CO₂ emission trading model with trading prices

Jarosław Stańczak · Paweł Bartoszczuk

Received: 5 January 2009 / Accepted: 15 June 2010 / Published online: 14 July 2010
© The Author(s) 2010. This article is published with open access at Springerlink.com

Abstract In this paper we consider the buying/selling prices of carbon dioxide (CO₂) emission permits in trading models with uncertainty. Permission prices, although usually omitted from standard models, may significantly influence the trading market. We thus undertook to construct a more realistic trade model and to compare it with the standard one. To do this, we introduced several important changes to the standard model, namely, (1) a new optimized quality function; and (2) transactions with price negotiations between regions. We also enhanced the model using methods described in the literature to allow it to deal with reported emissions uncertainty. Additionally, we used an original method of simulating this kind of market based on a specialized evolutionary algorithm (EA).

1 Introduction

It is claimed that the implementation of a tradable emission permit system can be an efficient strategy for achieving environmental goals. In permit systems a regulatory agency distributes emission permits to polluters in accordance with environmental goals. The permits are transferable among polluters, resulting in—to use simple, everyday trade model terminology—an equalization of marginal abatement costs among pollution sources.

J. Stańczak (✉)
Systems Research Institute, Polish Academy of Sciences,
01-447 Warszawa, ul. Newelska 6, Warsaw, Poland
e-mail: stanczak@ibspan.waw.pl

P. Bartoszczuk
Warsaw School of Economics, Enterprise Institute,
Al. Niepodległości 162, Warsaw, Poland
e-mail: pbarto1@sgh.waw.pl

National researchers build market models that optimize their ability to forecast the emission allowance process and the cost of emission reduction for different countries (Ermoliev et al. 2000) to stay fully informed about how obligations regarding greenhouse gas emissions will influence the world economy and about the rules governing the market. A major challenge is to build a transaction model and to solve many other problems associated with the credibility and uncertainty of emission level reports (Ermoliev et al. 1996; Godal 2000; Klaasen et al. 2001; Nahorski et al. 2007).

Earlier proposed models of emission permit trading for CO₂ do not really use transaction prices. Although the models calculate the equilibrium prices, they are not applied during the trade. The equilibrium permit price is used to calculate the emission reduction costs only. No negotiation of prices or any additional transaction costs are applied.

The problem-solving method proposed in earlier papers was original, but diverged from the real market situation. A more elaborated market model was thus considered, in which additional elements were included, such as the possibility of choice of price during negotiation and the influence of real prices on model solutions.

Application of the evolutionary algorithm (EA) method to simulate economic models is a very fast developing domain, mainly because it is quite easy to model economic systems using this tool. The evolutionary and agent-based approach to dynamic market modeling, which can be found in Bonatti et al. (1998), is used to simulate the very complicated information industry market. The evolutionary method is a natural way of performing computer simulations of the new model. This method is presented below.

2 New and earlier market models

The idea of emission permit trading is based on the assumption that some countries can save emission permits, which they can sell to countries wishing to emit more than their Kyoto obligations.

Trading is beneficial only when the price of permits is lower than the cost of emission reduction. Then, the country can reduce emissions below its obligation and sell the surplus as permits to another country (see Figs. 1 and 2).

Fig. 1 CO₂ emission reduction cost: without trade (c_{Ki}), and with trade (c_{ik}) for buying country, K_i —Kyoto limit, x_{ik} —emission after trade, x_{i0} —initial emission

Fig. 2 Without trade, the emission reduction costs of CO₂ are zero; with trade for the selling country (c_{ik}), K_i —Kyoto limit, x_{ik} —emission after trade, x_{i0} —initial emission

In the base model the total cost of holding emissions in region i down to x_i , is denoted by $C_i(x_i)$, which represents the abatement cost function. We assume that the cost functions $C_i(x_i)$ are positive, decreasing, and continuously differentiable for each region. The Kyoto target for each region i is indicated by K_i . The number of emission permits acquired, by source, is expressed by y_i (y_i is negative if region i is a net supplier of permits),

$$E = \min_{x_i} \sum_{i=1}^n C_i(x_i) \quad (1)$$

with the constraints:

$$x_i \leq K_i + y_i \quad (2)$$

$$\sum_{i=1}^n y_i = 0 \quad (3)$$

where:

- E is the minimum cost of decreasing emissions for all countries in the standard model;
- $C_i(x_i)$ is the cost of reducing emissions at region i down to x_i ;
- y_i is the number of emission permits acquired by region i ;
- K_i is Kyoto target for region i ;
- n is the number of regions;
- x_i is current emissions.

The goal is to minimize the cost of emission reduction in order to cut emissions to the required level in compliance with the Kyoto target.

This basic model does not deal with the problem of uncertainty in emissions, but it can be extended to do so, for instance, using ideas described in Nahorski et al. (2007), Horabik (2005), or Bartoszczuk and Horabik (2007). The only modification

required to the standard model for considering uncertainty is to replace formula (2) with formula (4) (Nahorski et al. 2007; Horabik 2005).

$$x_i + (1 - \zeta)(1 - 2\alpha)d_i n_i \leq K_i + y_i \quad (4)$$

where:

- ζ is the parameter that describes the correlation of emissions in the base year (1990) and the current year, usually $\zeta \in [0.65, 0.7]$;
- α is the parameter that describes the risk of noncompliance with the Kyoto target, $\alpha \in [0, 0.5]$;
- d_i is the inventory uncertainty for country i ;
- n_i is the base year emission for country i .

The modified model minimizes the sum of total emission reduction costs; however, the limits imposed are lower than the Kyoto target in order to take into account possible unreported emissions.

Normally, prices (the shadow price) are defined as the cost derivatives at a given point. However, in the real world, neither sellers nor buyers are apt to disclose their emission reduction costs. Moreover, the cost reduction function are usually not precisely known. Finally, they are not the only component of emission permit prices. Therefore, in the solution described we assume that a transaction is finalized only when the permit price that is negotiated is lower than the average cost of emission reduction for the buyer and higher than the average cost of emission reduction for the seller. It is obvious that each party wishes to maximize its profit, and this assumption is the basis for our new quality function.

In the evolutionary approach, which will now be further described, maximization over y_{ji} and π_{ji} is performed in each transaction by genetic operators (which try to simulate the price negotiation process to make the transaction more beneficial and reject any outcomes that are unprofitable), while the total maximization over x_{ji} is the EA task (selection of better market solutions with lower emission reduction costs). As mentioned earlier, the model presented contains important changes in relation to previous models. The most important change is a different goal function (5) which maximizes the difference between cost with no trade and cost with trade, where the buying/selling price of the permit is included. It considerably influences transaction profitability and the decision regarding buying/selling permits, including the decision, whether it is more advantageous to reduce emissions than to buy permits. Unfortunately, in a single-criterion version a large part of the information specific to participating parties is lost after the total sum over i is calculated, but this information is used by genetic operators and can be stored to research the market behavior. The goal function (5) is only a single-criterion version of a more sophisticated model with many criteria: one for each party. A multicriteria model designed for an agent or multicriteria evolutionary system, has not yet been built; however, the first steps have been taken toward applying such a system, through modeling of the Cournot game (Stańczak 2009).

Formulae (6–11) are constraints which assure that the market model created has realistic properties:

- A party is not allowed to emit more than its Kyoto obligation plus acquired permits (6);

- Additional permits can be bought only from parties participating in the market, and no extra permits are available (8);
- Buying/selling permits changes the level of allowable emissions of a party (7);
- The number of permits traded in one transaction is limited to $y_{\max}$ to avoid major fluctuations in permit prices (9);
- The price of the permit in a transaction is identical for the selling and buying party (10);
- The numbers of traded permits are treated as negative values for the selling parties and positive for the buying ones, and their absolute values are equal (11).

In the new model the goal function is given by the following formula:

$$G = \sum_{j=1}^T \max_{x_{ji}} \sum_{i=1}^n \max_{y_{ji}\pi_{ji}} (C_i(x_{j-1,i}) - (C_i(x_{ji}) - y_{ji} \cdot \pi_{ji})) \quad (5)$$

with the constraints:

$$x_{Ti} \leq K_i + \sum_{j=1}^T y_{ji} \quad (6)$$

$$x_{j-1,i} + y_{ji} = x_{ji} \text{ and } x_{0,i} = K_i \quad (7)$$

$$\sum_{i=1}^n \sum_{j=1}^T y_{ji} = 0 \quad (8)$$

$$0 \leq y_{ji} \leq y_{\max} \quad (9)$$

$$\pi_{ji} = \begin{cases} 0 & \text{for parties not trading in the transaction } j \\ \pi_{ji} & \text{for parties trading in the transaction } j \end{cases} \quad (10)$$

$$y_{ji} = \begin{cases} 0 & \text{for parties not trading in the transaction } j \\ -y_{ji} & \text{for party selling in the transaction } j \\ y_{ji} & \text{for party buying in the transaction } j \end{cases} \quad (11)$$

where:

- G is the minimum cost of decreasing emissions for all parties in the model;
 T is number of buying/selling transactions conducted;
 $C_i(x_{ji})$ are the costs of reducing emissions by the party i to x_{ji} after j transactions;
 K_i is the Kyoto target for the party i ;
 n is the number of parties;
 x_{ji} is the emissions of the party i after j transactions;
 y_{ji} is the number of emission permits acquired by the party i ;
 π_{ji} is the price of permits bought/sold in transaction j by the party i .

Using function (5) we look for a solution to maximize the difference between the cost when no trading takes place and the cost when permits are exchanged, in other words, the profit from emission trading. In the previous goal function we minimized the cost of emission reduction without including buying prices and expenditures for this goal: note that the cost of buying can be considerably greater than expenditure on CO₂ reduction if there is no trade. In the new approach, we also assume slightly different methods of permit price setting. The authority or market must set a minimum price below which the permit price cannot go. This is to exclude cases where countries reporting emissions below the Kyoto level have zero marginal abatement costs (compare Fig. 2). Therefore, the marginal cost (e.g., shadow price) is no longer a derivative of the abatement cost, but a derivative with minimal value. In practical cases the price negotiations prevent a situation arising where the price of a permit drops to zero. While no country would wish to “sell” permits at no cost, the model described should have some kind of protection against such cases.

The second important change is the introduction of transactions. Transactions are conducted iteratively until there is no further benefit for participants and all parties have dropped out. Prices and amounts of transferred permits are negotiated. Thus, unlike the static base model, our market model is dynamic.

The real price of permits and the number of permit is not known before computer simulation of market activity is conducted and the process of price negotiations among parties is emulated. As an approach to negotiations, in computer simulations presented in this paper the number of permits sold is randomly chosen from some interval. In a similar way the permit price is chosen from the interval between the maximum price (shadow price) of the buyer and minimum price (modified shadow price) of the seller.

Similar to the base model, the new formulae (5–10) do not deal with the problem of the uncertainty of reported emissions; however, this can be changed by introducing a new formula (12) instead of a formula (6) to consider possible unreported emissions:

$$x_{Ti} + (1 - \zeta) (1 - 2\alpha) d_i n_i \leq K_i + \sum_{j=1}^T y_{ji} \quad (12)$$

where all symbols have the same meaning as in formulae (4–11).

3 Evolutionary algorithm method in computer simulations

Although the standard evolutionary algorithm works as shown in Table 1, many problem-specific improvements are needed to make this simple scheme work efficiently. To adjust the genetic algorithm to the solved problem, there must be: (1) a proper encoding of solutions; (2) creation of specialized genetic operators for that problem and an accepted data structure; and (3) a fitness function that is optimized by the algorithm.

We thus use a specialized evolutionary algorithm to solve the problem: one individual contains information about all the parties participating in the market, making it a complete solution to our problem. Another method may also be applied whereby each party is treated as one independent individual (Alkemade et al. 2006; Clemens and Riechmann 2006). In the latter case we obtain only one solution, as

Table 1 The evolutionary algorithm

Random initialization of the population of solutions.
Reproduction and modification of solutions using genetic operators.
Valuation of the solutions obtained.
Selection of individuals for the next generation.
If a stop condition is not satisfied, return to step 2.

the population of solutions in evolutionary algorithm is limited to the number of parties participating in the trade. The case considered (five parties) is too small for an evolutionary algorithm to work efficiently, and is therefore not used.

We use the former approach. Thus, the whole individuals' population contains a number of solutions, as many as the number of individuals. The solutions need not be different, although they usually are. Thanks to this parallelism of evolutionary computations, we usually obtain several scenarios of possible market evolution.

The information needed to describe one party is as follows:

- Theoretical price of own permits (shadow price);
- The real price of current permits sold/bought;
- The value of current permits sold/bought;
- Number of permits currently sold/bought;
- The total sum of permits sold/bought;
- Current emissions;
- Previous emissions (before present transaction);
- Value of present and previous goal function.

To modify the solution, the following genetic operators were used:

- Competition: a chosen party offers a number of permits for sale, some other parties declare a willingness to buy them, next the best option is chosen, and the solution (the individual) is modified;
- Sale: the chosen parties conduct transactions.

The prices and numbers of permits traded are randomly chosen. The number of permits traded is chosen from the interval $\{1, \dots, 5\}$,¹ and the permit price is drawn from a given distribution as a value between the buying and selling offer, with the expected value of the distribution being the average of these two values.

In EA a goal function is called fitness function because it is often modified (scaled or moved) due to EA requirements. Thus the fitness function for EA is a direct goal function of a problem, as described by formula (5).

Population initializing procedures and genetic operators are designed so as to obey the constraints (6–12), and forbidden solutions cannot appear in the population of solutions.

As specialized genetic operators are used, some method of sampling them needs to be applied in all iterations of the algorithm. In the approach used (Mulawka and Stańczak 1999; Stańczak 2003), it is assumed that an operator that generates good

¹The limitation on the maximum amount of permits sold is introduced, as trading too large a number of permits in one transaction can have a large impact on permit prices and undermine profitability. One permit is an equivalent of 1MtC.

Table 2 The data applied for calculations for various regions

Country	Initial emission (x_{i0}) MtC/year	Cost function parameter (a) MUSD/(MtC/year) ²	Limit Kyoto (K_i) MtC/year
USA	1,820.3	0.2755	1,251
EU	1,038.0	0.9065	860
Japan	350.0	2.4665	258
CANZ	312.7	1.1080	215
FSU	898.6	0.7845	1,314

results should have greater probability and affect the population more frequently. As every individual may have its own preferences, every individual has a vector of floating point numbers, beside the encoded solution. Each number corresponds to one genetic operation. It is a measure of quality of the genetic operator. The higher the number, the higher the probability of operator execution by the individual.

This set of probabilities or, in other words, the ranking of qualities is also a basis of the experience of every individual, and each individual chooses an operator in each epoch of the algorithm according to it. Through the experience gathered, individuals can maximize the chances of their offspring surviving.

4 Computer simulation results

Computer simulations were conducted on data set, in line with other authors' papers, mainly Bartoszczuk and Horabik (2007), Horabik (2005), and Nahorski et al. (2007). We consider a group of the following countries: United States (USA); European Union (EU); Japan; Canada, Australia and New Zealand (CANZ); and former Soviet Union with Eastern Europe (FSU). We assume that cost depends on emission reductions in the following way (quadratic cost function) (Horabik 2005; Bartoszczuk and Horabik 2007).

$$C(x_i) = \begin{cases} a * (x_{i0} - x_i)^2 & \text{for } x_i < x_{i0} \\ 0 & \text{for } x_i \geq x_{i0} \end{cases} \quad (13)$$

Table 3 Results in scenario assuming perfect permit market model

Region/ country	Final emission MtC/year	Final price USD/tC	Number of imported permits Mt/year	Permits expenditures MUSD/year	Emission reduction cost MUSD/year
USA	1,561.0	143	310	11,974.3	18,523.7
EU	959.0	143	100	15,790.6	5,515.1
Japan	321.0	143	63	29,987.6	2,074.3
CANZ	248.0	143	33	16,077.6	4,638.2
FSU	808.0	143	−506	−73,830.1	6,439.5
Total	–	–	0	0	37,190.8

Table 4 The results of simulation from the new model; uncertainty not considered

Region/ country	Final emission MtC/year	Final price USD/tC	Number of imported permits Mt/year	Permits expenditures MUSD/year	Emission reduction cost MUSD/year
USA	1,562.0	142.3	311	47,486.4	18,381.1
EU	959.0	143.2	99	15,823.3	5,657.5
Japan	321.0	143.1	63	18,521.7	2,074.3
CANZ	248.0	143.4	33	4,010.5	4,638.2
FSU	808.0	142.2	−506	−85,841.9	6,439.5
Total	–	–	0	0	37,190.6

where:

- a cost function parameter;
- x_{i0} initial emission;
- x_i current emission.

In Table 2 we describe abatement cost function coefficients, which have special interpretation.

Table 5 Results obtained using the described new model; uncertainty considered with different values of risk parameter α , see formula (12)

Region	Reported emissions (MtC/year)	Final price USD/tC	Permits traded (Mt/year)	Cost of traded permits MUSD/year	Cost of emission reduction MUSD/year
Risk parameter $\alpha = 0.5$					
USA	1,559.0	144.0	308	44,834.0	18,810.5
EU	970.0	141.4	100	14,393.0	5,515.1
Japan	321.0	143.1	63	17,831.2	2,074.3
CANZ	249.0	141.2	34	4,049.0	4,495.9
FSU	809.0	140.6	−505	−81,107.2	6,298.1
$\alpha = 0.3$					
USA	1,538.0	167.1	287	45,975.4	25,340.5
EU	957.0	167.2	97	17,948.8	7,707.3
Japan	331.0	167.4	63	17,159.6	2,839.9
CANZ	242.0	168.2	27	5,406.2	6,384.3
FSU	840.0	167.5	−474	−86,489.9	8,936.8
$\alpha = 0.1$					
USA	1,514.0	191.9	263	51,350.1	33,414.8
EU	954.0	192.9	94	16,610.9	10,265.5
Japan	321.0	191.7	63	18,810.4	3,725.4
CANZ	237.0	190.8	22	2,737.3	8,216.9
FSU	872.0	192.8	−442	−89,508.7	11,842.5
$\alpha = 0.0$					
USA	1,502.0	204.3	251	51,801.7	37,870.0
EU	954.0	203.1	94	19,372.8	11,375.2
Japan	320.0	208.8	62	16,584.4	4,419.5
CANZ	234.0	203.3	19	1,403.0	9,321.1
FSU	888.0	205.4	−426	−89,161.9	13,448.4

In a traditional method (perfect market information) we obtain the results presented in Table 3.

In the case of new model application the results are presented in Table 4.

Application of the EA method to simulate the permits market provides some additional benefits, as the result is not just a single set of parameters, but a set of possible scenarios. EA operates on a population of mainly different individuals, and computations are conducted in a non-deterministic way. In particular, negotiation of permit prices is modeled as random number generation from a little modified normal distribution, cut to the desired interval—prices are generated from the interval that is profitable for both countries, and if there is no such interval, no transaction is made. Thus, different scenarios depend mainly on negotiated prices (i.e., prices randomly generated in the simulation). This non-deterministic aspect of EA can be seen if one compares results presented in Tables 4 and 5 for $\alpha = 0.5$: this value of risk parameter α makes the uncertainty component in (12) inactive, but results are not exactly the same. Thus, two different scenarios of market evolution can be observed. Such scenarios can be obtained and used as a basis for more sophisticated analyses of market behavior.

5 Conclusions

In contrast with the previous model (Bartoszczuk and Horabik 2007), our original permit market model is dynamic. The results show that including perfect permit prices is more cost-effective than in our dynamic market solution, but this fact is easy to explain—transactions are conducted at negotiated prices (free market assumptions), not at optimal prices calculated and imposed by some authority. The permit distribution is thus slightly different in the second model (column 4 in Tables 3 and 4), and the structure of buying parties is changed; it is more beneficial to reduce emissions than to buy permits in the case of the USA (column 6 in Tables 3 and 4). Obviously, the total cost of emission reduction in the second method (85,841.9 MUSD/year) is higher than in the first (73,830.1 MUSD/year), which can be explained by higher expenditure for permits and slightly higher necessary emission reduction. Results obtained using the new model with different values of risk parameter α show that in a case with full uncertainty, fewer permits are purchased and there is a greater reduction of emissions, which is more expensive. Moreover, the permit prices in the scenario with full uncertainty is 30% higher than in the scenario with parameter $\alpha = 0.5$ (no-uncertainty scenario). For all practical purposes we are able to anticipate that applying the dynamic model requires additional agreements among parties. While such activities are difficult to implement, our analysis proves that they are environmentally friendly, as they require emission reductions.

Open Access This article is distributed under the terms of the Creative Commons Attribution Noncommercial License which permits any noncommercial use, distribution, and reproduction in any medium, provided the original author(s) and source are credited.

References

- Alkemade F, La Poutre H, Amman M (2006) Robust evolutionary algorithm design for socio-economic simulation. *Comput Econ* 28:355–470

- Bartoszczuk P, Horabik J (2007) Tradable permit systems: considering uncertainty in emission estimates, water air soil pollution. *Focus* 7:573–579
- Bonatti M, Ermoliev Y, Gaivoronski A (1998) Modeling of multi-agent market systems in the presence of uncertainty: the case of information economy. *Robot Auton Syst* 24(3–4):93–113
- Clemens C, Riechmann T (2006) Evolutionary dynamics in public good games. *Comput Econ* 28:399–420
- Ermoliev Y, Klaassen G, Nentjes A (1996) Adaptive cost-effective ambient charges under incomplete information. *J Environ Econ Manage* 3:37–48
- Ermoliev Y, Michalevich M, Nentjes A (2000) Markets for tradable emission and ambient Permits: a dynamic approach. *Environ Resour Econ* 15:39–56
- Godal O (2000) Simulating the carbon permit market with imperfect observations of emissions: Approaching equilibrium through sequential bilateral trade. Interim Report IR-00-060, IIASA. Laxenburg, Austria
- Horabik J (2005) On the costs of reducing GHG emissions and its underlying uncertainties in the context of carbon trading. Raport Badawczy RB/34/2005, IBS PAN
- Klaassen G, Nentjes A, Smith M (2001) Testing the dynamic theory of emissions trading: experimental evidence for global carbon trading, Interim Report IR-01-063, IIASA. Laxenburg, Austria
- Mulawka J, Stańczak J (1999) Genetic algorithms with adaptive probabilities of operator selection. In: *Proceedings of ICCIMA '99*. New Delhi, India, pp 464–468
- Nahorski Z, Horabik J, Jonas M (2007) Compliance and emissions trading under the Kyoto protocol: rules for uncertain inventories. *Accounting for climate change. Uncertainty in greenhouse gas inventories—verification, compliance, and trading*. Springer, pp 119–138
- Stańczak J (2003) Biologically inspired methods for control of evolutionary algorithms. *Control Cybern* 32(2):411–433
- Stańczak (2009)) Evolutionary algorithm for market simulations. *Evolutionary computation and global optimization 2009*, *Prace Naukowe PW, Elektronika*, z. 169, Warszawa, pp 163–172