Reply to the Comments by J. C. Bergmann on the Paper: “Local Similarity in the Stable Boundary Layer and Mixing Length Approaches: Consistency of Concepts”

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In his comment J. C. Bergmann discusses the analysis of Van de Wiel et al. (2008; VDW08), acknowledges the conceptual merits of the work related to the introduced “apparent wall concept”, but identifies a factor of two error in the potential energy calculation. As a consequence (see Bergmann 2011; B11), the derived value (equal to 6.25) of the numerical constant (slope) in the Monin–Obukhov similarity function, respectively in its local-scaling equivalent (Nieuwstadt 1984), is reduced to half its value. The present authors fully agree with this re-analysis by B11, but wish to emphasise that the “apparent wall concept” is used to anticipate a plausible order of magnitude of the abovementioned constant, rather than predicting an exact value.

In the following, we briefly reanalyse the comment of B11. Consider a linear density profile (the very stable limit considered in VDW08). On average, parcels at a reference level $z' = 0$ with density ρ_0 will reach their ‘maximum’ displacement z_B when all their vertical kinetic energy is converted into potential energy (see Fig. 1).

B11 refers to z_B “as the root-mean-square displacement”. Thus we can find this length scale by computing the maximum work (in the mean sense: B11) that can be done by parcels, given their mean initial vertical kinetic energy $(1/2)\rho\sigma_w^2$ (per volume):

$$(1/2)\rho\sigma_w^2 = g \int_0^{z_B} (\rho_0 - \rho) dz' \quad (1)$$

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Fig. 1 Energy consideration of moving parcels in a stably stratified environment starting from the reference level $z' = 0$

With $\rho = \rho_0 + z' \partial \rho / \partial z'$ this becomes (with a constant density gradient):

$$(1/2) \rho \sigma_w^2 = -g \partial \rho / \partial z' \int_0^{z_B} z' dz' = -g \frac{\partial \rho}{\partial z'} \frac{1}{2} z_B^2, \quad (2)$$

and following VDW08 (with e.g. the Boussinesq assumption) we find indeed (as in B11):

$$z_B = \frac{\sigma_w}{N} \quad (3)$$

with N the buoyancy frequency. This result differs by a factor $1/\sqrt{2}$ from expression (13) in VDW08. Consequently, the estimated numerical constant α in the local similarity function $\phi_{m,h} = 1 + \alpha z/\Lambda$ will be reduced to half its original value. As VDW08 originally found $\alpha \approx 1/\kappa^2 = 6.25$ this implies that the correct estimate of α should be around 3.13.

So far the current authors agree with B11. However, in his section 3 B11 extends the findings to his own recent work. By assumptions on values of the empirical ratio σ_w/u_* and the turbulent Prandtl number, the estimate 3.13 is adjusted to 2.77 and B11 states: “*This value is very close to the author’s as yet unpublished result of 2.6–2.8 derived from energy balance and transport statistics.*” By placing emphasis on this exact number B11 suggests that he uses VDW08 beyond its validity, as their analysis was only meant as a plausible first-order estimate (their Section 3). Moreover in his Sections 3 and 4, where he discusses the fact that Bergmann’s $\alpha \approx 2.6$ –2.8 diverges from the ‘classical’ higher values of say $\alpha \approx 4$ –8, his analysis then becomes somewhat speculative stating: “*so that it is more than plausible to assume an as yet undiscovered stability-dependent systematic error in the experiments that yield large slopes*”. We dissociate ourselves from such speculations.

In fact, even in VDW08 recent results from direct numerical simulations of the Navier–Stokes equations for stationary conditions show higher α values than those indicated by B11. Though such a deviation of B11 theory could have several causes (including a Reynolds number dependence), it indicates that the discussion on α is still open. In this respect, Zilitinkevich et al. (2010), as discussed by B11, mention a typical α value of $C_u/\kappa \approx 5$ (note that their ‘classical’ Obukhov length does not include the von Karman constant). However, as different viewpoints on this topic improve our understanding of turbulent mixing in stably stratified flows, we encourage B11 to disseminate his scientific ideas related to this discussion.

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