varphi(s) - m$ for all j and all s where harvesting is economically viable (i.e. $x(s) \geq c$), the feedback Nash equilibrium of the unregulated game under technology* (P) *is characterized by a unique path where $x(s) = c$ (see Fig. 2b). Harvesting starts at $s = s_0$, defined by $x_0(s_0) = c$, and continues until $s = s_{max}$. The players make zero profits in equilibrium.*

Proof To prove Proposition 1 I need to show that the described exploitation path is a Nash equilibrium (Lemma 1) and that it is unique (Lemma 2). Total rent dissipation follows from the fact that the value of the harvest equals the cost of harvesting over the entire interval of harvesting. □

Define by $g^i(s)$ the individual contribution to the aggregate fishing intensity that keeps the biovalue at the level c. That is $\sum_i g^i(s) = \varphi(s) - m$ for all s at which $x(s) = c$.

Lemma 1 *The set of Markov-strategies*

$$f^{i*}(x(s)) = \begin{cases} f^i_{max} & \text{if} \quad x(s) > c \\ g^i(s) & \text{if} \quad x(s) = c \\ 0 & \text{if} \quad x(s) < c \end{cases} \tag{2}$$

constitutes a feedback Nash equilibrium.

Proof Suppose all players follow strategy (2), then $x(s)$ will grow until $x_0(s) = c$, and subsequently stay constant at $x(s) = c$ until s_{max}. Since $x(s) - c$ is never positive, no deviation can yield positive profits. As f^{i*} ensures that i gets zero profits, the strategy is an optimal response to the other players' strategy, hence it constitutes a Nash equilibrium. □

The intuition behind this result is analogous to the proverbial statement that there are no dollar bills laying on the street, for if there were, someone else would have taken them. For illustration, imagine that $[s_0, s_{max}]$ comprises a number of discrete intervals of length Δs. Any surplus value in an interval will be exploited instantly by choosing f^i_{max}, leading to a declining biovalue. At the start of the next interval, the biovalue will be below c and induce no fishing activity. In the interval after the next, the biovalue will be above c again, hence triggering maximum effort, etc. In the limit as $\Delta s \to 0$, the level of fishing intensity that keeps the biovalue at $x(s) = c$ from s_0 to s_{max} is given by $\sum_i g^i(s)$. Note that the individual g^i is not uniquely determined; what matters is that the path of $x(s)$ is unique.

(a) Optimal management
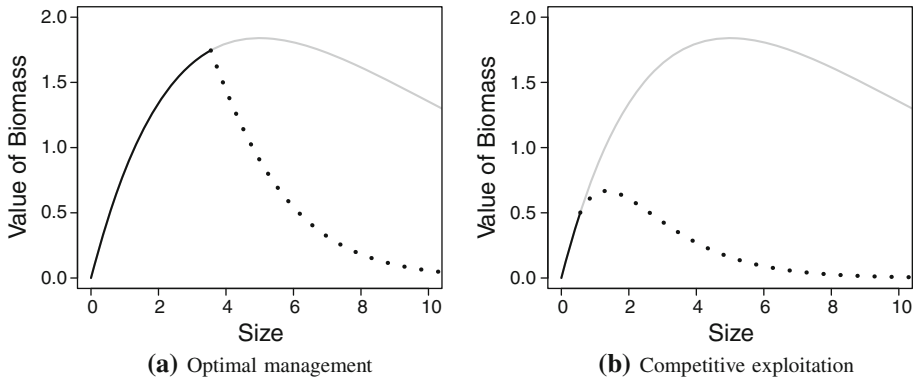
(b) Competitive exploitation

Fig. 3 Equilibrium harvesting patterns under knife-edge selectivity: the *black line* shows the development of biovalue over size $x(s)$, where it is *dotted* from the first-size-at-capture on. As before, the *grey line* indicates the development of biovalue in absence of fishing

Lemma 2 *The path described by the feedback Nash equilibrium* (2) *is unique.*

Proof The proof is stated in Appendix A.1.   □

Similar to the classical Bertrand-price-competition for perfect substitutes, the player has to share the obtainable profits when harvesting at the same size as his opponents, but obtains the full profits when harvesting at a smaller size. It is this mechanism which is at the heart of the "race to fish" along the dimension of size.[7] The outcome does not depend on the number of participating players, competition between two agents is enough to dissipate all rents.

4.2 Knife-Edge Selectivity

The harvest technology of the previous section might arguably be a special case. Another description of the harvest technology which is often found in empirical studies is knife-edge selectivity. This technology (where all fish above a given first-size-at-capture s_0 are subject to a general level of fishing mortality f) could be a realistic description of a distant water trawl fishery.

In general, it is clear that restricting the possibilities by which the fishing intensity can vary over size means that the different harvesting patterns are less pronounced (compare Figs. 3 to 2). Unfortunately, it also means that the results are less tractable. Nevertheless, the main results that optimal harvesting is symmetric around the size of maximum biovalue while competitive exploitation implies a "race to fish" carry over.

4.2.1 Optimal Management Under Knife-Edge Selectivity

The sole-owner problem to maximize (K) subject to (1) is essentially identical to the classical yield-per-recruit analysis from Beverton and Holt (1957). Although they do not explicitly model costs and prices, they do discuss economic aspects (*ibid.*, pp. 378).

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[7] In contrast to the Bertrand-game of course, the decision is not to pick one price out of an interval, but there are (infinitely) many sizes on the interval $[s_0, s_{max}]$ at which the players can harvest. As the biovalue in $[s_0, s_{max}]$ grows again if there is no further fishing, the players will harvest at any point in $[s_0, s_{max}]$: given that the other players harvest at every possible size, abstaining from harvesting at some size will not lead to any improvement in the obtainable biovalue at the other sizes.

Under the present formulation of knife-edge selectivity, both fishing mortality f and natural mortality m are independent of size. It is therefore possible to explicitly solve for $x(s) = v(s)n(s) = v(s)Re^{fs_0 - (m+f)s}$ and to reformulate this as the unconstrained problem to choose $s_0 \in [0, \infty)$; $f \in [0, f_{max}]$ so that $\pi(s_0, f)$ is maximized, where:

$$\pi(s_0, f) = f \int_{s_0}^{\infty} v(s) \cdot Re^{fs_0 - (m+f)s} \, ds - c(f) \tag{3}$$

with the corresponding first order conditions for an interior solution:

$$\frac{\partial \pi(s_0, f)}{\partial s_0} = f \int_{s_0}^{\infty} fv(s) \cdot Re^{fs_0 - (m+f)s} ds - fv(s_0) \cdot Re^{-ms_0} = 0$$

$$\Rightarrow f \int_{s_0}^{\infty} x(s) ds = x_0(s_0) \tag{4a}$$

$$\frac{\partial \pi(s_0, f)}{\partial f} = f \int_{s_0}^{\infty} (s_0 - s)x(s) ds + \int_{s_0}^{\infty} x(s) ds - c'(f) = 0$$

$$\Rightarrow \int_{s_0}^{\infty} x(s) ds = c'(f) + f \int_{s_0}^{\infty} (s - s_0)x(s) ds \tag{4b}$$

The first-order-condition for s_0, Eq. (4a), states that the harvested value, $f \int_{s_0}^{\infty} x(s) ds$, should equal the natural biovalue at the first-size-at-capture, $x_0(s_0)$. That there is a unique $s_0^* \in [0, s_{max}]$ which maximizes (3) for a given f can be seen by inspecting the corresponding second derivative:

$$\frac{\partial^2 \pi}{\partial s_0^2} = f \left[\frac{\partial \pi}{\partial s_0} - \frac{\partial x_0(s_0)}{\partial s_0} \right]$$

Since $\frac{\partial \pi}{\partial s_0} = 0$ at s_0^* and the natural value function $x_0(s_0)$ is increasing until s_{max}, the second derivative is negative at $s = s_0^*$. The first derivative is positive for $s_0^* - \epsilon$ and negative for $s_0^* + \epsilon$ (it crosses the zero-line from above). Now suppose there would be another value of s_0 before or after s_0^* for which $\frac{\partial \pi}{\partial s_0} = 0$. At this point, the first derivative would have to cross the zero-line from below. However, this is a contradiction since $\frac{\partial x^i(s_0)}{\partial s_0} > 0$ for $s_0 \in [0, s_{max}]$.

The first-order-condition for f, Eq. (4b), states that the marginal revenue from fishing, $\int_{s_0}^{\infty} x(s) ds$, should equal the marginal cost, $c'(f)$, plus the opportunity costs in terms of a reduced stock, $f \int_{s_0}^{\infty} (s - s_0)x(s) ds$. To see that there exists an f^* that maximizes (3) for a given $s_0 \in [0, s_{max}]$, note that $\frac{\partial \pi}{\partial f}$ is a continuous function which is larger than zero at $f = 0$ (if harvesting is at all profitable) and smaller than zero as as $f \to \infty$ (unless $c = 0$ and $s_0 = s_{max}$). Hence, it has to become zero at least once for $f > 0$.

As discussed by Beverton and Holt (1957), fishing intensity and first-size-at-capture are substitutes: The same harvest can be obtained by a high f and a large s_0 or by a low f and a small s_0 (though there is only one choice that maximizes profits). Consequently, costs have an indirect effect on the optimal first-size-at-capture: High cost lead to a relatively low f^*

and hence a relatively small s_0^*. Conversely, low costs yield a high f^* and hence a large s_0^*. If fishing is costless, it is optimal to harvest with unlimited intensity at s_{max}.

When the growth in value can be approximated by a linear function (for simplicity, I choose $v(s) = s$), the optimal first-size-at-capture can be expressed as a simple function of the fishing intensity f. Integrating the harvest function by parts yields:

$$\pi(s_0, f) = f \int_{s_0}^{\infty} s R e^{fs_0 - (m+f)s} \, ds - c(f)$$

$$= \frac{1 + (m+f)s_0}{(m+f)^2} f R e^{-ms_0} - c(f)$$

where the first-order-condition for s_0 is given by (5). It shows how the optimal first-size-at-capture is determined by the size of maximal unharvested biovalue ($s_{max} = \frac{1}{m}$ when $v(s) = s$), weighted by the ratio of fishing mortality f to total mortality $m + f$. Note that as $f \to \infty$, $\frac{f}{m+f} \to 1$, and $s_0^* \to s_{max}$.

$$\frac{\partial \pi}{\partial s_0} = 0 \Rightarrow s_0^* = \frac{1}{m} \cdot \frac{f}{m+f}. \tag{5}$$

Similarly, the optimal fishing intensity can be expressed as a function of s_0, but the expression is intricate and yields no insights beyond the intuition discussed above.[8] As it is tedious to work out the specific optimal level of f^*, I will concentrate on the first-size-at-capture and continue as if the level of fishing intensity was exogenously given.

4.2.2 Competitive Exploitation Under Knife-Edge Selectivity

How does non-cooperative exploitation compare to the efficient solution under knife-edge technology? Assume for the moment that there are N identical players with given individual fishing mortality f^i. In order to ensure that each player faces one size at which all his opponents start fishing, I furthermore restrict my attention to symmetric equilibria. Each agent i then has to choose whether or not to apply a smaller first-size-at-capture than his opponents. In doing so, the agent weighs the gain from harvesting smaller fish for himself against the loss from unduly reducing the stock before it has reached its highest value.[9]

Denote the first-size-at-capture from agent i's opponents by ς_0 and their fishing intensity by f^j (so that the total fishing intensity is given by $f = f^i + \sum_{j \neq i} f^j$). When agent i

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[8] For completeness, the first-order-condition yields a cubic function whose positive root can—with help of a computer—be found to be:

$$\frac{\partial \pi}{\partial f} = 0 \Rightarrow f^* = \frac{R}{3c} \sqrt[3]{c^2 \left(27m + 3\sqrt{(3 - 9ms_0 + 9m^2s_0^2 - 3m^3s_0^3 + 81cm^2)/c}\right)}$$

$$- \frac{1 - ms_0}{\sqrt[3]{c^2 \left(27m + 3\sqrt{(3 - 9ms_0 + 9m^2s_0^2 - 3m^3s_0^3 + 81cm^2)/c}\right)}} - m$$

[9] This game bears resemblance to innovation timing games (Hoppe and Lehmann-Grube 2005), where being early grants monopoly profits until other firms follow, but being late yields access to better technology. In contrast to these games, the profit function of a follower in the present game is not decreasing in the decision variable of the leader (the follower benefits from from a late first-size-at-capture of his opponents as this leaves a larger stock when he enters).

chooses to apply a smaller first-size-at-capture than his opponents ($s_0 < \varsigma_0$), his profit function π^A includes two integrals, one from s_0 to ς_0 where agent i is harvesting alone, and one from ς_0 onwards where all agents are harvesting.

$$\pi^A(s_0; \varsigma_0) = f^i \int_{s_0}^{\varsigma_0} v(s) R e^{f^i s_0 - (m + f^i)s} \, ds$$

$$+ f^i \int_{\varsigma_0}^{\infty} v(s) R e^{f^i s_0 + \sum_{j \neq i} f^j \varsigma_0 - (m + f^i + \sum_{j \neq i} f^j)s} \, ds - c(f^i) \qquad (A)$$

In contrast, when agent i chooses $s_0 \geq \varsigma_0$, his profit function π^B includes only one integral (from s_0 onwards) where all agents are harvesting:

$$\pi^B(s_0; \varsigma_0) = f^i \int_{s_0}^{\infty} v(s) R e^{\sum_{j \neq i} f^j \varsigma_0 + f^i s_0 - (m + \sum_{j \neq i} f^j + f^i)s} \, ds - c(f^i) \qquad (B)$$

Proposition 2 (Growth overfishing under knife-edge selectivity) *Under knife-edge technology, the game with N identical players and given fishing intensities has a unique symmetric Nash equilibrium first-size-at-capture. When the growth in value is linear, the Nash equilibrium first-size-at-capture is:*

$$s_0^{NE} = \frac{1}{m + \sum_{j \neq i}^N f^j} \cdot \frac{f^i}{m + \sum_{j \neq i}^N f^j + f^i} \ < \ s_0^*$$

Harvesting begins at a smaller size than is socially optimal, which implies a (incomplete) dissipation of rents.

Proof First note that the structure of problem (B) is identical to the sole-owner problem (3), only that $\widetilde{m} = m + \sum_{j \neq i} f^j$ has taken the place of m and $\widetilde{R} = R e^{\sum_{j \neq i} f^j \varsigma_0}$ has taken the place of R. Therefore, the fishing mortality of i's opponents acts as an addition to the natural mortality. The opponents' first-size-at-capture has no influence on player i's optimal choice of $\bar{s}_0$: The term $\widetilde{R} = R e^{\sum_{j \neq i} f^j \varsigma_0}$ can be taken outside the brackets of the first-order-condition.[10] Suppose the unique symmetric Nash equilibrium is the choice of s_0 that maximizes π^B, and denote $s_0^{NE} = \arg \max \pi^B$. Adapting the solution for the optimal first-size-at-capture (5), s_0^{NE} can in the linear case be written as: $\frac{1}{\widetilde{m}} \cdot \frac{f^i}{\widetilde{m} + f^i} = \frac{1}{m + \sum_{j \neq i}^N f^j} \cdot \frac{f^i}{m + \sum_{j \neq i}^N f^j + f^i}$. As players are identical by assumption, this is equivalent to $s_0^{NE} = \frac{1}{m + (N-1)f^i} \cdot \frac{f^i}{m + N f^i}$.

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[10] To see this note that

$$\frac{\partial \pi^B}{\partial s_0} = f^i \left(\int_{s_0}^{\infty} f^i v(s) R e^{\sum_{j \neq i} f^j \varsigma_0 + f^i s_0 - (m + \sum_{j \neq i} f^j + f^i)s} \, ds - v(s_0) R e^{\sum_{j \neq i} f^j \varsigma_0 - (m + \sum_{j \neq i} f^j)s_0} \right) = 0$$

implies

$$f^i \int_{s_0}^{\infty} v(s) e^{f^i s_0 - (m + \sum_{j \neq i} f^j + f^i)s} \, ds - v(s_0) e^{-(m + \sum_{j \neq i} f^j)s_0} = 0$$

Next, even if the individual fishing intensity f^i were at the efficient level, the non-cooperative first-size-at-capture is less than $\frac{1}{N}$th of the optimal one: $s_0^{NE} = \frac{1}{m+(N-1)f^i} \cdot \frac{f^i}{m+Nf^i} < \frac{1}{m} \cdot \frac{f^i}{m+Nf^i} = \frac{1}{N}s_0^* < \frac{1}{m} \cdot \frac{Nf^i}{m+Nf^i} = s_0^*$. Hence, rents are dissipated in the Nash equilibrium, although complete rent dissipation occurs only in the limit (i.e. as $N \to \infty$, $s_0^{NE} \to 0$).

Last, the fact that s_0^{NE} is indeed the unique symmetric Nash equilibrium is shown in Appendix A.2.   □

The individually optimal first-size-at-capture, s_0^{NE}, is decreasing in the total level of fishing intensity, whereas the socially optimal first-size-at-capture, s_0^*, is increasing in fishing intensity. The intuition is that collectively, it is the more important to spare small fish the more the number of fish is reduced with the onset of harvesting. But under unregulated competition, the individually rational argument is exactly the opposite: It is the more important to harvest small fish the more their number is reduced once harvesting from the other agents sets in.

The result that "the non-cooperative first-size-at-capture is less than $\frac{1}{N}$th of the optimal one" stems from the assumption that the value of a fish is a linear function of size which starts at the origin. In reality, there will probably be a minimum marketable size for fish which limits the extent of growth overfishing. Generally, the gravity of growth overfishing will depend on how much the economic and ecological value of fish increases with size for the specific fishery under consideration.

The Nash equilibrium first-size-at-capture was stated for given f^i and f^j, but these are also choice variables. I derive endogenous Nash equilibria for f numerically. The procedure and a table of exemplary values is provided in Appendix A.3. There is a stabilizing effect due to the substitutability between fishing intensity and first-size-at-capture: On the one hand, competition implies a higher f and hence a smaller first-size-at-capture. But on the other hand, there is an incentive to substitute a smaller s_0 for fishing intensity. As every agent economizes on the fishing intensity, the equilibrium first-size-at-capture is smaller. Indeed, higher costs are actually a blessing in this game; it leads to a more beneficial development of the state variable and in consequence to higher profits in the Nash equilibrium.

In contrast to the perfect selectivity case, fish are not targeted from the very moment it becomes economically viable. In a sense, the situation is akin to Bertrand competition with imperfect substitutes (compared to the price competition in perfect substitutes discussed earlier): The players face a trade-off between capturing a greater share of the market (by using a smaller mesh size) and reducing the size of the market (by diminishing the number of fish over a longer interval).

5 Regulation

What are the implications of selective harvesting for fisheries management? It is well known that an ITQ regulation—which is able to restore efficiency in the simple lumped-parameter model—does not fully eliminate the incentives for cost-increasing behaviour of the agents when there are unaccounted externalities in the production process or when the fish stock is heterogeneous (Boyce 1992; Costello and Deacon 2007). In this case, ITQs do not restore efficiency precisely because they are *not* able to "effectively separate the individual fishing decision from the development of the fish stocks" (Arnason 1990, p. 638). No undifferentiated quota system can avoid that the race to fish is played out along the dimension of size. However, I find that quotas in terms of numbers are superior to quotas in terms of weight or value.

5.1 ITQs Under Perfect Selectivity

There are two processes characterizing the development of a size-differentiated fishery. First, individual fish grow in value, creating an incentive to catch larger fish. Second, the number of fish is declining with size, which increases harvesting costs and creates an incentive to catch smaller fish. The introduction of undifferentiated ITQs does not break the second process: The quota price is independent of size and provides thus no incentive to spare small fish. However, the type of quota will be of importance. When it is given in terms of value, the individual agent is indifferent at which size he extracts one unit of quota. The agent is also indifferent when to use up a unit of quota in terms of weight if the price per kg does not change with size. In other words, only if the price increases with size will a weight-quota induce any incentive to postpone harvesting. Finally, the opportunity cost of using up one unit of number-quota decreases at the same rate as the individual fish gain value.

Let each individual i own a quota Q_i^t of type t. Introduce a new state variable $y(s)$ with $\partial y/\partial s = -f^i(s)q^t(s)$, and with $y(0) = Q_i^t$, and $\lim_{s\to\infty} y(s) \geq 0$. Denote by Q_i^v a quota in terms of value [i.e. $q^v(s) = x(s) = p(s)w(s)n(s)$], by Q_i^w a quota in terms of weight [i.e. $q^w(s) = w(s)n(s)$], and by Q_i^n a quota in terms of numbers [i.e. $q^n(s) = n(s)$]. The general problem for agent i is then to choose $f(s) \in [0, f_{max}]$ in order to maximize:

$$\pi^{qt}(f(s)) = \int_0^\infty f^i(s) \cdot [x(s) - c] \, ds$$

subject to $\quad x(0) = Rv(0)$, $\lim_{s\to\infty} x(s) \geq 0$, and

$$\frac{\partial x(s)}{\partial s} = \left(\varphi(s) - m - \sum_i f^i(s)\right) x(s),$$

$$y(0) = Q^t, \quad \lim_{s\to\infty} y(s) \geq 0, \quad \text{and}$$

$$\frac{\partial y(s)}{\partial s} = -f^i(s)q^t(s).$$

The corresponding Hamiltonian (dropping the size-subscripts) is:

$$\mathcal{H}^{qt} = [(1-\mu)x - \rho^t q^t - c]f^i + \mu x[\varphi - m - \sum_{j\neq i} f^j]$$

where $\rho^t(s)$ and $\mu(s)$ are the co-state variables associated with the quota of type t and the biovalue of the stock, respectively.

Competition implies that any surplus biovalue will be harvested with maximal intensity (see Proposition 1). This means that the future value of the stock is zero: $\mu = 0$. In contrast to the classical Bertrand game, where a quantity pre-commitment yields Cournot outcomes (Kreps and Scheinkman 1983), this is not the case here. The crucial difference is that the agents can harvest at more than one size (equivalent to naming more than one price), and the mechanism of undercutting the opponent's first-size-at-capture is hence not diluted. However, the agents do take the opportunity cost of depleting their quota into account. From the linearity of the Hamiltonian and $\mu = 0$, it can be seen that maximum effort is triggered when $[x - \rho^t q^t - c] \geq 0$.

According to the conventional maximum principle, the first-order condition relating to the co-state ρ^t would be: $\frac{\partial \rho^t}{\partial s} = -\frac{\partial \mathcal{H}^{qt}}{\partial y} = 0$, implying that ρ^t does not change over size. In so far as the maximum principle is—so to speak—a package, it is not guaranteed that the necessary conditions with respect to the co-state relating to y are valid when the necessary conditions

with respect to the co-state relating to x are not everywhere differentiable.[11] However, the marginal change in the value function as $y(0)$ increases does not depend on $x(0)$, so that the non-differentiability of the latter function has no influence on the shadow value of y. To see this note that $V(x(0), y(0)) = \int_0^\infty f^{i*}[x^*(s) - c]ds = \int_0^\infty f^{i*}[x^*(s) - c]ds = \int_0^\infty f^{i*}[(c + \rho^t q(s)) - c]ds = \int_0^\infty -\rho^t \frac{\partial y(s)}{\partial s}ds$. For a binding quota-constraint, we have $\lim_{s \to \infty} y(s) = 0$ and consequently $V(x(0), y(0)) = \rho^t y(0) = \rho^t Q^t$ and $\frac{\partial V(x(0), y(0))}{\partial y(0)} = \rho^t$. In other words, competition implies $x^*(s) = c + \rho^t q(s)$ for $s \in [s_0, s_{max}]$ and the entire value of participating in the fishery is derived from the quota.

Although the price that agent i is willing to pay for one additional quota does not depend on size, the price will depend on the type of quota. The right to harvest n fish is equivalent to the right to harvest $w(s)n$ kg of fish and $v(s)n$ Dollar of fish. As after s_{max} the increase in cost of extracting one fish exceeds the relative gain in value, the highest price that any agent would be willing to pay for the right to harvest n fish is $v(s_{max})n - c$. Consequently, $\rho^n = v(s_{max})\rho^v$ and $\rho^w = p(s_{max})\rho^v$.

For illustration, the various quota scenarios (Eqs. 6b–6d) are contrasted with unregulated competition (6a) and the social optimum (Eq. 6e, where μ^* is the shadow value of the stock under optimal management). Maximum effort is triggered when:

$$x(s) \geq \begin{cases} c & \text{non-cooperative game} & (6a) \\ \dfrac{c}{1 - \rho^v} & \text{quota in terms of value} & (6b) \\ \dfrac{c}{1 - [\rho^w/p(s)]} & \text{quota in terms of weight} & (6c) \\ \dfrac{c}{1 - [\rho^n/p(s)w(s)]} & \text{quota in terms of numbers} & (6d) \\ \dfrac{c}{1 - [\mu^*/p(s)w(s)n(s)]} & \text{social optimum} & (6e) \end{cases}$$

Under unregulated competition, harvesting sets in once the biovalue has reached the level of marginal costs. Under a regulation with ITQs in terms of value, the condition for the onset of harvesting is changed to $x(s) = \frac{c}{1 - \rho^v}$. As $\frac{c}{1 - \rho^v} > c$ (for $\rho^v \in (0, 1)$) the first-size-at-capture is larger than under unregulated competition. A value-quota works solely by constraining the extracted value, which leads to a later onset of harvesting and consequently to a somewhat more beneficial stock development. It does not provide any incentive to take the value growth of the fish stock into account. In particular, the quota is worthless ($\rho^v = 0$) when it exceeds the harvested value in the unregulated game. In other words, setting the quota to its optimal value cannot avoid complete rent dissipation along the dimension of size when the potential surplus exceeds the harvested value under non-cooperation.

An ITQ regulation in terms of weight implies yet a later first-size-at-capture when the price is increasing with size, and a number quota yields the largest first-size-at-capture. Still, no ITQ regulation restores the optimal first-size-at-capture. To see this, note that the condition for triggering maximum effort in the weight-quota case (6c) can be written as $c/(1 - \frac{\rho^w}{p(s)}) = c/(1 - \frac{p(s_{max})}{p(s)}\rho^v)$ which, since $\frac{p(s_{max})}{p(s)} > 1$ for $s < s_{max}$, is larger than $\frac{c}{1 - \rho^v}$. In contrast, when the price does not change with size, a weight-quota identical to a value-quota, since $p(s) = p(s_{max}) = p$ implies $c/(1 - \frac{p}{p}\rho^v) = \frac{c}{1 - \rho^v}$. The first-size-at-capture in a number-quota regime is determined by $x(s) = c/(1 - \frac{\rho^n}{p(s)w(s)})$ (Eq. 6d) which is larger

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[11] I would like to thank an anonymous reviewer for highlighting this aspect and Atle Seierstad for helpful discussions on this point.
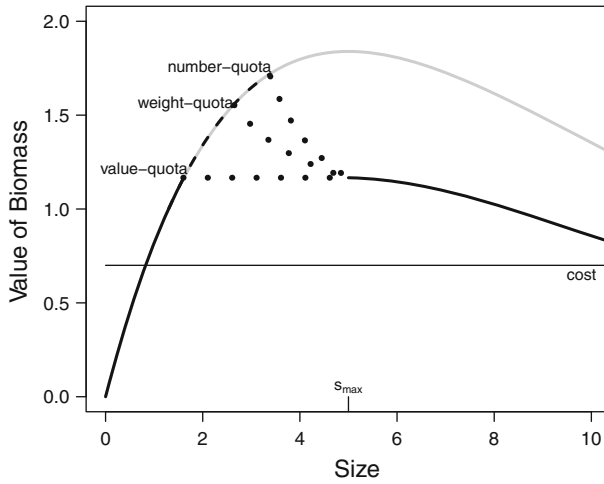
Fig. 4 Illustration of the quota regulated game

than the corresponding condition (6b) and (6c) since $\frac{p(s_{max})w(s_{max})}{p(s)w(s)}\rho^v > \frac{p(s_{max})}{p(s)}\rho^v > \rho^v$ for $s < s_{max}$ and $\rho^v \in (0, 1)$.

Figure 4 is an illustration of the stock development under the different quota regimes. Whereas unregulated competition would imply that the biovalue (thick black line) grows until it reaches the level of marginal costs (indicated by the horizontal thin black line), a value quota postpones the onset of harvesting until the biovalue reaches the level $\frac{c}{1-\rho^v}$. Since the right-hand-side of Eq. (6b) does not depend on size, the exploitation path is characterized by a straight line as in the unregulated game. The interval of harvesting is indicated by the dotted lines, it ends when the biovalue falls below the level of marginal costs of fishing plus the opportunity costs of holding a quota.

A quota in terms of weight yields an identical harvesting path when the price per kg does not depend on size (and obviously $\rho^w = \rho^v$). Otherwise, a weight-quota induces a falling harvesting pattern that starts at $x(s) = \frac{c}{1-(\rho^w/p(s))}$ and stops when the biovalue falls below the level of marginal costs of fishing plus the opportunity costs of holding a quota.

For parallel reasons as above, the harvesting pattern under a number quota is still steeper as under the other quota regimes (see Fig. 4). Such a quota regime is superior to a quota in terms of weight or value. However, from comparing (6d) to (6e) it is evident that also a number-quota falls short of the social optimum, precisely because it does not break the process of unduly forwarding harvest to exploit the lower cost of catching smaller and more abundant fish. This can be seen from the fact that even if a number-quota would incentivize an onset of harvesting at the social optimum, it would not lead to a sufficiently steep harvesting pattern.

5.2 ITQs Under Knife-Edge Selectivity

A quota regulation under knife-edge technology works essentially in the same way as under perfect selectivity since the underlying development of the fish stock is identical. As under perfect selectivity, denote the shadow price of a quota by ρ^t. Consider the effect of the different quota specifications in agent i's profit function:

$$\pi(s_0, f^i) = \begin{cases} f^i \displaystyle\int_0^\infty [1 - \rho^v]p(s)w(s)n(s)\mathrm{d}s - cf^i & \text{value} & (7a) \\[2em] f^i \displaystyle\int_0^\infty [p(s) - \rho^w]w(s)n(s)\mathrm{d}s - cf^i & \text{weight} & (7b) \\[2em] f^i \displaystyle\int_0^\infty [p(s)w(s) - \rho^n]n(s)\mathrm{d}s - cf^i & \text{numbers} & (7c) \end{cases}$$

To see that—for given f^i and f^j—a value quota does not change the individually optimal first-size-at-capture, note that $(1 - \rho^v)$ is constant and the pre-multiplication in the biovalue is the same as changing the arbitrary constant R to some R', which cancels from the first-order condition.

Moreover, to see that a weight quota has an effect on the individually optimal first-size-at-capture only if the price is increasing with size, note that when the price is constant, the same argument as above applies. Contrarily, when $p(s)$ is not constant, the first-order condition for s_0 can be written as:

$$[p(s_0) - \rho^w]w(s_0)n_0(s_0) = f^i \int_{s_0}^\infty [p(s) - \rho^w]w(s)n(s)\mathrm{d}s$$

Since the price at the first-size-at-capture is smallest when $p(s)$ increases with size, the equality between the biovalue at s_0 and the harvest will be reached at a later size, postponing the agent's first-size-at-capture.

A number-quota has the largest impact on the individually optimal first-size-at-capture. A number-quota picks up the full value growth of the fish stock over size (from weight and from price), while a weight-quota picks up only the value growth stemming from an increasing price, and a value-quota picks up neither.

Intuitively, when a fisherman is given a certain value that he can extract, he has no incentive to catch and sell high-priced fish. All the agent cares about is to harvest at lowest possible cost. If a fisherman is given a certain weight that he is allowed to land, he has some incentive to fill this quota with high-priced fish, but only if there is any price-differentiation. Finally, if a fisherman is given a quota in terms of numbers he has an incentive to avoid small fish and target larger and more valuable fish. Nevertheless, no quota accounts for the stock-dependence of the harvesting costs. In conclusion, no quota regime is able to fully eliminate growth overfishing, as none of them breaks the "rule of capture" (Boyce 1992).[12]

6 Discussion

Optimizing along one dimension may be fruitless as long as competition along the other dimension cannot be curbed. Growth overfishing could lead to a complete rent dissipation under ITQs in terms of value and weight if the price is constant and the total quota size is excessive. ITQs in terms of numbers fare better, but are also not sufficient to restore the social optimum. In general, ITQ regimes are often fraught with problems of discarding and

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[12] The result from Quaas et al. (2010) that number-quotas are sufficient to restore the first-best, is a special case when there are only two cohorts with equal mortality. It is implicit in their Corollary 2, that also in the case of only two harvestable cohorts a single quota instrument is not sufficient in general.

compliance. Introducing an ITQ regime in terms of numbers raises additional questions: Which is the appropriate quota size? On the one hand, if one simply converts the currently harvested biomass to numbers at the current unregulated size-structure, the quota will be significantly too large. On the other hand, if one sets the quota at the prospective regulated size-structure, the quota will be so small that it implies significant short-term losses until the regulated size-structure has established itself.

Other undifferentiated regulations, such as a restriction of the first-size-at-capture or the maximal fishing intensity, are simpler but similarly unable to restore the first-best. A gear restriction to not fish any fish smaller than s_{max} eliminates growth overfishing by definition. The optimum optimorum is reached, but if the number of players were endogenous, agents would enter the fishery until all rents are dissipated (Turvey 1964).

A regulation of the fishing intensity can retain some rents in the fishery. All agents still fish maximally as soon as $x(s) \geq c$, only now, by adequately limiting f_{max}^i, the condition $\sum_{i \neq j} f_{max}^i \geq \varphi(s) - m$ in Proposition 1 would not be fulfilled for small s. Not all economically profitable fish could be harvested in the beginning. Hence, the biovalue would initially rise over the level of costs. For larger fish, also the limited fishing mortality is sufficient to remove all economically profitable fish. Under knife-edge selectivity, an effort regulation restricts the effect of non-cooperative exploitation to the dimension of the first-size-at-capture. Yet, as shown above (Proposition 2), rents are dissipated even when effort is fixed to its optimal level. All in all, it is not surprising that a two-dimensional problem generally requires a two-dimensional tool to fix it. An ITQ scheme coupled with an appropriate gear regulation would restore efficiency in this model. Alternatively, one could complement the quantity-regulation via ITQs with a price-regulation via landing taxes as proposed by Smith and Gopalakrishnan (2010).

The present model is a highly simplified account of real-world fisheries, leaving many questions open for further research. First, the exact trade-offs under knife-edge selectivity warrant further investigation. For example, suppose that a regulator sets a total quota in terms of numbers so that—at the socially optimal first-size-at-capture—efficiency is restored. Then, every individual has an incentive to use a smaller mesh size in order to reduce his fishing intensity and hence his costs, still catching the same quantity. This would reduce the price he is willing to pay for an additional quota, and as this holds for all agents the quota price will not reflect the true shadow price of the resource.

Similarly, the stability of the Nash equilibrium described in section 4.1.2 has the property that any player would be willing to offer up to the entire surplus for *all* other players leaving the fishery, while any player would also accept any positive payment ε to leave the fishery. As the game is completely symmetric, it is of course stable in the sense that there is no solution to who would be the player that buys out all the other players. In general, a discussion of this aspect opens a lot of questions that are "outside the model": How would a stable number of participants emerge? How would the obtainable surplus be divided in a bargaining solution? How could it be enforced?

Moreover, a sharper selection pattern might have unanticipated consequences. For example, it is often argued that fecundity in many fish species increases with size and that it is therefore important to safeguard the so-called "super-spawners" (Smith and Gopalakrishnan 2010). Additionally, concerns about harvest induced evolution and ecosystem effects are sometimes raised (Zhou et al. 2010). Neither recruitment nor multi-species aspects were considered in the stylized model presented here. However, it has to be stressed that the number of large fish increases by a change in the gear selectivity. Since more fish are allowed to survive for a longer time, they are more abundant at the onset of harvesting and they will remain more numerous than under current selectivity.

Finally, the fundamental simplification of this model is to assume equilibrium along the dimension of time. A series of issues relate to the approach path to an equilibrium. The problem of how a quota should be specified so that the equilibrium is indeed reached, and the potential consequences of a drastically modified selection pattern have been discussed above. In general, the time it would take for a new size structure to establish itself will depend both on the specific life-histories of the fish stock and on the existing technological characteristics of the fishery in question. Full generality requires the use of partial differential equations. Cutting through this and analyzing the problem solely along the dimension of size made it possible to solve the non-cooperative game. The sole-owner solution is identical for the fully differentiated case (Brokate 1985), but although it appears intuitive, one can of course not be certain that the same holds true for the non-cooperative equilibrium. Modeling a dynamic non-cooperative game in a size- and time differentiated system is then the challenge of future work.

7 Conclusion

It is a fundamental fact that fish stocks consist of individual fish that each have a life history. Still, in economics there is a long tradition of describing the state of the fish stock by one variable. However, it is increasingly becoming clear that "fishing down the size structure" may have equally adverse effects on the health of fish stocks as "fishing down the food web" (Pauly et al. 1998) has on marine ecosystems. Moreover, harvesting fish that are inefficiently small may amount to a considerable socio-economic loss. In this paper, I have shown that growth overfishing can be seen as a universal feature emerging from competitive harvesting behavior.

This has crucial implications for fisheries management. In particular, indirect regulation with individual quotas will no longer be sufficient to restore the first-best. Furthermore, it makes a difference whether a quota is specified in terms of numbers, weight, or value. The former is the superior instrument in the setting discussed here. The reason is that fishing is a process which removes individuals from a population, but it is the value of these individuals that generates economic profits. Nevertheless, no ITQ regulation will restore efficiency as long as the agents target smaller sizes in order to reduce harvesting costs. Additional regulation will be necessary. Empirically, some sort of size limitation is probably in place in any fishery, but the problem is that these are often far away from the bio-economic optimum. There has been a lot of focus on ITQ management in the literature, and it is now high time that gear selectivity receives more attention.

A Appendix

A.1 Proof that the Non-Cooperative Exploitation Path is Unique (Lemma 2)

I show that any path different from (2), i.e. $f^i = g^i$ for $x(s) = c$ and $f^i = f^i_{max}$ for $x(s) > c$, cannot be a Nash equilibrium. Note that the surplus biovalue $x(s) - c$ is a upper-semi-continuous function defined on the compact set $[s_0, s_{max}]$ and it therefore has a maximum by the Extreme Value Theorem. Furthermore note that $f^i > g^i$ for $x(s) = c$ and $f^i > 0$ for $x(s) < c$ cannot feature any equilibrium path as it would imply negative profits for player i.

First, suppose that $f^i < g^i$ for $x(s) = c$ and $f^i = f^i_{max}$ for some $x(s) > c$. Let $s_i = \inf\{s : \sup\{s \le s_{max} : f^i(s) < f^i_{max}\}\}$. That is s_i is the first of all sizes over which a switch from non-maximal to maximal fishing intensity occurs. In other words, s_i is the first size at which the biovalue is reduced to c after it has passed the size s_0. As the interval $[s_0, s_i]$ is closed and bounded, the surplus biovalue has a maximum. If this maximum is in the interior, it would be optimal for i to deviate from the strategy $f^i < g^i$ and harvest maximally at this interior point. If the maximum is at s_i, then it would still be optimal for player i to deviate and harvest maximally one instant before s_i: The surplus is shared among all that participate in harvesting at s_i, while when harvesting one instant before s_i, the surplus accrues to player i alone. In the limit, $s_i \to s_0$, contradicting $f^i < g^i$. Hence this kind of strategy is no candidate for an equilibrium.

Then, suppose that $f^i = g^i$ for $x(s) = c$ and $f^i < f^i_{max}$ for $x(s) > c$. For strategies of this kind, the exploitation path would look identical to the one described by (2) as the development of the natural biovalue is continuous. The value $x(s) = c$ is reached before $x(s) > c$ and $\sum g^i$ keeps it at this level until s_{max}. Still strategies of this kind cannot be a feedback equilibrium: Provided all other players follow this strategy, and there was—for some reason—surplus available over some interval, then player i could deviate profitably from the strategy $f^i < f^i_{max}$ by harvesting maximally when the surplus is largest.

Finally, suppose that $f^i < g^i$ for $x(s) = c$ and $f^i < f^i_{max}$ for $x(s) > c$. The argument is the same as above, only here $f^i < g^i$ ensures that the surplus actually becomes positive. Again, given the other player's fishing intensity, there is an size at which the surplus is largest and player i could gain from deviating and harvesting maximally at this point. This exhausts the possibilities of different generic non-cooperative exploitation paths and leaves the path described by (2) as the unique equilibrium outcome.

A.2 Proof that s_0^{NE} is the Unique Nash Equilibrium in the Game Under Knife-Edge Selectivity (Proposition 2)

To prove that s_0^{NE} is indeed the unique symmetric Nash equilibrium, I show that player i prefers to be the follower (apply a larger mesh size than his opponents) and chooses $s_0^{NE} = \arg\max \pi^B$ when his opponents' first-size-at-capture is $\varsigma_0 \le s_0^{NE}$, and that he prefers to be the leader (apply a smaller mesh size than his opponents) and chooses some $s_0^L = \arg\max \pi^A$ when the first-size-at-capture of his opponents is $\varsigma_0 > s_0^{NE}$. That is, player i's best-reply function is a continuous mapping from $[0, \infty)$ to $[0, \infty)$ which crosses the 45-degree line only once at $\varsigma_0 = s_0^{NE}$ (Fig. 5 is an example for the linear case).

In order to do so, it is helpful to define three profit functions: First, the function π^i (Eq. 8), where only agent i is harvesting; second, the function π^N (Eq. 9), where there are N agents harvesting and all opponents of player i start harvesting at size $s = 0$; and third, the function π—Eq. (10), the profit function of interest—where N agents harvest, with player i opponents applying the first-size-at-capture ς_0. Let $\tilde{m}$ denote $m + \sum_{j \ne i} f^j$ and—for sake of the argument—let $s_0 \in [0, s_{max}]$ (It cannot be optimal to start harvesting when the value of the unharvested stock is already declining). In function π, the term $e^{\mathbf{1}_{s_0 > \varsigma_0}[-(\tilde{m}-m)(s-\varsigma_0)]}$ (where $\mathbf{1}_{s_0 > \varsigma_0}$ refers to the indicator function) equals $e^{-(f^j)(s-\varsigma_0)}$ for $s_0 > \varsigma_0$ and equals 1 otherwise, so that π combines π^A and π^B from the main text. The corresponding biovalues are $x^i(s; s_0)$, $x^N(s; s_0)$, and $x(s; s_0, \varsigma_0)$.

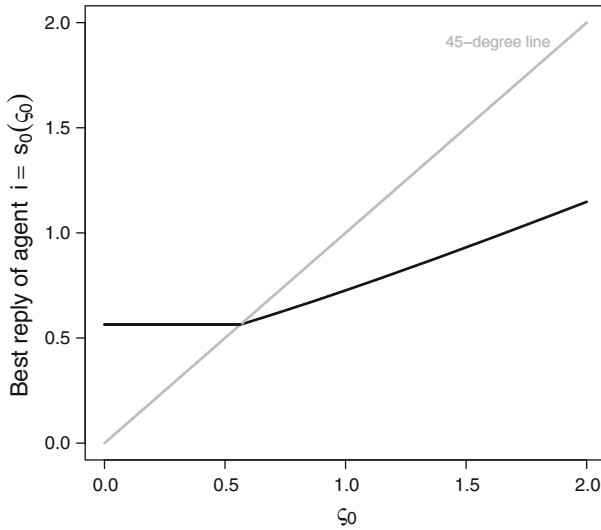$$\pi^i(s_0) = f^i \int_{s_0}^{\infty} v(s) R e^{f^i s_0 - (m+f^i)s} \, ds - c(f)$$

Fig. 5 Best reply function for $v(s) = s$, $f^i = 0.55$, $N = 2$

$$= f^i \int_{s_0}^{\infty} x^i(s; s_0) \, ds - c(f) \tag{8}$$

$$\pi^N(s_0) = f^i \int_{s_0}^{\infty} v(s) R e^{f^i s_0 - (\widetilde{m} + f^i) s} \, ds - c(f)$$

$$= f^i \int_{s_0}^{\infty} x^N(s; s_0) \, ds - c(f) \tag{9}$$

$$\pi(s_0; \varsigma_0) = f^i \int_{s_0}^{\infty} v(s) R e^{f^i s_0 - (m + f^i) s} e^{\mathbf{1}_{s_0 > \varsigma_0} [-(\widetilde{m} - m)(s - \varsigma_0)]} \, ds - c(f)$$

$$= f^i \int_{s_0}^{\infty} x(s; s_0, \varsigma_0) \, ds - c(f) \tag{10}$$

The derivatives of these three functions are:

$$\frac{\partial \pi^i}{\partial s_0} = f^i \left[-x^i(s_0) + f^i \int_{s_0}^{\infty} x^i(s; s_0) \, ds \right]$$

$$\frac{\partial \pi^N}{\partial s_0} = f^i \left[-x^N(s_0) + f^i \int_{s_0}^{\infty} x^N(s; s_0) \, ds \right]$$

$$\frac{\partial \pi}{\partial s_0} = f^i \left[-x(s_0, \varsigma_0) + f^i \int_{s_0}^{\infty} x(s; s_0, \varsigma_0) \, ds \right]$$

First note that π and its derivative are in between π^i and π^N and their respective derivatives as $x^N(s_0) \leq x(s_0) \leq x^i(s_0)$. In fact, $\pi^i(s_0)$ can be written as $\pi(s_0, \infty)$ and $\pi^N(s_0)$ could be written as $\pi(s_0, 0)$. When player i applies a smaller first-size-at-capture than his opponents $s_0 < \varsigma_0$, the stock has not been depleted by their mortality yet, so that $x^i(s_0) = x(s_0, \varsigma_0)$. However, the fishing mortality of i's opponents does kick in at a later size, so that $\int_{s_0}^{\infty} x^i(s; s_0) ds > \int_{s_0}^{\infty} x(s; s_0, \varsigma_0) ds$. A larger ς_0 implies a lighter harvesting pressure on the stock, so that $\int_{s_0}^{\infty} x(s; s_0, \varsigma_0) ds$ is increasing in ς_0.

The function π^i is the sole-owner profit function, which has a unique maximum at s_0^* (see the discussion of Eq. 4a in the main text). The structure of π^N is identical to π^i (only that m is replaced by $\widetilde{m}$ and the property of the biovalue to have a unique peak does not depend on the level of natural mortality), so that there is also a unique value of s_0 that maximizes π^N. Denote this value s_0^{NE}.

Now, if $s_0 > \varsigma_0$, it follows that $x(s_0, \varsigma_0) = x^N(s_0)e^{(\widetilde{m}-m)\varsigma_0}$ for all $s \geq \varsigma_0$. Hence

$$\frac{\partial \pi}{\partial s_0} = \frac{\partial \pi^N}{\partial s_0} e^{(\widetilde{m}-m)\varsigma_0}.$$

The profit function π and π^N are proportional if $s_0 > \varsigma_0$. Provided the maximum is achieved for $s_0 > \varsigma_0$, it is the same:

$$\frac{\partial \pi^N(s_0)}{\partial s_0} = 0 \text{ for } s_0 > \varsigma_0 \quad \Rightarrow \quad \frac{\partial \pi(s_0)}{\partial s_0} = 0$$

Thus the best-reply to $\varsigma_0 \leq s_0^{NE}$ is $s_0 = s_0^{NE}$.

Conversely, if $\varsigma_0 > s_0^{NE}$, it follows that for $s_0 > \varsigma_0$

$$\frac{\partial \pi^N(\varsigma_0)}{\partial s_0} e^{(\widetilde{m}-m)\varsigma_0} = \frac{\partial \pi(\varsigma_0)}{\partial s_0} < 0.$$

As the function π^N has no second maximum, π is not maximized by any choice of $s_0 > \varsigma_0$. However, π attains a maximum at some $s_0^L < \varsigma_0$:

Recall that π is continuously differentiable and in between π^N and π^i. At $s_0 = s_0^{NE}$, the derivative of π^N is zero and the derivative of π^i is positive, so that also the derivative of π is non-negative. At $s_0 = \varsigma_0$ the derivative of π is negative. By continuity, it must be zero somewhere in $[s_0^{NE}, \varsigma_0)$.

In conclusion, player i prefers to apply a smaller first-size-at-capture than his opponents if they choose $\varsigma_0 > s_0^{NE}$. This completes the proof.

A.3 Nash Equilibria in Knife-Edge Selectivity Game

Table 1 gives the results from the simulations of the linear knife-edge game (with $R = 1$, $m = 0.2$) for several players. $N = 1$ is the sole-owner reference case. The pseudocode of the algorithm is presented below. When the number of players exceeds 4 (5 when costs $c = 0.03$), the fishery becomes economically inviable for the given parameter values and the algorithm does not converge.

```
ς₀, fʲ ⇐ random initial values
Do Until |σ − ς₀| & |e − fʲ| ≤ ε
        max πᴬ(ς₀, fʲ) by choosing s₀ < ς₀, fᴬ
        max πᴮ(ς₀, fʲ) by choosing s̄₀ ≥ ς₀, fᴮ
```

Table 1 Simulations of knife-edge game

Cost = 0.03	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$
s_0^*	4.29	0.22	0.10	0.06	0.05
Total $f = \sum_i f^{i*}$	1.21	3.88	4.53	4.80	4.95
f^{i*}		1.94	1.51	1.20	0.99
Profit	1.78	0.15	0.05	0.02	0.01
Cost = 0.3	$N = 1$	$N = 2$	$N = 3$	$N = 4$	
s_0^*	3.55	0.56	0.27	0.18	
Total $f = \sum_i f^{i*}$	0.49	1.11	1.29	1.40	
f^{i*}		0.55	0.43	0.35	
Profit	1.60	0.33	0.13	0.07	
Cost = 3	$N = 1$	$N = 2$	$N = 3$	$N = 4$	
s_0^*	2.15	0.85	0.50	0.35	
Total $f = \sum_i f^{i*}$	0.15	0.26	0.30	0.32	
f^{i*}		0.13	0.10	0.08	
Profit	0.95	0.34	0.16	0.09	

```
If          π^A > π^B
Then        ς_0 ⇐ s_0
            f^j ⇐ f^A
Else        ς_0 ⇐ s̄_0
            f^j ⇐ f^B
EndIf
σ, e ⇐ ς_0, f^j
```

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