Analytical solution of polymer slug injection with viscous fingering

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Abstract

We present an analytical solution to estimate the minimum polymer slug size needed to ensure that viscous fingering of chase water does not cause its breakdown during secondary oil recovery. Polymer flooding is typically used to improve oil recovery from more viscous oil reservoirs. The polymer is injected as a slug followed by chase water to reduce costs; however, the water is less viscous than the oil. This can result in miscible viscous fingering of the water into the polymer, breaking down the slug and reducing recovery. The solution assumes that the average effect of fingering can be represented by the empirical Todd and Longstaff model. The analytical calculation of minimum slug size is compared against numerical solutions using the Todd and Longstaff model as well as high resolution first contact miscible simulation of the fingering. The ability to rapidly determine the minimum polymer slug size is potentially very useful during enhanced oil recovery (EOR) screening studies.

Keywords Polymer slug · Viscous fingering · Enhanced oil recovery · Method of characteristics

1 Introduction

Polymer flooding is the most widely used chemical enhanced oil recovery (EOR) technique in the world, with more incremental oil recovery attributed to this method than all other types of chemical EOR combined [21]. It is typically used in more viscous oil reservoirs ($1 < \mu_o < 100cp$) as mixing polymer into the injected water increases the viscosity of the aqueous phase and in turn reduces its mobility relative to the oil. It is thus sometimes referred to as augmented waterflooding (e.g., [2]) or water-based flooding, as the physical properties of the injected water are modified by adding the polymer. It is particularly relevant and attractive today, because oil companies are increasingly looking to develop more viscous oil fields as the fields with lighter crudes become mature.

Injecting polymer solution rather than water results in a higher shock front saturation compared to an ordinary

waterflood, reducing watercut until the polymer front breaks through, as well as improving the overall macroscopic sweep efficiency in heterogeneous reservoirs [9]. It can also help stabilize displacements for which the oil-water shock front mobility ratio is greater than 1 [5], thus reducing or preventing the degree of viscous fingering. An example of this is shown in Fig. 1. Here, the oil is 50 times more viscous than water, so the injected water front is unstable and immiscible fingering between the injected water and resident oil is observed (see Fig. 1a). Dissolving polymer in the injected water, so that the viscosity of the aqueous phase matches that of the oil, makes the polymer front stable although the leading shock front between the connate water bank and the oil may still be unstable (Fig. 1b). Figure 1c shows the average saturation profiles between the injection and production wells for the two cases.

As for other EOR techniques, the additional costs associated with polymer injection are typically higher than waterflooding. The main extra cost is the polymer itself, incurred continuously as operating expenditure (OPEX). Sheng et al. [24] reported that the average cost of polymer (excluding related costs such as processing cost) is around USD4.00 per barrel of incremental oil achieved, which can be a substantial addition to the oil lifting cost.

Costs can be reduced by injecting a fixed volume (or slug) of polymer solution, followed by water. However, care has to be taken to ensure that the polymer slug is sufficiently

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Fig. 1 Illustration of the stabilization of viscous fingering by polymer injection in a line drive case with an oil-water viscosity ratio of 50 and a polymer-oil viscosity ratio of 1. **a** Saturation distribution seen in water injection. **b** Saturation distribution seen in polymer injection. **c** Average water saturation between the injector and the producer from (a), (b). **d** Oil recovery curves. All shown at 0.1 PVI. All the parameters used in this simulation are as per Table 1

large that it maintains its integrity from the injection to the production well.

Various factors including adsorption and viscous fingering can potentially destroy the slug. Adsorption means that polymer is lost progressively from the slug as it moves through the reservoir. If the slug is too small then all the polymer may be adsorbed before it reaches the production well. Viscous fingering occurs at the trailing edge of the polymer slug because the chase water is less viscous and thus more mobile than the polymer solution. Miscible viscous fingers of water tend to form and grow into the polymer slug. If these fingers reach the leading edge of the slug then the slug integrity is destroyed and the benefits of polymer injection are lost (Fig. 2). A number of authors have presented analytical solutions that can be used to estimate the impact of adsorption on a polymer slug including [3, 20, 22, 24], and [8], but to date no-one has derived a solution to describe the impact of viscous fingering.

Very high resolution simulations are required to properly capture the growth and development of viscous fingers (e.g., [10, 13]). Using the required number of grid cells is impractical, if not impossible, especially in field-scale studies. This has driven the development of empirical fingering models which capture the average behavior of a

fingering front. These were originally derived for application in miscible gas injection ([14, 16, 25]) but Bondor et al. [7] proposed that the Todd and Longstaff model [25] could also be used to describe the fingering of water into the rear of a polymer slug. One drawback of these fingering models, however, is that the fitting parameters in their formulation may need to be calibrated by comparison with detailed simulation.

Using such empirical models to describe fingering has enabled various authors to subsequently derive analytical solutions to describe various miscible gas injection processes. For example, Blunt and Christie [6] and Juanes and Blunt [15] have shown how it is possible to predict the behavior of water alternating gas (WAG) displacements using the Todd and Longstaff model. Such analytical models enable rapid estimation of the best ratio of water to gas to inject to maximize recovery as well as providing a means for validating numerical models.

This paper presents an analytical expression to estimate the minimum polymer slug size needed to maintain the effectiveness of the polymer flood in the event of viscous fingering of the chase water case. This is obtained from a new semi-analytical solution that describes the average effects of the fingering of chase water into a polymer

Fig. 2 Illustration of the chase water fingering into a polymer slug and destroying the slug integrity. This reduces or destroys the benefits of polymer injection. **a** Map of polymer concentration. **b** Map of water phase saturation. **c** Average water saturation between the injector and the producer from (a), (b). **d** Oil recovery curves. All shown at 0.5 PVI. All the parameters used in this simulation are as per Table 1

slug in 1D. We first briefly review the analytical solution of continuous polymer injection, following which the derivation describing the dynamics of a stable chase water front position as a function of time is made. These solutions are then combined and extended to capture the effect of miscible viscous fingering of the chase water into the polymer slug. This is achieved by assuming that the average effect of fingering can be represented by the empirical Todd and Longstaff model. Finally, the validity of the analytical technique is demonstrated by comparing it against its numerical equivalent as well as high resolution first contact miscibility simulation.

2 Analytical solution for continuous polymer injection

Let us first consider continuous polymer injection into a homogeneous 1D model so that we can understand the key features of the displacement. We assume that the system is incompressible, there is no adsorption of polymer onto the rock, and physical diffusion and dispersion are negligible. We define a normalized concentration of polymer solution in the water phase,

$$C_p = \frac{c_p}{c_{p,\text{inj}}} \quad (1)$$

where c_p is the polymer concentration in mass of polymer per volume of water and $c_{p,\text{inj}}$ is the injected concentration of polymer in the same units. Using this definition $\mu_w(C_p = 0) = \mu_w$ and $\mu_w(C_p = 1) = \mu_p$, where μ_w is the water viscosity and μ_p is the polymer viscosity at the injection well.

As shown in Appendix A, the conservation of aqueous phase saturation S_w and C_p can be expressed as two hyperbolic, first order equations

$$\frac{\partial S_w}{\partial t_d} + \frac{\partial F_w}{\partial x_d} = 0 \quad (2)$$

$$\frac{\partial C_p S_w}{\partial t_d} + \frac{\partial f_p F_w}{\partial x_d} = 0 \quad (3)$$

where F_w is the fractional flow of the water phase and f_p is the fractional flow of the polymer component in the water phase.

If we assume the effects of gravity and capillarity are negligible, the fractional flow of the aqueous phase F_w is given by

$$F_w = \frac{Q_w}{Q} = \frac{1}{1 + \frac{\mu_w k_{ro}}{k_{rw} \mu_o}} \quad (4)$$

and the fractional flow of the polymer component in the water phase is given by

$$f_p = \frac{Q_p}{Q_w} \quad (5)$$

where Q_w is the flow rate of the water phase, Q_p is the flow rate of the polymer component in the water phase, k_{rw} is the relative permeability of water, μ_w is the water viscosity, k_{ro} is the relative permeability to oil and μ_o is the viscosity of oil. In this paper, we assume k_{rw} and k_{ro} curves depend only on water phase saturation (shown in Fig. 6) and any changes in the fractional flow function (4) are due only to changes in μ_w when the polymer is added to the water phase. In the absence of viscous fingering, $f_p = C_p$ as the polymer solution and the chase water are first contact miscible. t_d and x_d are dimensionless time and dimensionless distance in the direction of flow, respectively. They are defined by the following expressions

$$x_d = \frac{x}{L} \quad (6)$$

$$t_d = \frac{Qt}{\phi AL} \quad (7)$$

where L is the distance between the injection and production wells, A is the cross-sectional area (perpendicular to flow), ϕ is the porosity and Q is the injection rate.

For a line drive in which polymer is injected continuously into a reservoir containing oil and immobile connate water (at saturation S_{wc}), the initial conditions are

$$S_w = S_{wc}, C_p = 0; \text{ for all } x_d$$

If the injected water phase's viscosity remains constant (i.e., continuous water injection without polymer or continuous secondary polymer solution injection), we then have a Riemann problem in the half-plane which can be solved by using the method of characteristics (MOC), with x_d/t_d as the self-similar variable. Following Pope [19], the solution to this problem can be obtained graphically using two fractional flow curves, for water-oil ($F_w(S_w, C_p = 0)$) and polymer-oil ($F_w(S_w, C_p = 1)$) as shown in Fig. 3a. Note that we use $C_p = 1$ here for simplicity. The following solutions are valid for any values of $0 \leq C_p \leq 1$ as long as the F_w curves are correct for the selected concentration values.

The resulting solution for S_w against x_d is shown in Fig. 3b. Unlike the Buckley-Leverett solution for waterflooding, the profile of S_w against x_d is characterized by the existence of two shocks and a spreading wave. The first shock with saturation S_1 exists due to the formation of a connate water bank ($C_p = 0$) that has been displaced by the more viscous polymer. This is followed by another shock, at which C_p increases from 0 to 1, and water saturation increases from S_1 to S_2 . S_2 is found by the tangent to the polymer fractional flow curve that goes through origin

Fig. 3 1D solution for continuous secondary polymer injection. **a** Fractional flow curves for water-oil and polymer-oil that are used to construct the solution shown in **(b)**. **b** Secondary polymer flooding is different from waterflooding (in dashed green line) as there are two shocks, one at the beginning of the connate water bank with saturation S_1 and another at the beginning of the polymer solution with saturation S_2

as illustrated in Fig. 3a, while the “jump” from water to polymer curves (S_1 to S_2) must satisfy the following condition which imposes the conservation of mass at the vicinity of the shock

$$\frac{F_w(S_2) - F_w(S_1)}{S_2 - S_1} = \frac{F_w(S_2)}{S_2} \quad (8)$$

3 Analytical solution for polymer slug injection

Having understood the key features of continuous polymer injection into an oil reservoir with no mobile water initially, we now examine how this is modified when we inject a slug of polymer followed by chase water. This analytical solution is complicated by the fact that there is a third discontinuity that occurs between the chase water and the trailing edge of the polymer slug, in addition to the two shocks described in the previous section. As noted in the introduction, analytical solutions to this problem have been presented by Bedrikovetsky [3], Ribeiro et al. [22], de Paula and Pires [20], Borazjani et al. [8], and Vicente et al. [26]. However, these solutions are more mathematically complex than those presented here; moreover, they focused on assessing the effects of adsorption and did not consider the effect of viscous instability.

3.1 No fingering

We first consider the case when there is no fingering of the chase water into the polymer slug. Polymer injection

is stopped at time $t_{d,slug}$ and followed by chase water. The injection rate of the chase water is the same as the injection rate used to inject the polymer solution.

In the case of chase water injection, a further discontinuity forms at the trailing edge of the polymer slug, in addition to the shock at the front of the connate water bank and the shock at the leading edge of the polymer slug. This discontinuity does not travel at constant speed, unlike the first two shocks. This was first analyzed by Bedrikovetsky [3] using Green’s theorem; however, we shall explain the dynamics of this discontinuity more simply here using a geometric construction in the graph of fractional flow against water saturation. This will enable us to describe the late time behavior of the discontinuity and subsequently derive an analytical expression for estimating the minimum slug size in the presence of viscous fingering in the next subsection.

The boundary conditions corresponding to the injection of the chase water are

$$S_w = 1, C_p = 1; \quad 0 < t_d < t_{d,slug}$$

$$S_w = 1, C_p = 0; \quad t_d > t_{d,slug}$$

The discontinuity between the chase water and the polymer slug is found at distance x_3 from the inlet. The water saturation immediately downstream of the discontinuity is S_3 and the water saturation immediately upstream is S_4 .

Let us now find x_3 and saturation S_3 . First, select a value S_3 on the polymer-oil fractional flow curve remembering that $S_2 < S_3 < 1 - S_{or}$. We then draw a line tangent to this fractional flow curve with a gradient of $v_3 = dF_w(S_3)/dS_w$. This line passes through the y-axis and x-axis at points A and B respectively as illustrated in Fig. 4a.

Now consider Fig. 4b. The area under the curve representing the slug is given as the sum of A_1 and A_2 . For the polymer volume to be conserved (assuming no adsorption), this area must equal the injected volume of $t_{d,slug}$. Hence, we have

$$t_{d,slug} = S_2(v_2 t_d - x_3) + \int_{S_2}^{S_3} x_d dS_w - (S_3 - S_2)x_3$$

where v_2 is the characteristic velocity of the leading edge of the polymer slug.

Since for $x_d \geq x_3$, $x_d = \frac{(dF_w(S_w, C_p=1))}{dS_w} t_d$, we then have

$$t_{d,slug} = S_2 v_2 t_d - S_2 x_3 + (F_3 - F_2) t_d - (S_3 - S_2) x_3$$

where $F_2 = F_w(S_w = S_2, C_p = 1)$ and $F_3 = F_w(S_w = S_3, C_p = 1)$.

From Buckley-Leverett theory and the Welge construction (Fig. 4a), $F_2 = S_2 v_2$. Subsequently,

$$t_{d,slug} = S_2 v_2 t_d + F_3 t_d - S_2 v_2 t_d - S_3 x_3$$

$$t_{d,slug} = F_3 t_d - S_3 x_3 \quad (9)$$

Since $x_3 = v_3 t_d$, (6) can be rearranged to give

$$\frac{t_{d,\text{slug}}}{t_d} = F_3 - S_3 v_3 = A \quad (10)$$

$$x_3 = v_3 t_d = \frac{A t_d}{B} \quad (11)$$

Note that when $t_d = t_{d,\text{slug}}$, $x_3 = 0$ and so $F_3 = 1$. This indicates that the trailing edge of the slug is at the injection well and the water saturation at the injection well is 1, as expected. As t_d increases, $F_3 - S_3 v_3$ decreases reflecting a decreasing value of S_3 which causes F_3 to decrease and v_3 to increase (remember that S_2 the leading shock front saturation of the polymer slug remains constant). To find the limiting value of S_3 (the saturation at the trailing edge of the polymer slug) and v_3 , we let t_d go to infinity in Eq. 10. This gives $v_3 = F_3/S_3$. This can only be true if S_3 has decreased to S_2 so the trailing edge of the polymer slug is traveling at the same speed as the leading front, i.e.,

$$v_3 = \frac{F_3}{S_3} = \frac{F_2}{S_2} = v_2 \quad (12)$$

This means that at late times, we have a rectangular saturation profile for the slug. The length of the slug, $x_2 - x_3$, is simply given by

$$x_2 - x_3 = \frac{t_{d,\text{slug}}}{S_2} \quad (13)$$

At the chase water front, the value of S_4 can be determined graphically from S_3 at any given time by performing a jump from polymer-oil curve to water-oil curve in Fig. 4a similar to Eq. 8

$$\frac{F_w(S_4) - F_w(S_3)}{S_4 - S_3} = \frac{F_w(S_4)}{S_4} \quad (14)$$

Fig. 4 1D solution for polymer slug injection. **a** Fractional flow curves for water-oil and polymer-oil. The blue dashed line with a gradient of v_3 that goes through points A and B can be used to calculate x_3 using (11). The change in saturation from S_3 to S_4 can be determined by taking a jump between the two curves using a line that goes through the origin, S_3 and S_4 . **b** The position of slug trailing edge x_3 is determined by considering the area $A_1 + A_2$ which should be equal to $t_{d,\text{slug}}$

3.2 Empirical model of miscible viscous fingering

The discussion thus far assumes that the interface between chase water and slug tail is stable without any occurrence of viscous instability. In reality, the interface is unstable and viscous fingering of the chase water into the polymer slug is expected as the chase water is less viscous than the polymer slug and the water and polymer solution are first contact miscible. In principle, high resolution simulation is required to model the evolution of each of the fingers; however, as noted previously, Bondor et al. [7] proposed using the Todd and Longstaff model to describe the average behavior of the water and polymer components when this fingering occurs. This is now the standard way of representing the effects of viscous fingering in commercial simulations because (a) it can be implemented relatively easily in the existing black oil simulator framework and (b) it provides a fast way of approximating the likely impact of viscous fingering of the chase water on a polymer slug. Bondor et al. [7] proposed that fingering of the chase water into the polymer slug could be treated as the fractional flow of two components of the water phase: the chase water and the polymer slug. The effective polymer slug and chase water viscosities in each grid block, $\mu_{p,\text{eff}}$ and $\mu_{w,e}$, should be calculated using:

$$\mu_{p,\text{eff}} = \mu_m(C_p)^\omega \mu_p^{(1-\omega)} \quad (15)$$

$$\mu_{w,e} = \mu_m(C_p)^\omega \mu_w^{(1-\omega)} \quad (16)$$

where μ_p is the viscosity of the polymerized water at maximum polymer concentration ($C_p = 1$), μ_w is the pure water viscosity, and $\mu_m(C_p)$ is the viscosity of a mixture of pure water and polymer solution as a function of the polymer concentration. ω is a mixing parameter that can vary between 0 (no fingering) and 1 (complete mixing) but is typically set to 0.67 when there is fingering mainly based on the calibration made against experimental results made earlier by Blackwell et al. [4]. In general, this needs to be calibrated by comparison with detailed fingering simulations. The effective aqueous phase viscosity, $\mu_{w,\text{eff}}$, used in the 2-phase black oil model is then calculated using

$$\frac{1}{\mu_{w,\text{eff}}} = \frac{1 - C_p}{\mu_{w,e}} + \frac{C_p}{\mu_{p,\text{eff}}} \quad (17)$$

3.3 Analytical solution with fingering

As noted above, existing analytical solutions describing the dynamics of injecting a polymer slug followed by chase water ignore the impact of the chase water fingering into the trailing edge of the polymer slug. We now develop an approximate analytical solution that captures this effect and show that the resulting solution can be used to estimate the minimum polymer slug size needed to ensure that it is not

destroyed by fingering before it reaches the injection well. We achieve this by using the Todd and Longstaff model to represent the average effects of the fingering. To incorporate miscible fingering between the polymer slug and the chase water using the Todd and Longstaff model, we take the mass balance Eq. 3 and replace the fractional flow f_p with

$$f_p = \frac{1}{1 + \frac{\mu_{p,e} \text{eff}}{\mu_w} \frac{1-C_p}{C_p}} = \frac{1}{1 + \frac{\mu_p (1-\omega)}{\mu_w} \frac{1-C_p}{C_p}} \quad (18)$$

using Eqs. 15–16. An analogous formulation for the water-oil-solvent system was used by Blunt and Christie [6] and more recently by Juanes and Blunt [14] to derive an analytical solution for WAG injection, although in WAG injection the fingering takes place at the front between the solvent (gas) and the displaced oil. They found exact solutions by simultaneously solving the conservation Eqs. 2 and 3.

From Eqs. 12–13, we assume that the water phase saturation in the polymer slug is constant and given by the saturation at the leading edge of the polymer slug. We further justify this assumption with the observation that this saturation is typically high and the change in saturation across the slug, from trailing to leading edge, is relatively small. This assumption means that, we can reduce (3) to

$$\begin{aligned} S_2 \frac{\partial C_p}{\partial t_d} + F_2 \frac{\partial f_p}{\partial x_d} &= 0 \\ S_2 \frac{\partial C_p}{\partial t_d} + F_2 \frac{df_p}{dC_p} \frac{\partial C_p}{\partial x_d} &= 0 \end{aligned} \quad (19)$$

In this case, we can obtain the dimensionless position of a given mean concentration of polymer C_p , from the injection well, using

$$x_{d,C_p} \cong v_2(t_d - t_{d,\text{slug}}) \left. \frac{df_p}{dC_p} \right|_{C_p} \quad (20)$$

For example, by using $\omega = 0.67$ in Eq. 20 allows visualization of the spreading wave at the back of the slug as shown in Fig. 5. The front of the spreading wave ($C_p = 1$) characterizes the finger tips, which travel at a faster speed than the slug front. Its approximated velocity is given by

$$v_{\text{tips}} = v_2 \left. \frac{df_p}{dC_p} \right|_{C_p=1} \quad (21)$$

Eventually, the finger tips will reach the slug front and cause the slug to break down. The time at this occurs when the blue line (giving the approximate characteristic speed of the leading fingers) intersects the red line (giving the characteristic speed of the leading edge of the polymer slug).

Fig. 5 Incorporation of Todd and Longstaff model into the slug trailing edge using Eq. 16, assuming $\omega = 0.67$ and using a slug size of $t_{d,\text{slug}} = 0.5$ PV. The red line shows the evolution of the leading edge of the polymer slug while the solid black line shows the evolution of the trailing edge using the analytical solution derived in Section 3.1. The dashed black line shows the motion of the trailing edge if the water saturation in the polymer slug is constant and equal to the leading shock front saturation. The upper blue line shows the motion of the finger tips using the Todd and Longstaff model, which indicates that the fingers will reach the leading edge of the polymer slug at approximately $t_d = 0.69$. The lower blue line shows the motion of the trailing edge of the fingering. All data used in this calculation is from Table 1

The minimum slug size needed to ensure that the chase water fingers only just cross the polymer slug by the time the polymer breaks through is given by

$$t_{d,\text{slug,min}} = \frac{S_2}{F_2} \left(1 - \frac{1}{\left. \frac{df_p}{dC_p} \right|_{C_p=1}} \right) \quad (22)$$

Table 1 Summary of data used in this study

Parameters	Values
Fluid viscosities	
Oil	$\mu_o = 50$
Water	$\mu_w = 1$
Polymer	$\mu_p = 50$
Water-polymer mixture viscosity	$\mu_m = \left(\frac{1-C_p}{\mu_w} + \frac{C_p}{\mu_p} \right)^{-4}$
Relative permeabilities	
Oil	$\left(\frac{1-S_w-S_{or}}{1-S_{wc}-S_{or}} \right)^2, S_{or} = 0.2$
Water, polymer	$\left(\frac{S_w-S_{wc}}{1-S_{wc}-S_{or}} \right)^2, S_{wc} = 0.2$
Reservoir initial state	$S_w = 0.2, S_o = 0.8$
Injection state	
$0 < t_d < t_{d,\text{slug}}$	$S_w = 1, C_p = 1$
$t_d > t_{d,\text{slug}}$	$S_w = 1, C_p = 0$
Grid blocks	
MRST (1D model)	1000×1
ECLIPSE (1D model)	10000×1
FCM (2D model)	300×150
	(aspect ratio=1:0.5)

Fig. 6 Relative permeabilities as defined in Table 1 used in this study

Further detail is given in Appendix B. Equation 22 can also be derived graphically as presented in Section 5.

4 Evaluation of the analytical approach

We now evaluate the above analytical solution by comparing its predictions with results from detailed simulation of the viscous fingering of chase water into a polymer slug and black oil simulation in which the Todd and Longstaff model has been implemented in the polymer options.

4.1 Methodology

To model miscible viscous fingering into the trailing edge of the slug, we used a higher order, IMPES (implicit pressure, explicit saturation), finite difference simulator developed to model the details of viscous fingering in three component, two phase flows ([10, 11]). This was originally written for miscible gas applications but was adapted to model polymer slug injection. Its ability to predict viscous fingering in miscible displacements has previously been validated by

Fig. 7 Polymer front simulation in ECLIPSE and MRST. **a** Comparison between MRST, ECLIPSE, and analytical solution. **b** Grid refinement in ECLIPSE for $\omega = 1$. It can be seen that many more grid blocks (~ 10000) are needed to resolve the leading front of the polymer slug when $\omega = 1$. Note that the connate water bank has already broken through in this case

Fig. 8 Comparison of the concentration and saturation profiles obtained with ECLIPSE and the modified MRST model, when simulating a polymer flood using 100 grid blocks. The modification to the Todd and Longstaff parameter reduces smearing of the leading polymer front

comparison with experimental results by Christie [11], Christie and Jones [12], and Al-Shuraiqi et al. [1], among others.

The commercial simulator ECLIPSE [23] and an open-source simulator called MATLAB Reservoir Simulation Tools, MRST [17] were used for the black oil simulations using the Todd and Longstaff model to capture the average effects of fingering. An identical set of equations governs both simulators, and we chose to use the fully implicit scheme in both these simulators throughout the study.

All simulations used the data summarized in Table 1, unless stated otherwise. All the symbols and nomenclature are defined in Table 2. The oil and water relative permeability curves are illustrated in Fig. 6. Grid dimensions were chosen following a grid refinement study.

4.2 The modified Todd and Longstaff model

Standard implementations of polymer flooding models in commercial simulators, such as ECLIPSE, include the Todd

Fig. 9 Comparison of the concentration and saturation profiles obtained with ECLIPSE and the modified MRST model, when simulating a polymer flood using 100 grid blocks. The modification to the Todd and Longstaff parameter reduces smearing of the leading polymer front

Fig. 10 Comparison between FCM simulators, MRST and analytical model with $\omega = 0.67$, showing that the new analytical model can predict the development of the viscous fingers into the trailing edge of the slug

and Longstaff model to describe the effect of mixing between polymer solution and water and its effect on water phase viscosity. This is presumably intended to allow engineers to model the fingering of chase water into polymer as proposed by Bondor et al. [7]; however, using $\omega > 0$ can also result in unphysical spreading of the shock front at the leading edge of the polymer slug in the absence of adsorption. The front at the leading edge of the polymer slug is no longer self-sharpening and considerably more grid blocks are then required to resolve it.

This effect is illustrated for a continuous polymer flood, in Fig. 7. Figure 7a shows the polymer front obtained from 1D ECLIPSE and MRST simulations with the analytical solution for $\omega = 0$ (no mixing between polymer and water) and $\omega = 1$ (complete mixing). One hundred grid blocks were used in both cases. Both simulators give similar results in terms of numerical accuracy (first order) and in both cases the polymer front is smeared when $\omega = 1$. Figure 7b shows that 10,000 grid blocks are needed in this case to obtain a shock front close to the analytical solution. Clearly, this is impractical for field-scale simulations. One solution is to use higher-order solvers ([18, 19]), but these are more computationally expensive. Alsofi and Blunt [2] used a simpler method in streamline-based simulator, whereby the weighted average fractional flow is used to segregate the flow between the regions with and without polymer.

In this study, we adapted MRST using an approach comparable to the method proposed by Alsofi and Blunt [2]. At every time step, we calculate the maximum value of polymer concentration, $C_{p,\max}$ reached in each of the grid blocks since the beginning of injection. We then modify the Todd and Longstaff mixing parameter using

$$\omega' = \omega C_{p,\max} \quad (23)$$

where ω is the input value of the Todd and Longstaff mixing parameter. This means that the mixing parameter is zero in cells that have never seen polymer and reduced in cells that see polymer increasing (as the polymer slug advances) but is equal to the input value at the trailing edge of the polymer slug. This strategy was found to give better resolution of the leading polymer front in MRST as shown in Fig. 8.

4.3 Results—no viscous fingering

We first verify that the black oil simulators can reproduce the analytical solution derived in Section 3.1 for polymer slug injection for the case without viscous fingering. Figure 9 shows the comparison between the analytical solution and the predictions of the modified MRST using $\omega = 1$ for a slug volume of 0.4 PV. Other input data is as per Table 1. A very high resolution ECLIPSE model (DX = 10000) is also shown here.

The simulators correctly capture the location of the trailing edge x_3 and the relatively small jump in saturation from S_3 to S_4 at x_3 . The immobile oil region, formed by the locus of chase water front saturation S_4 , is also observed.

4.4 Results—viscous fingering

We now evaluate the ability of the new analytical model to predict viscous fingering of the chase water into the trailing edge of the polymer slug by comparing its predictions with those obtained when we model the fingering explicitly. Here, we consider two cases, μ_p/μ_w , of 10 and 50. We keep $\mu_p = \mu_o$ in all cases so that the potential impact of immiscible fingering at the slug front can be ruled out. The polymer slug size is 0.5 PV. We also compare these two predictions with those obtained from the modified MRST using the Todd and Longstaff model with $\omega = 0.67$.

Figure 10 shows that there is very good agreement between the three approaches suggesting that the assumptions made in the derivation of the analytical model are appropriate. In particular, the time when the finger tips reach the polymer front is successfully predicted analytically.

5 Minimum slug size computation

We now show how the analytical model can be used to calculate the minimum polymer slug size graphically that will maintain its integrity between injection and production wells even if there is fingering of the chase water into the trailing edge of the slug.

Fig. 11 Optimal slug size estimation for polymer-water viscosity ratio of 50. The red line shows the evolution of the leading edge of the polymer slug while the solid black line shows the evolution of the trailing edge for $t_{d,\text{slug}} = 0.3$ using the analytical solution derived in Section 3.1. The upper blue line shows the motion of the corresponding finger tips using the Todd and Longstaff model (assuming $\omega = 0.67$) which indicates $t_{d,\text{slug}} = 0.3$ is too small and causes the slug to break down in the middle of the reservoir. The optimal slug size can be determined by drawing a line with a gradient of v_{tips} given in Eq. 21 that meets x_2 at $x_d = 1$. All data used in this calculation is from Table 1

Fig. 12 Oil recovery for different slug sizes. **a** A slug size of 0.3 PV is too small as fingering of the chase results in the breakdown of the polymer slug before the slug reaches the production well. **b** The fingering pattern seen at the same time for a 0.52 PV slug. The fingers have just reached the leading edge of the slug as the polymer front reaches the

production well. **c** Very little fingering seen for a slug size of 0.65 PV. **d** The recovery curves obtained from the 0.52 and 0.65 PV slugs are virtually identical, whereas recovery is reduced for the 0.3 PV slug. The minimum slug size that will not be destroyed by viscous fingering at polymer breakthrough is 0.52 PV

Figure 11 shows the evolution of the fingering for $t_{d,slug} = 0.3$ PV on an $x_d - t_d$ plot. The red line shows the evolution of the leading edge of the polymer slug while the solid black line shows the evolution of the trailing edge using the analytical solution derived in Section 3.1. The dashed black line shows the motion of the trailing edge if the water saturation in the polymer slug is constant and equal to the leading shock front saturation. The upper blue line shows the motion of the finger tips using the Todd and Longstaff model, assuming $\omega = 0.67$. The lower blue line shows the motion of the trailing edge of the fingering. Clearly, with $t_{d,slug} = 0.3$ PV, we expect the slug will start to breakdown in the middle of the reservoir at around $t_d = 0.41$ PV.

The smallest slug size to minimize the breakdown of the slug by fingering should be selected such that the finger tips reach the production well ($x_d = 1$) at the same time as the polymer front. This can be graphically determined by drawing a line with a gradient of v_{tips} given in Eq. 21 that

meets x_2 at $x_d = 1$. The green line in Fig. 11 illustrates this. Then, $t_{d,slug,min}$ can be directly determined from the value of the x -intercept. Assuming $\omega = 0.67$, we found that $t_{d,slug,min} = 0.52$ PV.

To verify this, we ran the FCM simulation for $t_{d,slug}$ of 0.3, 0.52, and 0.65 PV. The results are shown in Fig. 12. As expected, $t_{d,slug} = 0.3$ PV is too small and viscous fingering of the chase water causes the slug to break down in the middle of the reservoir. This is reflected in the oil recovery plot in Fig. 12d which indicates that the polymer breakthrough at the production well occurs rather early, at around $t_d = 0.5$ PV. We also observe that the recovery is less efficient as the increase in oil is very gradual post polymer breakthrough. The recoveries obtained for $t_{d,slug} = 0.65$ PV and $t_{d,slug} = 0.52$ PV are found to be almost identical, which means that, in this case, the minimum slug size needed to ensure fingering of the chase water does not destroy the slug is $t_{d,slug} = 0.52$ PV.

Fig. 13 Optimum slug size as a function ω and viscosity ratios. All data used in this calculation is from Table 1, using Eq. 22

The minimum slug size needed to ensure fingering of the chase water does not completely break down the polymer slug for various values of ω and viscosity ratios calculated using Eq. 22 are shown in Fig. 13. We can see here the expected trend showing that high oil viscosities or low ω values necessitate the use of larger slug size. Generally in polymer flooding, polymer viscosity needs to be high enough that the oil can be swept more efficiently than waterflooding. However, interestingly, too high a polymer-oil viscosity ratio, μ_p/μ_o , has an adverse effect instead—a large slug size is required as the chase water tends to finger more through the polymer slug as we can see in Fig. 13b.

Although we need to consider many parameters to calculate the required slug size, for moderate values of ω of say around 0.4 to 0.8, Fig. 13 suggests that $t_{d,\text{slug},\text{min}}$ of around 0.5 to 0.6 PV is a good first approximation that may be used in the field.

6 Conclusion

We have investigated the fingering of chase water into a polymer slug during secondary polymer flooding in the absence of adsorption using a mixture of numerical simulations and analytical approaches. Both detailed numerical simulations, describing the fingers explicitly, and black oil simulations using a Todd and Longstaff model to represent the average effects of the fingering were performed. We have reviewed the existing analytical solutions that predict both continuous polymer flooding and a slug of polymer followed by chase water in the absence of fingering.

Existing solutions describing the injection of chase water following a polymer slug assume a stable interface between chase water and polymer slug. A graphical solution based on Welge analysis was presented in Section 3.1 and compared successfully against numerical simulation. We showed analytically that at late times the water saturation throughout the polymer slug tends to the saturation of the leading shock of the polymer slug.

We have extended this analysis to obtain an approximate method for predicting the growth of the fingering of chase water into the polymer slug due to fingering, in the absence of adsorption. This analysis provides a simple analytical expression that can be used to estimate the minimum polymer slug size needed to ensure that it is not destroyed by fingering of the chase water before polymer break through.

Comparison of detailed numerical simulations of the fingering of chase water with black oil simulations using the Todd and Longstaff model to represent the average effects of the fingering have shown that a value of $\omega = 0.67$ can be used, for oil-water viscosity ratios of 10 and 50. However, due to the empirical nature of the Todd

Table 2 Nomenclature

Symbol/abbreviation	Definition
A	Reservoir cross-sectional area
c_p	Polymer concentration in mass of polymer per volume of water
C_p	Normalized polymer concentration
$c_{p,\text{inj}}$	Injected polymer concentration in mass of polymer per volume of water
$C_{p,\text{max}}$	Maximum value of normalized polymer concentration in each of the grid blocks
DX	Number of grid blocks in x direction
DY	Number of grid blocks in y direction
DZ	Number of grid blocks in z direction
f_p	Fractional flow of polymer
F_w	Fractional flow of aqueous phase
L	Reservoir length
M	Mobility ratio
M_e	Effective mobility
N_p	Oil production
PV	Pore volume
Q	Total injection flow rate
Q_p	Polymer injection flow rate
Q_w	Water injection flow rate
S_1	Connate water bank saturation
S_2	Slug front water saturation
S_3	Slug trailing edge water saturation
S_4	Chase water front saturation
S_o	Oil phase saturation
S_{or}	Irreducible oil saturation
S_w	Water saturation
S_{wc}	Connate water saturation
t	Time
t_d	Dimensionless time
$t_{d,\text{slug}}$	Dimensionless slug size
$t_{d,\text{slug},\text{min}}$	Minimum dimensionless slug size
v_1	Connate water bank speed
v_2	Slug front water speed
v_3	Slug trailing edge water speed
v_4	Chase water front speed
v_d	Characteristic speed
v_{tips}	Approximated finger tips velocity
v_h	Hydrocarbon phase velocity
v_s	Spreading wave speed
v_t	Total velocity
v_w	Aqueous phase velocity
x_1	Connate water front position
x_2	Slug front position
x_3	Slug trailing edge position
x_d	Dimensionless distance in direction of flow
μ_m	Mixture viscosity

Table 2 (continued)

Symbol/ abbreviation	Definition
μ_o	Oil viscosity
μ_p	Polymer viscosity
$\mu_{p,eff}$	Polymer effective viscosity
$\mu_{w,e}$	Water effective viscosity
$\mu_{w,eff}$	Aqueous effective viscosity
ϕ	Fractional rock pore volume
ω	Todd and Longstaff mixing parameter
ω'	Modified ω

and Longstaff model, this value may not work for cases with heterogeneous reservoirs, very high viscosity ratios, or polymer-oil viscosity ratios that are less than 1. We expect calibration of ω is required for more general problems. When ω is provided, the optimum slug size can be rapidly determined using the approximate analytical solution. Such results can potentially be used during EOR screening or feasibility studies, during which only order of magnitude estimations are required and more accurate but highly expensive computational study may not be necessary.

We expect that the analytical solution presented can be extended to include the effects of adsorption and possibly to estimate flow behavior when polymer is injected subsequent to a waterflood.

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Appendix A

In this appendix we derive the conservation equations for the water phase and the polymer solution. Consider the conservation of mass for the water phase in one dimension

$$\frac{\partial S_w}{\partial t} + \frac{\partial v_w}{\partial x} = 0. \quad (\text{A.1})$$

where we have assumed that water is incompressible. We define the fractional flow F_w of the water phase as

$$F_w = \frac{Q_w}{Q} = \frac{v_w}{v_t} \quad (\text{A.2})$$

Fig. 14 Fractional flow f_p as a function of polymer concentration C_p for $\mu_p/\mu_w = 50$

where v_t is the sum of aqueous and hydrocarbon phase velocities, $v_t = v_w + v_h$. Substituting (A.2) into (A.1), we then have

$$\frac{\partial S_w}{\partial t} + \frac{\partial F_w v_t}{\partial x} = 0. \quad (\text{A.3})$$

Similarly, the conservation of the polymer component C_p in the water phase is given by

$$\frac{\partial C_p S_w}{\partial t} + \frac{\partial v_p}{\partial x} = 0. \quad (\text{A.4})$$

where v_p is the velocity of the polymer component. We can also define the fractional flow of polymer component in the water phase as

$$f_p = \frac{v_p}{v_w} = \frac{v_p}{F_w v_t} \quad (\text{A.5})$$

We can therefore rewrite (A.4) as

$$\frac{\partial C_p S_w}{\partial t} + \frac{\partial f_p F_w v_t}{\partial x} = 0 \quad (\text{A.6})$$

In the absence of fingering then the fractional flow of the polymer solution is simply the dimensionless concentration of the polymer in the water phase, C_p , as the polymer solution is first contact miscible with the injected chase water. In the presence of viscous fingering, the average fractional flow f_p can be described using the Todd-Longstaff formulation in Eq. 18. A plot of f_p as a function of C_p for μ_p/μ_w of 50 is shown in Fig. 14. Note that for $\omega = 1$, we have $f_p = C_p$ which models a fully-mixed, piston-like displacement.

Appendix B

We present here the estimation of the minimum slug required in order to maintain its integrity. Recall that the slug front travels at velocity $v_2 = F_2 S_2$ and it arrives at the production well when $x_2 = 1$, hence

$$x_2 = v_2 t_d = 1 \quad (\text{B.1})$$

With no fingering, the back of the slug will have approximately the same velocity as the slug front (as discussed in Section 3.1) so $v_3 = v_2$. Hence, we have

$$x_3 \cong v_2(t_d - t_{d,slug}) \quad (\text{B.2})$$

To take the fingering into account, we multiply x_3 in (B.2) by df_p/dC_p . The fastest wave occurs when $C_p = 1$, shown as the upper blue line in Fig. 5. At the production well, we have

$$x_{d,C_p=1} \cong v_2(t_d - t_{d,slug}) \frac{df_p}{dC_p} \bigg|_{C_p=1} = 1 \quad (\text{B.3})$$

The minimum slug size needed to ensure that the chase water fingers only just cross the polymer slug by the time the polymer breaks through is found by substituting (B.1) into (B.3)

$$t_{d,slug,min} = \frac{1}{v_2} \left(\frac{\frac{df_p}{dC_p} \big|_{C_p=1} - 1}{\frac{df_p}{dC_p} \big|_{C_p=1}} \right) \quad (\text{B.4})$$

$$t_{d,slug,min} = \frac{S_2}{F_2} \left(1 - \frac{1}{\frac{df_p}{dC_p} \big|_{C_p=1}} \right) \quad (\text{B.5})$$

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