Erratum to: Existence and stability of symmetric periodic simultaneous binary collision orbits in the planar pairwise symmetric four-body problem

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The conclusions of the numerical results on the linear stability of the periodic orbits remain valid for $0.27 < m \leq 1$ where the error in the numerical estimate of the repeated trivial characteristic multiplier 1 remains small. However, there is an error in the stated algebraic multiplicity of the trivial characteristic multiplier 1 for each periodic orbit. Instead of at least two, it should be at least four.

In Lemma 3.1 the final sentence should read as: “The angular momentum A and the $SO(2)$ symmetry of the PPS4BP contribute to the algebraic multiplicity of the trivial characteristic multiplier 1 of $\gamma_\epsilon(s)$. Proving the following result is routine (see Meyer and Hall 1992).”

Lemma 3.2 should read as: *If $\gamma(s)$ is a periodic orbit of the Hamiltonian system of equations with Hamiltonian $\hat{H}$ on the level set $\hat{H} = 0$, then for each $\epsilon > 0$ the periodic orbit $\gamma_\epsilon(s)$ has 1 as a defective characteristic multiplier with algebraic multiplicity at least four.*

For each $\epsilon > 0$, the periodic orbit $\gamma_\epsilon(s)$ is *spectrally stable* if all of its characteristic multipliers have modulus one. By Lemma 3.2, the periodic orbit $\gamma_\epsilon(s)$ has 1 as

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a defective characteristic multiplier with algebraic multiplicity at least four, and so the monodromy matrix $X_\epsilon(T_\epsilon)$ is not semisimple. However, there is a four-dimensional subspace U_1 containing

$$\text{Span} \left(\gamma'_\epsilon(0), \left(\frac{\partial}{\partial \epsilon} \gamma_\epsilon \right) (0) \right)$$

that is $X_\epsilon(T_\epsilon)$ -invariant. The periodic orbit $\gamma_\epsilon(s)$ is said to be *linearly stable* if it is spectrally stable and there exists a 4-dimensional $X_\epsilon(T_\epsilon)$ -invariant subspace U_2 such that $U_1 + U_2 = \mathbb{R}^8$ and $X_\epsilon(T_\epsilon)$ restricted to U_2 is semisimple. A proof of the following is routine.

Theorem 3.3 *Suppose $\gamma(s)$ is a periodic orbit of the Hamiltonian system of equations with Hamiltonian $\hat{\Gamma}$ on the level set $\hat{\Gamma} = 0$. Then $\gamma_\epsilon(s)$ is spectrally (linearly) stable for some $\epsilon > 0$ if and only if $\gamma_\epsilon(s)$ is spectrally (linearly) stable for all $\epsilon > 0$.*

In the heading 6 Numerical estimates in the equal mass case, the paragraph should read as: The components of the scaled periodic orbit $\gamma_\epsilon(s)$ for an $\epsilon > 1$, shown in Fig. 2, satisfy the symmetries $S_F \gamma_\epsilon(s) = \gamma_\epsilon(s + T_\epsilon/4)$ and $S_G \gamma_\epsilon(s) = \gamma_\epsilon(T_\epsilon/4 - s)$. We choose ϵ so that $T_\epsilon = 2\pi$, and check the linear stability of $\gamma_\epsilon(s)$ (see Theorem 3.3). Using a Runge–Kutta order 4 algorithm with a fixed time step of $2\pi/50000$, we computed $X_\epsilon(2\pi)$. Four of the eigenvalues of $X_\epsilon(2\pi)$ are 1 by Theorem 3.2. Numerical estimates of the remaining eigenvalues of $X_\epsilon(2\pi)$ are

$$\begin{aligned} & -0.9888731375 \pm 0.1487612779i, \\ & -0.9973584383 \pm 0.07263708002i, \end{aligned}$$

that have modulus one. Thus numerically, the periodic orbits $\gamma_\epsilon(s)$ are linearly stable for all $\epsilon > 0$. The first complex conjugate pair of eigenvalues for $X_\epsilon(2\pi)$ matches the complex conjugate pair of characteristic multipliers for the periodic orbit $Q_1(\sigma)$, $Q_2(\sigma)$, $P_1(\sigma)$, $P_2(\sigma)$ of the Hamiltonian system of equations (14) with Hamiltonian Γ , corresponding to $\gamma_1(s) = \gamma(s)$, where we computed (Bakker et al. 2010) the real part of the complex conjugate pair of modulus one to be -0.9888840619 . Because, by a lengthy computation, $J\nabla^2 \hat{\Gamma}(\gamma_\epsilon(s))$ is not block diagonal, the second complex conjugate pair of eigenvalues of $X_\epsilon(2\pi)$ are not repeats of the characteristic multipliers of the periodic orbit $Q_1(\sigma)$, $Q_2(\sigma)$, $P_1(\sigma)$, $P_2(\sigma)$.

In the heading 7 Numerical estimates in the unequal mass case, the final paragraph should read as: We numerically estimate the monodromy matrices $X(2\pi; m)$ and their eigenvalues for $\gamma(s; m)$ with $0 < m \leq 1$. We used a Runge–Kutta order 4 algorithm coded in MATLAB with a fixed time step of $2\pi/50000$ at increments of 0.01 in m . In Fig. 6 we graph the maximum modulus of the eight eigenvalues of $X(2\pi; m)$ over $0.2 < m \leq 1$. The maximum modulus of the eigenvalues of $X(2\pi; m)$ for $0 < m \leq 0.2$ becomes large as $m \rightarrow 0$, and are therefore not included in Fig. 6. The maximum modulus for the eight eigenvalues shows that $\gamma(s; m)$ is linearly unstable for $0 < m \leq 0.53$. The error in the numerical estimation of the repeated eigenvalue 1 of $X(2\pi; m)$ is large enough to account for the graph in Fig. 6 not lying at 1 over $0.54 \leq m \leq 1$. However, for $0.54 \leq m \leq 1$, the other four eigenvalues come in two distinct complex conjugate pairs that are each of modulus one and located away from ± 1 . This shows that $\gamma(s; m)$ is linearly stable for $0.54 \leq m \leq 1$.