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Enabling greener DSL access networks by their stabilization with artificial noise and SNR margin

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Abstract Low-power modes (LPM) are a standardized means in asymmetric digital subscriber lines (ADSL) 2 for reducing the power consumption at the central office. However, the activation of LPMs is hampered by the operators' concern for instability introduced by frequent transmit power changes. The injection of artificial noise (AN) has been proposed as a standard-compliant stabilization technique. We develop an analytical solution for setting the AN power spectrum. Based on this solution we jointly optimize the AN power spectrum and the signal-to-noise ratio (SNR) margin. Simulation results show the performance gain in terms of rate and energy compared to heuristic rules for setting the AN power spectrum. We propose and demonstrate three approaches for evaluating the performance of AN-enabled DSL systems, including (a) joint spectrum balancing, AN, and margin optimization, (b) single-user worst-case-stable optimization, and (c) worst-case-stable optimization based on sequential initialization. Simulation results confirm a strong dependency of the performance under AN on the selected SNR margins, and highlight the total AN power consumption as well as the residual energy savings under low-power modes stabilized by AN.

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1 Introduction

The activation of low-power modes (LPM) [14] in the digital subscriber line (DSL) access network implies frequent transmit power changes, resulting in signal-to-noise ratio (SNR) variations with which current DSL systems can hardly cope [22]. We study the problem of stabilizing DSL systems when using LPMs. One commercially available [1] solution for protecting legacy DSL systems is the injection of "artificial noise" (AN) at the transmitter in order to shift eventual variations of the crosstalk noise inside the specified SNR margin which is considered during the initialization of the modems [22]. Besides the empirical rules in [14, 22], to the best of our knowledge the problem of setting the AN power spectrum has not been studied so far.

Our contribution is the solution of various optimization problems related to AN-enabled DSL networks and their performance evaluation, considering also regulatory and system constraints. Under a worst-case crosstalk noise assumption similar to that made in practice [22] we demonstrate an analytical AN power solution and show the gain by performing a standard-compliant joint optimization of the AN power spectrum and the SNR margin. However, we embed the problem in a more general mathematical programming framework, which in addition to this worst-case optimization allows us to derive upper performance bounds for AN-enabled DSL systems. Related to the computation of this bound we find that AN seamlessly integrates into previous dynamic spectrum management (DSM) algorithms and show *provably* near-optimal results for this bound. Our key messages are that (a) the performance under AN heavily depends on the selected SNR margin and hence AN needs to be optimized jointly with the margin; and (b) the injected AN improves the stable rate levels and energy-efficiency compared to systems which are stabilized by SNR margins only, but shows a performance loss for longer loops (e.g., above 1.5 km) compared to an ideal, frequency selective margin setting (commonly referred to as "virtual noise" [24]).

The work on LPMs in the field of DSL is motivated by governmental initiatives [9] and the substantial energy savings of 60–80 % in terms of the line-driver's (LD) power consumption which are achievable by currently standardized LPMs in downstream transmission direction [28]. The (class AB) LD is responsible for 40–50 % of the total power consumption of an ADSL2 transceiver [15] and the component which is most affected by a reduction of the transmit power. These savings are further doubled when considering support equipment such as cooling [23]. The main concern brought up against the activation of LPMs is the fluctuation in the crosstalk noise received on a line when other users enter or leave LPMs. This may lead to an increase in bit-error rate which forces the modems to reinitialize. The two proposals in [8] to resolve this issue partly require changes in standards or do not allow for immediate transitions back into the full-power mode (e.g., in less than 3 s as foreseen in [18]). The most promising proposal made in [14] is to physically inject additional "artificial noise" (AN) at the transmitter (i.e., at the central office) and thereby to reduce the relative impact of future crosstalk variations on the perceived SNR. A related technique is "virtual noise" (VN) [24] which is a frequency selective, tunable receiver-noise parameter masking the crosstalk noise. However, VN is currently not standardized for ADSL systems. Differently to VN, the AN is not masking the changing noise scenario but leads to additional received noise which makes these disturbances fit inside the used SNR margin. In other words, increasing the background noise decreases the SNR reduction when further crosstalk noise is added on top. This approach does not need any standardization effort and supports fast power state transitions. In [22] it is suggested to set the AN power spectrum to the worst-case crosstalk noise a line may experience. The only theoretical disadvantage of AN is that it leads to higher transmit power, background noise, and crosstalk noise levels. This may reduce the achievable bit-rate and conflict with the initial intention behind the usage of AN: namely to enable LPMs and thereby to reduce the energy consumption in DSL. In [7] it is argued that spectrum balancing in combination with bit-swapping is the more energy efficient solution compared to VN, and hence also compared to AN. However, this assumes the availability of the corresponding features in the modems. Furthermore, we deem the majority of possible power savings in DSL coming from the intensive usage of LPMs, which incurs much stronger crosstalk fluctuations than occurring in current networks. Hence, we study the impact of AN on energy-efficiency and bit-rate, concluding that AN significantly improves the stable system performance compared to current ADSL2 systems, especially when the currently used SNR margin is not optimized for each line individually. In this study we are concerned about the energy consumption in DSL systems operating at fixed bit-rates. However, note once more that AN enables LPMs and hence also the implicated energy savings by exploiting the variation in the users' actual rate demand, cf. [28] for a quantitative analysis thereof.

We begin in Sect. 2 by defining the system constraints and the theoretical multi-user optimization problem for jointly setting the AN power spectra, the transmit power spectra, and the bit allocations of all users. This multi-user formulation is notably more general than actually needed for the single-user problem faced in practice [22]: that is the stabilization of a line for the worst-case crosstalk noise. However, the observed "worst-case" noise during the initialization of a DSL connection depends not only on the channel but, for example, also on the lines' target rates, SNR margins, the user behavior (line usage), and the sequence in which the modems are activated. In order to facilitate a deterministic performance evaluation of AN-enabled networks we study three approaches: (a) the joint optimization of AN with the transmit power spectra, bit-allocations, and SNR margins, (b) the single-user bit-loading problem stabilizing the line for the worst-case crosstalk noise, and (c) the multi-user sequential initialization of the lines assuming the same worst-case noise as in (b) but considering the actual crosstalk noise levels at initialization. Approach (a) effectively allows to compute an optimization based bound on the performance under approach (b), and is studied in Sect. 3. In this theoretical setting we further investigate in Sect. 3.2 the impact of setting different SNR margins for different lines by means of a novel margin-search heuristic. Note that this heuristic has also applicability in networks which are not AN-enabled. Similarly, the simplified DSM approach for large networks in Sect. 3.4, based on the assumption of an identical spectral power allocation for collocated lines, is applicable to the performance evaluation in networks with and without AN capabilities. In Sect. 4 we study approaches (b) and (c), and derive an analytical solution for the AN spectrum and an optimal bit-loading algorithm for AN-enabled networks. The performance (in terms of bit-rate and energy-efficiency) under the proposed approaches is compared and the value of explicit SNR margin optimization demonstrated by simulations. Our conclusions from this work are summarized in Sect. 5.

We use bold-faced lower/upper-case letters $\mathbf{a}$ and $\mathbf{A}$ to denote vectors and matrices, respectively. Moreover, $\mathbb{R}$ and $\mathbb{C}$ mean the sets of real and complex numbers, respectively, whereas $\mathcal{CN}(\mu, \sigma^2)$ represents a circular symmetric complex normal distribution with mean μ and variance σ^2.

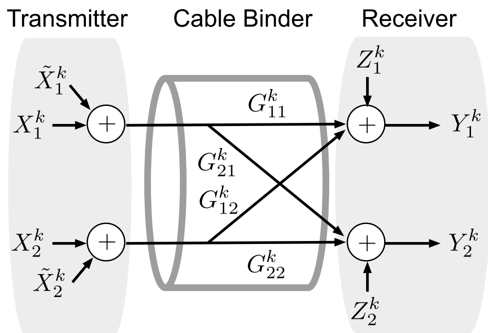

Fig. 1 Downstream signal model for the DSL interference channel on subcarrier k for $U=2$ users highlighting the addition of the artificial noise $\tilde{X}_u^k$ to the transmitted symbol X_u^k, $u \in \{1,2\}$

2 System and multi-user optimization model

2.1 System constraints

Current DSL systems use discrete multi-tone (DMT) modulation and frequency-division duplexing (FDD), effectively leading to independent, far-end crosstalk limited transmissions on the U lines over K subcarriers. On each subcarrier $k \in \mathcal{K} = \{1, \dots, K\}$ we assume a complex AN signal $\tilde{X}_u^k \sim \mathcal{CN}(0, a_u^k)$ being added to the transmitted symbol $X_u^k \sim \mathcal{CN}(0, p_u^k)$ of all users $u \in \mathcal{U}$ indexed by $\mathcal{U} = \{1, \dots, U\}$, cf. Fig. 1. Denoting the channel coefficient from user i to user u, $i, u \in \mathcal{U}$, by G_{ui}^k, the received symbol on line u and subcarrier k is given as

$$Y_u^k = G_{uu}^k X_u^k + \sum_{i \in \mathcal{U} \setminus u} G_{ui}^k \bigl(X_i^k + \tilde{X}_i^k\bigr) + G_{uu}^k \tilde{X}_u^k + Z_u^k, \quad (1)$$

where $Z_u^k \sim \mathcal{CN}(0, n_u^k)$ is the additive background noise. Assuming independence among all random variables, the actual number of bits per DMT symbol which can be transmitted on line $u \in \mathcal{U}$ and subcarrier $k \in \mathcal{K}$ with a given bit-error probability can be derived as [11]

$$r_u^k(\mathbf{p}^k, \mathbf{a}^k) = \log_2\left(1 + \frac{H_{uu}^k p_u^k}{\Gamma(\sum_{i \in \mathcal{U} \setminus u} H_{ui}^k (p_i^k + a_i^k) + H_{uu}^k a_u^k + n_u^k)}\right), \quad (2)$$

where $H_{ui}^k = |G_{ui}^k|^2$, $\mathbf{p}^k \in \mathbb{R}^U$ and $\mathbf{a}^k \in \mathbb{R}^U$ are all users' transmit and AN powers, respectively, and Γ is the SNR gap [11] reflecting the modulation scheme, the targeted bit-error rate and the coding gain. As can be seen in Fig. 1 and in (2) the added AN also generates additional crosstalk noise $\sum_{i \in \mathcal{U} \setminus u} H_{ui}^k a_i^k$ among the users. The reverse, i.e., the powers $\mathbf{p}^k(\mathbf{r}^k)$ for given rates $\mathbf{r}^k$ and constant AN $\mathbf{a}^k$, $k \in \mathcal{K}$, are known to be computable by the solution of a linear matrix equation [6].

However, AN is variable and should be chosen such that the worst SNR a user may experience with respect to crosstalk noise (i.e., when all other users are transmitting) does not exceed the SNR which was targeted at the initialization of the line. This constraint can be precisely written as

$$\tilde{r}_u^k(p_u^k, a_u^k) \le r_u^k(\mathbf{p}^k, \mathbf{a}^k), \quad (3)$$

where $\tilde{r}_u^k(p_u^k, a_u^k)$ denotes the rate at initialization. The rate $\tilde{r}_u^k(p_u^k, a_u^k)$ will be the largest, and therefore the constraint in (3) the most restrictive, if no crosstalk noise is present at the initialization phase. Hence we define

$$\tilde{r}_u^k(p_u^k, a_u^k) = \log_2\left(1 + \frac{H_{uu}^k p_u^k}{\gamma_u \Gamma(n_u^k + H_{uu}^k a_u^k)}\right), \quad (4)$$

where γ_u is the extra SNR margin which is set at the line initialization phase and used for protection against fluctuations in crosstalk noise. Recommended margin values used in practice are in the range between 6 dB and 10 dB [30]. Based on a specific setting of the AN power spectrum a value of 3 dB is suggested in [22]. However, in later sections we will optimize the margin value specifically for the considered networks. Another constraint concerns the modulation schemes used in practice, which limit the rate $\tilde{r}_u^k(\mathbf{p}^k, \mathbf{a}^k)$ to lie in the discrete set $\mathcal{B} = \{0, \Delta, \dots, \hat{B}\}$, Δ and $\hat{B}$ being the bit-loading step-size and bit cap, respectively. Furthermore, regulatory power spectral limitations in the form of masks $\hat{p}_u^k$, $k \in \mathcal{K}$, $u \in \mathcal{U}$, can be described by the constraint $(p_u^k + a_u^k) \le \hat{p}_u^k$.

2.2 Formulation for the optimization of artificial noise

The two most widely used optimization objectives in the DSL literature are the minimization of the total transmit power[1] and the maximization of the sum-rate. As our work is applicable to both of these two goals they are unified in a weighted objective function

$$f(\mathbf{p}^k, \mathbf{a}^k, \hat{\mathbf{w}}, \check{\mathbf{w}}) = \sum_{u \in \mathcal{U}} \bigl(\hat{w}_u(p_u^k + a_u^k) - \check{w}_u \tilde{r}_u^k(\mathbf{p}^k, \mathbf{a}^k)\bigr), \quad (5)$$

on subcarrier $k \in \mathcal{K}$, where $\hat{\mathbf{w}}, \check{\mathbf{w}} \in \mathbb{R}^U$ are constant weights which reflect the optimization target. Furthermore, we consider a minimal per-user target-rate R_u and maximum total transmit-power P_u, $\forall u \in \mathcal{U}$, respectively. The problem of

[1] While the line-driver power consumption is a more important objective in terms of its share in the transceiver's total power budget [12, 13, 27], the work in [27] motivates approaching it by a transmit power minimization, showing that it incurs a negligible performance loss in terms of line-driver power.

jointly optimizing transmit power and artificial noise considering the constraints of Sect. 2.1 can be cast as (cf. the formulations in [6, 21, 26] for the case without AN)

$$\underset{\mathbf{p}^k \geq \mathbf{0}, \mathbf{a}^k \geq \mathbf{0}, k \in \mathcal{K}}{\text{minimize}} \quad \sum_{k \in \mathcal{K}} f\left(\mathbf{p}^k, \mathbf{a}^k, \hat{\mathbf{w}}, \check{\mathbf{w}}\right) \tag{6a}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} \tilde{r}_u^k\left(\mathbf{p}^k\right) \geq R_u, \quad \forall u \in \mathcal{U}, \tag{6b}$$

$$\sum_{k \in \mathcal{K}} \left(p_u^k + a_u^k\right) \leq P_u, \quad \forall u \in \mathcal{U}, \tag{6c}$$

$$\tilde{r}_u^k\left(\mathbf{p}^k\right) \in \mathcal{B}, \quad \forall k \in \mathcal{K}, \ u \in \mathcal{U}, \tag{6d}$$

$$\tilde{r}_u^k\left(p_u^k, a_u^k\right) \leq r_u^k\left(\mathbf{p}^k, \mathbf{a}^k\right), \quad \forall k \in \mathcal{K}, u \in \mathcal{U}, \tag{6e}$$

$$\left(p_u^k + a_u^k\right) \leq \hat{p}_u^k, \quad \forall k \in \mathcal{K}, u \in \mathcal{U}. \tag{6f}$$

We emphasize once more that the purpose of formulating this problem is to derive a performance bound for AN-enabled DSL networks which can be compared to other stabilization techniques. Before presenting an algorithm for the solution of this problem in Sect. 3 we proceed by studying one of its essential building blocks, that is the per-subcarrier bit and power allocation problem.

2.3 Stabilized power control with artificial noise

The classical single-subcarrier power control problem [10, 29] for fixed, minimum bit allocation $\mathbf{r}^k$ can be cast as a linear program (LP) [3] and its solution $\mathbf{p}^k(\mathbf{r}^k)$ obtained by solving a linear system, as already mentioned in Sect. 2. Similarly, we will show that the single-subcarrier power control problem including AN remains an LP. The value of this observation is that discrete-rate DSM algorithms as in [21, 25], which rely on the solution of a series of such power control problems, can be extended in a straightforward way to cover also the optimization of the AN problem in (6a)–(6f), cf. the next section for details. Regarding only the per-subcarrier constraints in (6d)–(6f) we can write the power control problem on subcarrier $k \in \mathcal{K}$ for the joint optimization of transmit power and AN under a fixed bit load $\mathbf{r}^k \in \mathbb{R}^U$ as

$$\underset{p_u^k \geq 0, a_u^k, \forall u \in \mathcal{U}}{\text{minimize}} \quad \sum_{u \in \mathcal{U}} w_u\left(p_u^k + a_u^k\right) \tag{7a}$$

$$\text{s.t.} \quad H_{uu}^k p_u^k \geq \left(2^{r_u^k} - 1\right)\gamma_u \Gamma\left(n_u^k + H_{uu}^k a_u^k\right), \quad \forall u \in \mathcal{U}, \tag{7b}$$

$$\sum_{i \in \mathcal{U} \setminus u} H_{ui}^k\left(p_i^k + a_i^k\right) + H_{uu}^k a_u^k + n_u^k \leq \gamma_u\left(n_u^k + H_{uu}^k a_u^k\right),$$

$$\forall (k, u) \in \left\{(k \in \mathcal{K}, u \in \mathcal{U}) | r_u^k > 0\right\}, \tag{7c}$$

$$\left(p_u^k + a_u^k\right) \leq \hat{p}_u^k, \quad \forall u \in \mathcal{U}, \tag{7d}$$

where $\mathbf{w} = (\hat{\mathbf{w}} + \boldsymbol{\nu})$, $\boldsymbol{\nu} \in \mathbb{R}^U$ are additional weights which will be specified in Sect. 3, and the initial bit-loading constraint in (7b) is simply a reformulation of the constraint $\tilde{r}_u^k(p_u^k, a_u^k) \geq r_u^k$ using (4). Moreover, the SNR variation constraint in (7c) is a reformulation of (6e) using the rate-definitions in (2) and (4). The fact that in (7c) we possibly only constrain a subset of the users comes from the observation that the constraint in (6e) is only active when a user u transmits on subcarrier k. Furthermore, in this case the constraint in (6e) only restricts the denominators of the SNR terms in (2) and (4), cf. (7c). Note that $\gamma_u > 1$ has to hold strictly in order for (7c) to be feasible under non-zero crosstalk noise and bit-load. The problem in (7a)–(7d) is an efficiently solvable LP [3].

2.4 Stabilized power control without extra noise

For networks which are not AN-enabled one can formulate yet another "stabilized" power control problem (below referred to as "Margin Only") simply by dropping the AN terms in (7a)–(7d), leading to the formulation

$$\underset{0 \leq p_u^k \leq \hat{p}_u^k, \forall u \in \mathcal{U}}{\text{minimize}} \quad \sum_{u \in \mathcal{U}} w_u p_u^k \tag{8a}$$

$$\text{s.t.} \quad H_{uu}^k p_u^k \geq \left(2^{r_u^k} - 1\right)\gamma_u \Gamma n_u^k, \quad \forall u \in \mathcal{U}, \tag{8b}$$

$$\sum_{i \in \mathcal{U} \setminus u} H_{ui}^k p_i^k + n_u^k \leq \gamma_u n_u^k,$$

$$\forall (k, u) \in \left\{(k \in \mathcal{K}, u \in \mathcal{U}) | r_u^k > 0\right\}. \tag{8c}$$

In this formulation the stabilization comes solely from the selective bit allocation, as can be seen in the constraint in (8c) which limits the total crosstalk noise received on each line. The solution of the problem in (8a)–(8c) can be given analytically, as the constraint in (8b), when changed to an equality, provides us with the lowest power values p_u^k, independently for each user $u \in \mathcal{U}$. All that remains to be done is to evaluate feasibility of these values for the constraints in (8a) and (8c), which are the loosest for the found smallest values of p_u^k. The optimal objective can then be evaluated in (8a), or assigned infinity in case of infeasibility.

2.5 Stabilized power control with virtual noise

As mentioned in the introduction, the "ideal" alternative to AN, standardized for VDSL2 systems [17], is "virtual noise" (VN) [24]. VN has a similar effect as AN but is only a transmission parameter, i.e., not physically present on the line. The optimal "receiver-referred" VN equals the crosstalk noise spectrum a line experiences when all lines are active, and can hence be computed by current DSM schemes [6, 21, 25], i.e., one first calculates the optimized received crosstalk noise levels by multi-user DSM (setting the redundant margin to $\gamma = 1$), and then sets $v_u^k = \sum_{i \in \mathcal{U} \setminus u} H_{ui}^k p_i^k$, where $v_u^k, u \in \mathcal{U}, k \in \mathcal{K}$, is the received VN power level considered at the initialization and added to the background noise n_u^k. For comparison to AN we will however also study the (only theoretically relevant) case where $\gamma_u > 1$ and we enforce $v_u^k \geq 0$. In this case we need to solve linear subproblems similarly as in (8a)–(8c), only differing in the variable VN terms v_u^k which are added to the background noise in (8b) and on the right-hand side of the inequality in (8c). Another definition of VN which is used in standardization [17] and which will be used in later sections comes from referring to it as a transmit power level, giving the "transmitter-referred" VN v_u^k / H_{uu}^k. However, this is only a scaling applied to v_u^k and does otherwise not alter the way we compute the VN.

After having studied the per-subcarrier power control problem for the cases with AN, with VN, and without any additional noise parameters (i.e., using the SNR margin only) we proceed with a Lagrange relaxation based approach for the near-optimal solution of the original multi-carrier AN problem in (6a)–(6f).

3 Performance bound computation for AN-enabled networks

In this section we approach the problem in (6a)–(6f) by solving its partial Lagrange dual problem [2]. The Lagrange relaxation is motivated by the typically large number of subcarriers $|\mathcal{K}|$ which become independent in terms of the power allocation after the relaxation. The dual problem is, similarly as in [6, 25, 26] for the case without AN, defined as

$$\underset{\boldsymbol{\nu}, \boldsymbol{\lambda}}{\text{maximize}} \; \underset{\mathbf{p}^k \geq \mathbf{0}, \mathbf{a}^k \geq \mathbf{0}, k \in \mathcal{K}}{\text{minimize}} \; \sum_{k \in \mathcal{K}} f\left(\mathbf{p}^k, \mathbf{a}^k, (\hat{\mathbf{w}} + \boldsymbol{\nu}), (\check{\mathbf{w}} + \boldsymbol{\lambda})\right) + \sum_{u \in \mathcal{U}} \nu_u P_u - \lambda_u R_u \tag{9a}$$

$$\text{s.t.} \quad \text{Per-subcarrier constraints in (6d)–(6f)}, \tag{9b}$$

where $\boldsymbol{\nu}, \boldsymbol{\lambda} \in \mathbb{R}^U$ are the Lagrange multipliers associated with the relaxed constraints in (6b) and (6c), respectively.

We refer to Appendix A for a generic algorithm to solve the dual problem in (9a)–(9b). While there are numerous algorithmic options for approaching the maximization and minimization in (9a), our implementation relies on the schemes described in [25, 26]. However, at its center is the problem specific LP in (7a)–(7d) which needs to be solved repeatedly for different rate allocations.

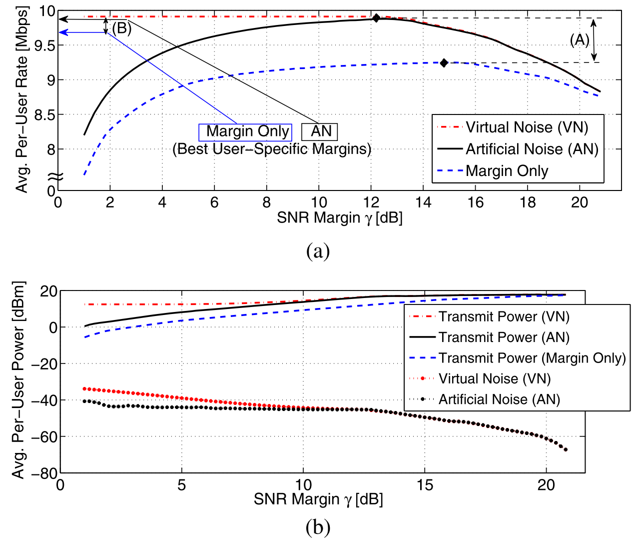

Fig. 2 Dependency of the mean-rate (**a**) and transmit powers (**b**) of three stabilization techniques on the set SNR margin $\gamma_u = \gamma, \forall u \in \mathcal{U}$ in ADSL2

3.1 Simulation results for multiple users and fixed margins

We evaluate our performance bound for AN-enabled networks using the proposed scheme in Algorithm 2 in a 3-user downstream ADSL2 scenario[2] with loop-lengths of 800, 1100, and 1400 m, respectively. By weak-duality [2] we find that the suboptimality of the primal solutions for the original problem in (6a)–(6f) obtained by the application of the heuristic in [26] subsequent to our dual-optimal algorithm (cf. Line 7 in Algorithm 2) is below 10^{-4} % in all the simulation results shown in this section. This means that the shown solutions, which apply to the original problem in (6a)–(6f), are provably near-optimal and our performance bound fairly tight. Figure 2(a) shows the optimal mean-rate among all users over the SNR margin $\gamma_u = \gamma$ set equal for all users $u \in \mathcal{U}$ and increased in steps of 0.2 dB. We see that the performance under AN increases up to a certain margin value (at around $\gamma = 12.6$ dB), beyond which it decreases again. This behavior can be intuitively explained by the constraints in (7c) which limit the SNR variation under crosstalk and therefore, for small values of γ, the bit-allocation of heavily interfering lines. Differently, under VN we have constant performance up to $\gamma = 12.2$ dB which is explained by the fact that VN can fully replace the role of

[2]The parameters for ADSL2 follow the standard in [18, Annex B], using non-overlapping spectra with ISDN, $\Gamma = 6.8$ dB, $\hat{B} = 15$, $\Delta = 1$, and $n_u^k = -120$ dBm/Hz, $\forall k \in \mathcal{K}, u \in \mathcal{U}$. The crosstalk model is the commonly used 99 % worst-case model [11]. Weights for rate maximization are set to $\hat{w}_u = 0$, $\check{w}_u = 1$, $u \in \mathcal{U}$, and to $\hat{w}_u = 1$, $\check{w}_u = 0$, $u \in \mathcal{U}$, for sum-power minimization. The parameters for ADSL2+ follow the standard in [16, Annex A], using non-overlapping spectra with ISDN and other settings as for ADSL2.

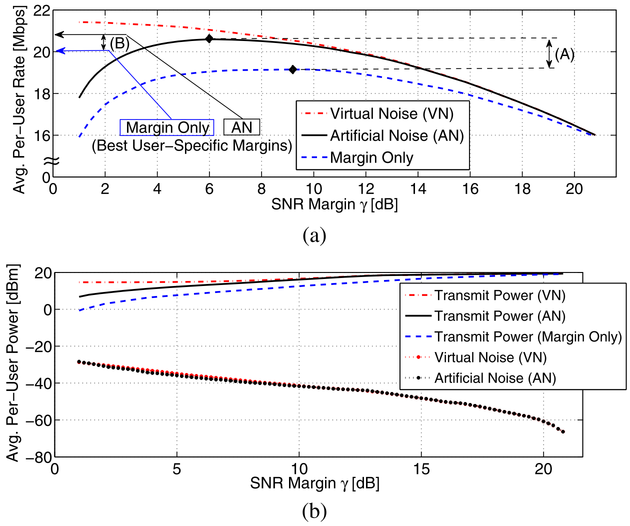

Fig. 3 Dependency of the mean-rate (**a**) and transmit powers (**b**) of three stabilization techniques on the set SNR margin $\gamma_u = \gamma, \forall u \in \mathcal{U}$ in ADSL2+

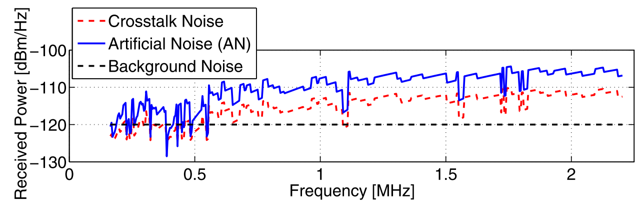

Fig. 4 Relation between the crosstalk and artificial noise received on the shortest (800 m long) line for a margin of $\gamma_u = 1$ dB, $\forall u \in \mathcal{U}$, in ADSL2+

the SNR margin. Note that for better comparability to AN we always plot the transmitter-referred VN v_u^k / H_{uu}^k. Considering the third curve where the lines are stabilized by an adequate bit allocation and the SNR margin only, we see that the optimal margin is higher than under AN (at around $\gamma = 14.8$ dB). The gain in sum-rate by AN compared to the optimal margin setting without extra noise is approximately 6.8 % (cf. the interval "(A)" in Fig. 2(a)), while that of VN compared to AN is less than 0.4 %. Note that this is a simplified evaluation of the gain by AN as we set the SNR margin equal for all users. We will perform a heuristic setting of the margin for each user separately in Sect. 3.2. Figure 2(b) further shows how for large values of γ the total used AN (as well as transmitter-referred VN) decreases. This is intuitive as for large values of the margin γ the initial bit-loading constraint in (7b) becomes more and more active while the SNR variation constraint in (7c) becomes less and less active and AN hence meaningless.

We repeat the simulation under the same simulation setup for ADSL2+ which uses approximately double the spectral bandwidth, cf. Fig. 3. The best found SNR margin values are now 9.2 dB and 6 dB under no extra noise ("margin only") and AN, respectively, cf. Fig. 3(a). This means that the optimal margin values are now smaller than for ADSL2, while the AN and VN sum-power values slightly increased, cf. Figs. 2(b) and 3(b). This can be explained by the higher bandwidth used in ADSL2+ compared to ADSL2 and the frequency selectivity of the channel, which results in a more frequency selective crosstalk noise in ADSL2+. The gain at the optimal margin value γ by AN compared to the case with no extra noise is now 7.5 % (cf. the interval "(A)" in Fig. 3(a)), and that by VN compared to AN 4 %, cf. Fig. 3(a). Concluding, as seen in Figs. 2(b) and 3(b) the total power spent for AN is negligible compared to the total transmit power.

Next we exemplarly have a look at the spectral shape of the optimized AN at our solution in Fig. 3(a) for the margin $\gamma_u = 1$ dB, $\forall u \in \mathcal{U}$, in ADSL2+, cf. Fig. 4 for an illustration. In [14] it is recommended that the (received) AN power levels should follow the spectral shape of the crosstalk noise (i.e., the optimal VN setting), and set at (or slightly below) the crosstalk power levels, as also described in [22]. However, from Fig. 4 we see that this simple rule does not hold for every setting of the SNR margin. In addition, our simulations will show that the optimal margin value can vary widely for different loop lengths, cf. Sect. 3.3.

3.2 Heuristic for the joint optimization of spectral power allocation, margin, and artificial noise

Motivated by the dependency of the optimal spectral AN and transmit power allocation on the SNR margin we propose a heuristic for jointly optimizing all three variable sets. However, the proposed scheme can also be applied to efficiently search for the single best margin for all users, as found in Sect. 3.1 by an exhaustive search. In order to obtain a low-complexity scheme we embed a multi-user bit-search technique in a search for the margins γ_u, performed sequentially one user after the other. A more explicit summary is given in Appendix B. We repeat the simulation setup of Sect. 3.1, this time optimizing the SNR margins on a per-user basis using Algorithm 3, initialized at $\gamma_u = 10$ dB, $\forall u \in \mathcal{U}$. Beginning with ADSL2 (cf. the results for equal margins among users in Fig. 2) we find that the gain by AN compared to no extra noise ("margin only") is now less than 1.8 %, cf. the interval "(B)" in Fig. 2(a). The little rate gains for AN by per-user margin optimization are explicable by the fact that VN upper-bounds the AN performance for any margin setting under AN, and the performance under AN and a single margin was already close to that of VN in Fig. 2(a). The results for user-specific margin optimization in Fig. 3(a) show again higher improvements for the margin-only scheme, making the difference between the two schemes shrink to less than 3.8 %, cf. the interval "(B)" in Fig. 3(a). Summarizing this section, AN seems to be able to partially compensate for the

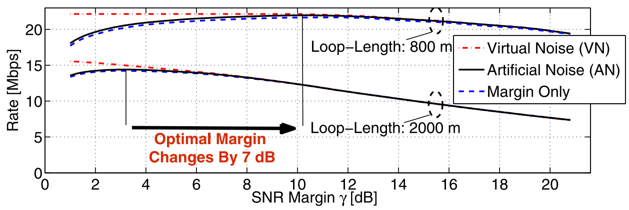

Fig. 5 Dependency of the mean-rate on the single SNR margin $\gamma_u = \gamma, \forall u \in \mathcal{U}$, for three stabilization techniques in two collocated scenarios

performance loss incurred by setting the SNR margin values of all users equal. However, also the single margin value needs to be optimized under AN based on the actual network topology, a point we investigate further in the next section.

3.3 Margin optimization in collocated scenarios

Summarizing our simulation results so far, they indicate that a near-optimal (with reference to the VN scheme) stable rate can, in networks with a limited distribution of users, already be achieved by an adequate setting of a single network-wide SNR margin. In this section we investigate scenarios with 3 collocated users and loop-lengths of 800 m and 2000 m, respectively, and apply again the near-optimal algorithm of Appendix A for a varying SNR margin $\gamma_u = \gamma, \forall u \in \mathcal{U}$. We find that the per-user margin optimization heuristic of this section does not significantly improve the sum-rate compared to the optimal single margin γ in this collocated setup, and omit these results for this reason. However, as seen in Fig. 5, there is a large gap between the optimal single-margin settings for the two loop-lengths, supporting our point that the optimization of AN and the SNR margin need to be done jointly.

3.4 Assuming equal power spectra on collocated lines

The simplification of multi-user DSM by the use of "virtual lines" has been proposed in [5]. Another approximation is given by enforcing an identical power allocation (in terms of AN/VN and transmit power/bit allocation) for all collocated lines. This leads to small modifications of the subproblems in (7a)–(7d) or (8a)–(8c) in the sense that the crosstalk from each optimized user is multiplied by the number of lines collocated with the crosstalker, and additionally we need to consider the crosstalk from the lines that are collocated with the victim line. The latter crosstalk is determined by the power allocation of the victim line, and can hence be interpreted as a "self-noise". As the collocated lines share the same solution, the optimization of a single line enforces the power and sum-rate constraints in (6b) and (6c) for all collocated lines, assuming that the target-rates, sum-power budgets, channel, DSL technology, etc., are equal (which we

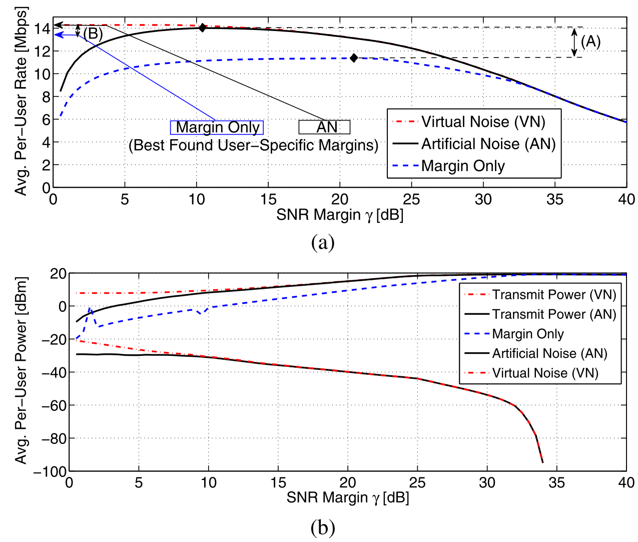

Fig. 6 Dependency of the mean-rate (**a**) and transmit powers (**b**) of three stabilization techniques on the set SNR margin $\gamma_u = \gamma, \forall u \in \mathcal{U}$ in ADSL2+ with 33 lines

do here). The main benefit from this simplification is that it allows us to make a performance evaluation under optimal DSM in networks of a more realistic size. We consider three collocated ADSL2+ user groups of 11 lines each (i.e., 33 lines in total), located, as above, at 800 m, 1100 m and 1400 m distance from the central office, respectively, and other simulation parameters as specified in Sect. 3.1.

The results shown in Fig. 6 qualitatively resemble those in Fig. 3. For the best single-margin setting we observe a rate and transmit power under AN which is similar to that under VN. Furthermore, again we see that the margin-only scheme profits most from a user-specific SNR margin, where the extra benefit by AN in terms of mean-rate drops from more than 23 % to less than 5 %. Comparing Figs. 6 and 3 we find that the higher crosstalk level in this 33-user example compared to the 3-user example in Sect. 3.1 results in a higher optimal SNR margin and higher AN sum-power at the optimal margin. Note also that the extra benefit by VN compared to AN is below 2 % (or 0.3 Mbps). We repeated this simulation, this time with all 33-users being collocated at a distance of 2000 m from the deployment point (results not shown). While the optimal SNR margin for AN approximately halved, the benefit by VN compared to AN is now more than 11 % (or more than 1.1 Mbps). Regarding once more Fig. 5 for 3 collocated, separately optimized ADSL2+ lines, we can draw a similar conclusion. There we find a benefit by VN of around 0.8 % (or 0.18 Mbps) for a loop-length of 800 m, and of more than 8 % (or 1.17 Mbps) in case of 2000 m. Summarizing, compared to the "ideal" frequency selective SNR margins (VN) the stable performance suffers from the injection of AN especially at longer loop-lengths. This effect will be further analyzed in Sect. 4.2 by

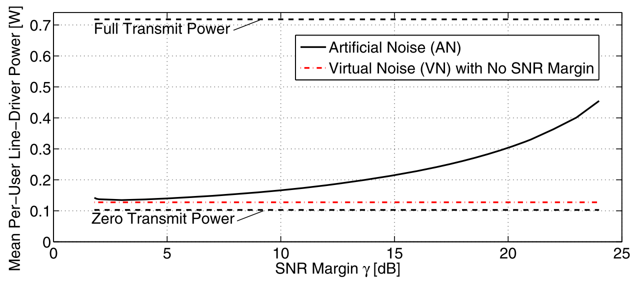

Fig. 7 Dependency of the mean line-driver power consumption on the single SNR margin $\gamma_u = \gamma, \forall u \in \mathcal{U}$, under AN compared to the optimal power under VN in a 33-user ADSL2+ scenario

loop-reach simulations, where we will also give an intuitive explanation for it.

In Fig. 7 we look at the mean line-driver power consumption for supporting 80 % of the per-user rates achieved by a sum-rate maximization under VN. While the power under AN is seen to be fairly margin dependent, under the best shown margin setting (at $\gamma = 3$ dB) it is close (less than 6 % higher) to that under VN. Note that the energy savings by AN-enabled LPMs, that is the difference between the line-driver power at full transmit power (the top line in Fig. 7) and the line-driver power under AN for optimized SNR margin in Fig. 7, is around 0.58 W (i.e., over 80 %) in this example. Furthermore, the optimal margin value is far below that in the corresponding sum-rate maximization problem (cf. Fig. 6), meaning that the optimal margin setting depends not only on the channel and the network topology, but also on the target-rate. A word of caution is needed at this point, as the typically encountered low AN sum-power level (in comparison to the total transmit power) does *not* automatically imply that the AN technique is energy efficient, as transmit power may be also wasted due to the needed frequency flat SNR margin. We will see an example supporting this claim in the next section where a power-mask based crosstalk noise is assumed in order to decouple and therefore simplify the performance evaluation based on the original multi-user DSM problem in (6a)–(6f).

4 Performance analysis for worst-case crosstalk

A commonly applied simplification in performance evaluation for multi-user DSL networks is to decouple the users by assuming the highest possible crosstalk noise t_u into line $u \in \mathcal{U}$ based on the spectral mask constraints[3] in (7d), given

[3]Note however that estimates of the worst-case noise encountered in real networks are more commonly based on long-term network observations [22].

Algorithm 1 Single-user AN and Bit-Allocation Scheme

for all users $u \in \mathcal{U}$ **do**
  while (e.g., exhaustive) search for γ_u **do**
    Compute the constant received AN $[\tilde{a}_u^k]_+$ as in (11)
    Solve the problem in (13a)–(13d) by greedy bit-loading [4]

as

$$\sum_{i \in \mathcal{U}\backslash u} H_{ui}^k \left(p_i^k + a_i^k\right) \le t_u^k = \sum_{i \in \mathcal{U}\backslash u} H_{ui}^k \hat{p}_i^k. \tag{10}$$

Replacing the crosstalk terms by these (constant) upper-bounds in the per-subcarrier problem in (7a)–(7d), we see that the users' AN and power allocation become decoupled problems, and the constraint in (7c) can be simplified to

$$H_{uu}^k a_u^k \ge \tilde{a}_u^k \doteq \left(t_u^k/(\gamma_u - 1) - n_u^k\right). \tag{11}$$

As a decrease in the AN value a_u^k makes the constraints in (7b) and (7d) less restrictive for the power allocation variables p_u^k and the goal is to minimize the total transmit power in (7a), we can give the optimal AN setting explicitly as

$$a_u^k = \left[\tilde{a}_u^k\right]_+ / H_{uu}^k, \tag{12}$$

where $[x]_+ = x$ for $x \ge 0$, and $[x]_+ = 0$ otherwise.

4.1 Optimal algorithm for worst-case-stable bit-loading

We investigate the setting of the AN jointly with the power allocation based on the decoupling worst-case assumption in (10), which, as will be shown, can be solved efficiently and optimally through a (modified) greedy bit-loading procedure. Inserting (12) back into (7a)–(7c) and regarding the original problem in (6a)–(6f) we see that one recovers the modified single-user bit-loading problem

$$\underset{\tilde{r}_u^k \in \mathcal{B}, k \in \mathcal{K}}{\text{minimize}} \quad \sum_{k \in \mathcal{K}} \hat{w}_u \left(p_u^k(\tilde{r}_u^k) + \left[\tilde{a}_u^k\right]_+ / H_{uu}^k\right) - \check{w}_u \tilde{r}_u^k \tag{13a}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} \tilde{r}_u^k \ge R_u, \qquad \sum_{k \in \mathcal{K}} p_u^k(\tilde{r}_u^k) \le P_u, \tag{13b}$$

$$p_u^k(\tilde{r}_u^k) \le \hat{p}_u^k - \left[\tilde{a}_u^k\right]_+ / H_{uu}^k, \quad \forall k \in \mathcal{K}, \tag{13c}$$

$$\text{where} \quad p_u^k(\tilde{r}_u^k) \doteq \left(2^{\tilde{r}_u^k} - 1\right) \frac{\gamma_u}{H_{uu}^k} \Gamma \left(n_u^k + \left[\tilde{a}_u^k\right]_+\right), \tag{13d}$$

which we write in terms of the variables $\tilde{r}_u^k$ instead of p_u^k to emphasize the relation to bit-loading algorithms. AN solely leads to a modified (but constant) additive objective term, mask, and background noise in (13a), (13c), and (13d), respectively. Hence, we find that greedy bit-loading [4, 25] (with sum-power objective) is optimal when applied to the

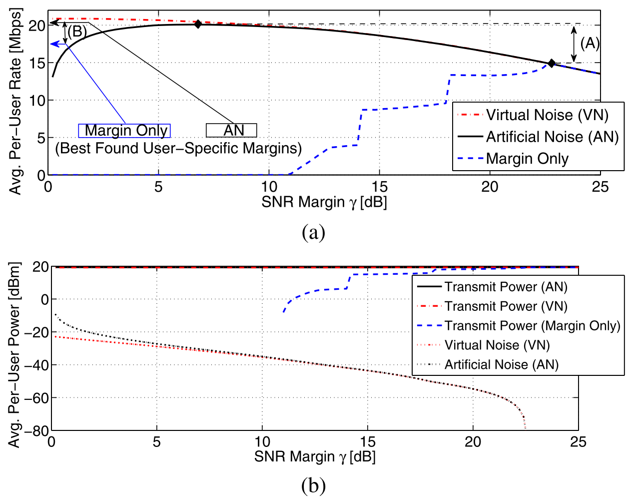

Fig. 8 Average rates (**a**) and power levels (**b**) under three stabilization techniques and a worst-case crosstalk assumption in a distributed 3-user ADSL2+ scenario

problem in (13a)–(13d). We refer to [4] for the corresponding result under a sum-power objective, and to [25] for a proof which is applicable to the cost function in (13a). The intuition is that the weights in (13a) are equal for all subcarriers and hence do not alter the optimal decisions the classical single-user bit-loading algorithm [4] takes. This optimal greedy bit-loading procedure needs then to be embedded in a (e.g., exhaustive) search loop for the SNR margin value γ_u, cf. Algorithm 1 for a generic description.

The same arguments can be applied to show that the single-user bit-loading problem of optimizing VN (under general margins $\gamma_u \geq 1$ and $v_u^k \geq 0$) is readily solvable by a (modified) greedy bit-loading algorithm, and the (receiver-referred) VN power allocation explicitly given as $v_u^k \doteq [\frac{t_u^k}{\gamma_u} - \frac{\gamma_u - 1}{\gamma_u} n_u^k]_+$, which for $\gamma_u > 1$ is equivalent to $(\gamma_u - 1)/\gamma_u \tilde{a}_u^k$. This implies that, assuming worst-case crosstalk as in (10) and an identical SNR margin, the VN spectral power allocation is always below that of the AN. Assuming the (for VN) optimal selection $\gamma_u = 1$, the VN v_u^k equals the crosstalk noise t_u^k, as already remarked in Sect. 2.5. As in the case of AN, the bit-loading considers a background noise increased by the additional noise v_u^k, while the objective and power mask constraints are, differently to (13a) and (13c), not altered by the VN.

We proceed with simulation results based on the proposed modified greedy bit-loading procedure applied to a single user, showing the optimal rate and sum-power levels under the conservative crosstalk assumption in (10).

4.2 Worst-case-stable single-user evaluation

We repeat the 3-user ADSL2+ example of Sect. 3.1, this time using the worst-case crosstalk assumption of Sect. 4

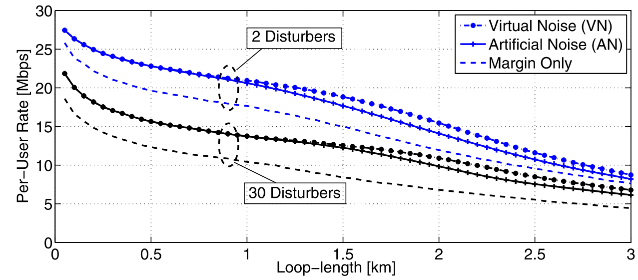

Fig. 9 Rate under the three stabilization techniques in ADSL2+ for 2 and 30 collocated disturbers, respectively

and Algorithm 2 applied to each user individually. The results are shown in Fig. 8, cf. Fig. 3 for the corresponding results under our near-optimal multi-user DSM scheme. First note that without the extra noise (AN or VN) and fixed crosstalk noise the feasibility of the stability constraint in (7c) solely depends on the set SNR margin. Hence, in Fig. 8 we obtain no (stable) rate below a certain threshold margin. Considering the larger crosstalk noise considered here it comes as no surprise that the achieved rates are lower and the AN and VN power levels higher in Fig. 8 compared to Fig. 3. In Fig. 8(a) we additionally show the mean-rate achieved when we are allowed to select the SNR margin for each user individually and optimally (up to the selected granularity of 0.2 dB). Similarly as in Sect. 3.2 we find that this additional freedom in margin setting mostly improves the performance in the case when no extra noise (AN or VN) is used. Note that due to the user-independence under the worst-case assumption in (10) it comes at no additional effort to find the optimal user-specific margins compared to finding the single optimal margin set equal for all users.

The main advantage of the conservative simplification made in this section is that it allows to do a performance evaluation in networks with a large number of users. Hence, we do now assume numerous disturbers collocated with an ADSL2+ system, and simulate Algorithm 2 with an exhaustive search over the margin with a granularity of 0.2 dB. The results in Fig. 9 represent the rate performance for a certain loop-length and the best SNR margin selection in the above sense for 2 and 30 collocated disturbers, respectively.[4] Compared to the scenario without extra noise we find that AN provides a gain in rate of between 6.3 % and 18.9 % in case of 2 disturbers, between 13.4 % and 36.7 % in the case of 10 disturbers, and between 17.4 % and 48.2 % in case of 30 disturbers. Furthermore, AN gives a (worst-case crosstalk) rate performance similar to VN, at least for loop-lengths below 1 km. Above this length we find a maximum rate loss compared to VN of 8.9 %, 11.3 %, and 12.4 % for 2, 10, and 30 disturbers, respectively.

[4]Results obtained for 5 and 10 disturbers are qualitatively similar (although shifted in terms of rates), and therefore omitted.

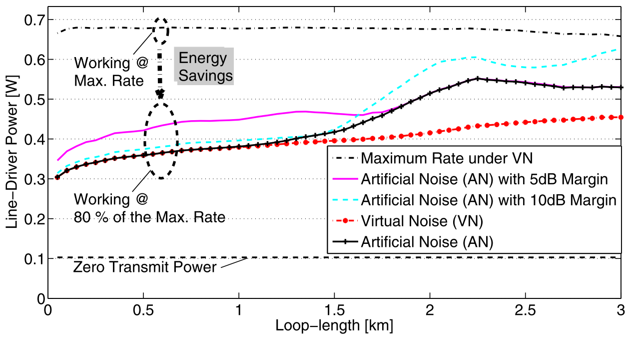

Fig. 10 Line-driver power consumption of a single line under study operating at 80 % of the maximum (stable) achievable rate, under the three studied stabilization techniques and 30 collocated disturbers in ADSL2+

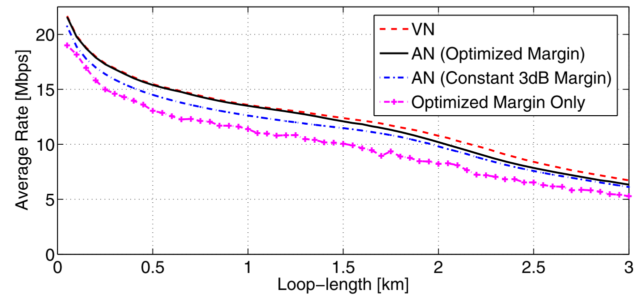

Fig. 11 Average rate of 33 stabilized ADSL2+ lines under sequential initialization

In order to further study the energy-efficiency of AN we repeat this simulation versus loop-length, but show instead of the achieved rate the line-driver power[5] consumed at 80 % of the maximum rate achieved by the VN technique. The results depicted in Fig. 10 show a loss in terms of line-driver power consumption by AN compared to VN for long loops, similar as seen for the rate performance in Fig. 9. This can be explained intuitively as for long loops the performance becomes less constrained by the crosstalk noise t_u^k but more constrained by the noise $H_{uu}^k a_u^k + n_u^k$, while VN does not suffer from the artificial noise term $H_{uu}^k a_u^k$. The maximum extra power cost by AN compared to VN is around 27 % (or 119 mW, respectively) for a loop-length of 2.25 km. Note however that the extra "power cost" by AN becomes less evident for lower target-rates (e.g., below 50 % of the maximum rates, results omitted) where the power consumption is lower as well. Figure 10 also shows the power consumption for two fixed values of the SNR margin. The comparison to the line-driver power consumption under AN and the best selected margin shows once more the gain by SNR margin optimization under varying loop-lengths. Furthermore, Fig. 10 shows energy savings of between 18 % and 55 % by margin-optimized AN operation at 80 % of the maximum rates compared to the energy consumption at 100 % rate (cf. the top line in Fig. 10).

4.3 Worst-case-stable sequential initialization

In Sect. 4.2 we evaluated the performance of a line seeing no crosstalk noise during its initialization, which is stabilized for the worst-case crosstalk noise in (10). Differently, in case a line u initializes when other lines are already active we need to consider the additional crosstalk noise $\tilde{t}_u^k$, $k \in \mathcal{K}$, that is caused by the active disturbers. Hence, the crosstalk noise present at initialization also needs to be added to the right-hand side of the stabilization criterion in (7c). This leads to a lower optimal AN level (cf. (12))

$$a_u^k \doteq \frac{1}{H_{uu}^k}\left[\left(\frac{t_u^k - \gamma_u \tilde{t}_u^k}{\gamma_u - 1} - n_u^k\right)\right]_+. \tag{14}$$

Using this definition the optimal modified bit-loading problem is given similarly as in (13a)–(13d), differing only in the higher background noise ($n_u^k + \tilde{t}_u^k$) and the altered AN levels in (14).

The noise levels $\tilde{t}_u^k$ at initialization depend on many factors as mentioned in Sect. 1, including the users' line usage behavior. As a simple possible rule for performance simulation we assume that the lines initialize sequentially, one user after the other [22]. In a collocated scenario this leads to a unique average rate per line, where we apply the modified bit-loading algorithm in total U times with sequentially updated received crosstalk levels $\tilde{t}_u^k$, and AN levels in (14).

Note that the sum-rate of the first initializing user is not necessarily the highest among users, as we will observe in the simulation results of the following section. This counterintuitive behavior stems from the stability constraint in (7c) which demands for higher levels of AN in case of low (background and crosstalk) noise levels, and can be intuitively analyzed for the special case where the AN level in (14) is strictly positive. In this case we see by insertion of (14) in (13d) that the total received noise (i.e., including the AN) is decreasing for an increasing crosstalk noise $\tilde{t}_u^k$ at initialization.

In Fig. 11 we show the obtained simulation results for collocated 33-user scenarios with varying loop-length. The lowest and highest curves depict the performance without AN/VN and with VN, respectively. Especially at lower loop-lengths we find a noticeable gain by margin optimization under AN. More precisely, we find a maximum gain in average bit-rate in Fig. 11 compared to the fixed margin setting at 3 dB suggested in [22] of over 7 %. In terms of reach the gain is even beyond 32 % up to a loop-length of 1 km.

[5]The used model is that of a class AB line-driver and uses the parameterization specified in [27].

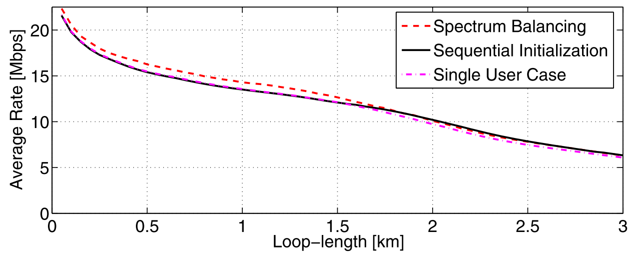

Fig. 12 Average rate of 33 AN-enabled ADSL2+ lines under different performance evaluation techniques

4.4 Comparison of performance evaluation techniques

In Fig. 12 we compare the three proposed evaluation techniques in terms of loop-reach simulations: The DSM based AN setting for collocated scenarios of Sect. 3.4, the single-user optimization under worst-case crosstalk noise stabilization criterion in Sect. 4.1, and the sequential initialization scheme with modified bit-loading in Sect. 4.3. While the combination of AN and DSM gives rates that are partly (for longer loops) below those under those obtained by sequential initialization and worst-case crosstalk noise stabilization, they form an upper bound for the single-user worst-case stabilization scheme. The explanation is that while for both, DSM and single-user optimization, we consider a stabilization criterion for the case where no crosstalk noise is present at initialization, the DSM scheme results in an optimized crosstalk noise spectrum and hence higher performance. Differently, all but one line in the sequential evaluation scheme see crosstalk noise at initialization, and the stabilization criteria are hence less restrictive for most lines. Note however that the stabilization constraints in (7c) are easily adaptable to this scenario and DSM hence also usable to bound the performance under sequential initialization.

5 Conclusions

We proposed three deterministic approaches for the performance evaluation in artificial noise (AN) enabled asymmetric digital subscriber line (ADSL) networks. In all three cases the simulation results confirm the gain by performing the standard-compliant joint optimization of the AN power spectrum and the SNR margin, e.g., by over 7 % in terms of bit-rate and beyond 32 % in terms of reach up to 1 km loop-length. We observe that the usage of AN leads in certain scenarios to an increased power consumption and reduced rates compared to an ideal solution, that is a frequency selective SNR margin (commonly referred to as "virtual noise"). For example, for longer loop-lengths (above 1.5 km) and higher bit-rates (e.g., above 50 % of the maximum rates) we identify a gap of up to 27 % in terms of line-driver power consumption and 12 % in terms of bit-rates compared to the ideal solution. However, simulations also show a substantial gain in sum-rate by using AN and margin optimization compared to the classical approach where only the SNR margin is used for line stabilization. Concerning the motivation of AN by its application to stabilize low-power mode enabled networks, we find that a significant power reduction can be achieved despite the additional power consumption caused by AN.

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Appendix A: An optimal algorithm for the dual problem in (9a)–(9b)

Our scheme for optimally solving the problem in (9a)–(9b) is summarized in Algorithm 2. The minimization part in (9a) is solved by a discrete search over the bit-allocation $\mathbf{r}^k$, $k \in \mathcal{K}$, instead of an optimization over the (continuous) spectral power allocation variables as suggested by (9a). For this purpose we use the problem-specific, efficient depth-first branch-and-bound (BnB) scheme proposed in [25], where we note that any other discrete search scheme such as that in [21] can in principle be used instead. Furthermore, such a discrete search involves the evaluation of the powers $\mathbf{p}^k(\mathbf{r}^k)$ by solving the LP in (7a)–(7d) in order to evaluate feasibility in (9b) and the objective in (9a). On top of the discrete search for the bit-allocation comes an iterative scheme

Algorithm 2 DSM Scheme for the Joint Optimization of AN and Transmit Power

1: **while** Master Problem (Maximization in (9a)–(9b)) not Solved **do**
2: Generate a New Set of Dual Variables $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ (e.g., By the LP based Column-Generation Scheme in [26])
3: **for** All Subcarriers $k \in \mathcal{K}$ **do**
4: **while** Optimal Bit- and Power Allocation Not Found **do**
5: Follow an optimal, discrete search method (e.g., the BnB Search in [25]) to Obtain Another $\mathbf{r}^k$
6: Evaluate $\mathbf{p}^k(\mathbf{r}^k)$ and the Objective in (9a) by Solving the LP in (7a)–(7d) with weights $\mathbf{w} = (\hat{\mathbf{w}} + \boldsymbol{\nu})$
7: *Optional* (for the specific multiplier search scheme in [26], cf. Line 2): Recover a solution to the problem in (6a)–(6f) by the Heuristic in [26]

targeting the maximization in (9a) (the "master problem"), where we use the LP based column-generation scheme proposed in [26]. Again we note that other algorithms for non-differentiable optimization problems [2, Chap. 6] can in principle be applied instead, such as an exhaustive search [6], a subgradient search [21], or the ellipsoid method suggested in [31]. Altogether, we see that the optimization of AN jointly with the spectral transmit power allocation neatly integrates into previous DSM approaches, notably also into low-complexity heuristics as in [19] or [26], cf. Sect. 3.2 for further details. While in this section we assumed a fixed margin $\gamma_u, u \in \mathcal{U}$, the spectral AN allocation and the SNR margin are coupled and are therefore jointly optimized in Sect. 3.2. The optimal scheme of this section is only applicable to problems with a few users. However, simplifications as the introduction of "virtual disturber lines" [5, 20] reduce the number of lines which are jointly optimized and therefore make such an optimal scheme also relevant for the performance evaluation in large networks. Similarly, in Sect. 3.4 we simplify the multi-user optimization problem by assuming equal AN and transmit power allocation for collocated lines.

Appendix B: Heuristic for joint margin, AN, and transmit power optimization

Algorithm 3 summarizes an algorithm for the joint optimization of the SNR margin and the spectral AN (or VN) and transmit power allocation. In Line 6 we run a greedy multi-user discrete bit-loading algorithm, differing from that in [19] solely by the initialization which considers the currently best found margin, AN, and transmit power allocation, and the procedure for computing the cost in (5) of a specific bit-allocation, namely by solving the LP in (7a)–(7d) (or the problem in (8a)–(8c) for the "margin-only" case, or a similar problem as in (7a)–(7d) in case of using VN for $\gamma_u > 1$ and $v_u^k \geq 0$, cf. Sects. 2.3–2.5 for details). In Lines 4–6 we perform a kind of line-search for parameter γ_u of user u, which is repeated for all users. As commented in Line 8 one may choose to call in the end a more complex DSM algorithm for the found incumbent, fixed SNR margins, as that in Algorithm 2 or a similar scheme using suboptimal heuristics, cf. the discussion of alternatives in Sect. 3.

Algorithm 3 Low-Complexity DSM Heuristic for Joint SNR Margin, AN and Transmit Power Optimization

1: *Optional Initialization:* Run Greedy Multi-User Bit-Loading [19] under the Initial SNR margin and Use a Heuristic$^{\otimes}$ to Recover a *Stable* (i.e., feasible w.r.t. (7c)) Initial Bit and Power Allocation
2: **repeat**
3:   **for** $u = 1$ to U **do**
4:     **while** Line-Search not Finished **do**
5:       Update γ_u Inside a Line-Search*
6:       Modified$^{\otimes}$ Greedy Multi-User Bit-Loading [19]
        – Initialization based on Incumbent Allocation
        – Calculate the Cost in (5) by Solving (7a)–(7d)
7: **until** Incumbent Objective Change is Below a Threshold
8: *Optional:* Call Algorithm 2 for the Found Incumbent SNR Margins to Obtain the Final Bit, Power, and AN Allocation

$^{\otimes}$ The incumbent power allocation (as currently available for any SNR margins in Line 6 or in Line 1 the unstable power allocation obtained by the algorithm in [19]) is used to produce an initial discrete bit-allocation, which is decreased by Δ for all users until it is feasible in (7a)–(7d) under the actual SNR margins.

* Initially we halve δ until $\gamma_u + \delta$ or $\gamma_u - \delta$ gives an improving objective ("Local Search"), determining a search direction $d = +1$ or $d = -1$, respectively, and an update $\gamma_u = \gamma_u + d \cdot \delta$. Next we continue by searching in the improving direction ("directed search"), i.e., we repeat $\gamma_u = \gamma_u + d \cdot \delta$ while an improvement in objective is made. Otherwise we halve δ and go back to the "local search" step. The process finishes when the step-size δ is below a threshold $\delta^{\min}$ and hence is guaranteed to converge.

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