ERRATA OF THE PAPER “ON THE H^p - L^q BOUNDEDNESS OF SOME FRACTIONAL INTEGRAL OPERATORS”

PABLO ROCHA, MARTA URCIUOLO, Córdoba

(Received February 18, 2014)

In the proof of Theorem 3.1 in [4], we state the following assertion:

If $0 \leq r < 1$, $0 < p \leq 1$, $1/q = 1/p - r$ and $f \in H^p(\mathbb{R}^n)$ we write $f = \sum_{j \in \mathbb{N}} \lambda_j a_j$, where a_j is a p -atom and $\sum_{j \in \mathbb{N}} |\lambda_j|^p \leq c \|f\|_{H^p}^p$. So the theorem will be proved if we obtain that there exists $c > 0$ such that $\|Ta\|_{L^q} \leq c$ with c independent of the p -atom a , since this estimate and the inequality $\left(\sum_{j \in \mathbb{N}} |\lambda_j|^q\right)^{1/q} \leq \left(\sum_{j \in \mathbb{N}} |\lambda_j|^p\right)^{1/p}$ give $\|Tf\|_q \leq c \|f\|_{H^p}$.

Although the final inequality holds, the assertion is not completely correct. Indeed, in [1], M. Bownik gives an example of a linear functional defined on a dense subspace of the Hardy space $H^1(\mathbb{R}^n)$ and he shows that although this functional is uniformly bounded on atoms, it does not extend to a bounded functional on the whole $H^1(\mathbb{R}^n)$. So in general it is not enough to verify that an operator or a functional is bounded on atoms to conclude that it extends boundedly to the whole space. See also [2].

By Proposition 2 in [4] we have that T is a well defined bounded operator from $L^s(\mathbb{R}^n)$ into $L^q(\mathbb{R}^n)$, $1/q = 1/s - r$, $1 < s < 1/r$. Also, from Remark 4.12 in [3], we obtain that the equality $f = \sum_{j \in \mathbb{N}} \lambda_j a_j$ holds in $L^s(\mathbb{R}^n)$ for $f \in H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$. So, taking a subsequence if necessary, we get

$$(1) \quad |Tf(x)| \leq \sum_{j \in \mathbb{N}} |\lambda_j| |T(a_j)(x)|$$

a.e. $x \in \mathbb{R}^n$.

So the correct assertion should be:

If $0 \leq r < 1$, $0 < p \leq 1$, $1/q = 1/p - r$, taking $1 < s < 1/r$ and $f \in H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$ we write $f = \sum_{j \in \mathbb{N}} \lambda_j a_j$, where a_j is a p -atom, the convergence

is in $H^p(\mathbb{R}^n)$ and in $L^s(\mathbb{R}^n)$, with $\sum_{j \in \mathbb{N}} |\lambda_j|^p \leq c \|f\|_{H^p}^p$. So the theorem will be proved if we obtain that there exists $c > 0$ such that $\|Ta\|_{L^q} \leq c$ with c independent of the p -atom a , since this estimate, (1) and the inequality $\left(\sum_{j \in \mathbb{N}} |\lambda_j|^q\right)^{1/q} \leq \left(\sum_{j \in \mathbb{N}} |\lambda_j|^p\right)^{1/p}$ give $\|Tf\|_q \leq c \|f\|_{H^p}$ for $f \in H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$, so the theorem follows from the density of $H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$ in $H^p(\mathbb{R}^n)$.

In [5], Theorem 1, we make a similar assertion. An analogous argument works.

References

- [1] *M. Bownik*: Boundedness of operators on Hardy spaces via atomic decompositions. Proc. Am. Math. Soc. (electronic) *133* (2005), 3535–3542. zbl MR
- [2] *S. Meda, P. Sjögren, M. Vallarino*: On the H^1 - L^1 boundedness of operators. Proc. Am. Math. Soc. *136* (2008), 2921–2931. zbl MR
- [3] *E. Nakai, Y. Sawano*: Hardy spaces with variable exponents and generalized Campanato spaces. J. Funct. Anal. *262* (2012), 3665–3748. zbl MR
- [4] *P. Rocha, M. Urciuolo*: On the H^p - L^q boundedness of some fractional integral operators. Czech. Math. J. *62* (2012), 625–635. zbl MR
- [5] *P. Rocha, M. Urciuolo*: On the H^p - L^p -boundedness of some integral operators. Georgian Math. J. *18* (2011), 801–808. zbl MR

Authors' address: Pablo Rocha, Marta Urciuolo, Famaf-UNC, Ciem-Conicet, Medina Allende s/n, Ciudad Universitaria, 5000 Córdoba, Argentina, e-mail: rp@famaf.unc.edu.ar, urciuolo@famaf.unc.edu.ar.