

# Galaxy rotations from quantised inertia and visible matter only

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Received: 3 March 2017 / Accepted: 26 June 2017 / Published online: 2 August 2017  
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**Abstract** It is shown here that a model for inertial mass, called quantised inertia, or MiHsC (Modified inertia by a Hubble-scale Casimir effect) predicts the rotational acceleration of the 153 good quality galaxies in the SPARC dataset (2016 AJ 152 157), with a large range of scales and mass, from just their visible baryonic matter, the speed of light and the co-moving diameter of the observable universe. No dark matter is needed. The performance of quantised inertia is comparable to that of MoND, yet it needs no adjustable parameter. As a further critical test, quantised inertia uniquely predicts a specific increase in the galaxy rotation anomaly at higher redshifts. This test is now becoming possible and new data shows that galaxy rotational accelerations do increase with redshift in the predicted manner, at least up to  $Z = 2.2$ .

**Keywords** Celestial mechanics

## 1 Introduction

It has been well known since van Oort (1932), Zwicky (1933) and Rubin et al. (1980), that galaxies rotate far too fast to be gravitationally stable. The usual solution for this is to add dark matter to the galactic haloes to hold stars in with more gravitational force. This solution is ad hoc since it has to be added to different galaxies in different amounts. It is also difficult to falsify so it is therefore unsatisfying, and it has recently been shown by McGaugh et al. (2016) that the acceleration of stars in galaxies is correlated with the distribution of the visible matter only, which implies there is no dark matter.

One alternative to dark matter is MoND (Modified Newtonian Dynamics) (Milgrom 1983) in which either the gravitational force on, or the inertial mass of, orbiting stars is changed for very low accelerations. MoND is an empirical hypothesis that has no physical model and relies on its adjustable parameter ( $a_0$ ) which is fitted to the data by hand, which is unsatisfactory since no justification is given for this parameter.

Fulling (1973), Davies (1975) and Unruh (1976) proposed that when an object accelerates it perceives Unruh radiation, a dynamical equivalent of Hawking radiation (Hawking 1974), and Unruh radiation may now have been seen in experiments (Smolyaninov 2008).

McCulloch (2007, 2013) has proposed a new model for inertia that assumes that when an object accelerates, say, to the right, an information horizon forms to its left and it perceives Unruh radiation which is also suppressed by the horizon on the left. Therefore there is a radiation imbalance, and net Unruh radiation pressure that pushes the object back against its initial acceleration, predicting standard inertia (McCulloch 2013; Gine and McCulloch 2016). Furthermore, this model predicts that some of the Unruh radiation will also be suppressed, this time isotropically, by the distant cosmic horizon which will make this mechanism less efficient, reducing inertial mass in a new way, especially for very low accelerations for which Unruh waves are very long (McCulloch 2007). The complete model, called MiHsC (Modified inertia by a Hubble-scale Casimir effect) or quantised inertia modifies the standard inertial mass ( $m$ ) as follows:

$$m_i = m \left( 1 - \frac{2c^2}{|a|\Theta} \right) \quad (1)$$

where  $c$  is the speed of light,  $\Theta$  is the Hubble diameter and  $|a|$  is the magnitude of the acceleration of the object relative to surrounding matter. Equation (1) predicts that for

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terrestrial accelerations (e.g.: 9.8 m/s<sup>2</sup>) the second term in the bracket is tiny and standard inertia is recovered, but in environments where the mutual acceleration is of order 10<sup>-10</sup> m/s<sup>2</sup>, for example at the edges of galaxies or in dwarf galaxies, the second term becomes larger and the inertial mass decreases in a new way. This modification does not affect equivalence principle tests using torsion balances or free fall since the predicted inertial change is independent of the mass.

In this way quantised inertia explains galaxy rotation without the need for dark matter (McCulloch 2012, 2017) because it reduces the inertial mass of outlying stars and allows them to be bound even by the gravity from visible matter. It also explains the recently observed cosmic acceleration (McCulloch 2010) and the experimental tests on the emdrive (McCulloch 2015).

Recently, McGaugh et al. (2016) analysed 153 galaxies taken from the SPARCs database across a large range of scales and showed that the actual acceleration of the stars within them, as determined from the stars' observed motion, was correlated only with the acceleration that would be expected given the visible matter in the galaxy. This result has now also been shown to apply to elliptical galaxies (Lelli et al. 2017). As mentioned above, these results argue against the existence of dark matter. McGaugh et al. (2016) also found that the relationship between the observed acceleration (*a<sub>obs</sub>*) and that expected from the visible or baryonic matter (*a<sub>bar</sub>*) could be described quite well by the function

$$a_{obs} = \frac{a_{bar}}{1 - e^{-\sqrt{a_{bar}/a_0}}} \tag{2}$$

It has been implied that this function can be obtained from some versions of Modified Newtonian Dynamics (MoND) of Milgrom (1983), though this empirical model needs to be adjusted to fit, and has no supporting physical model.

In this paper, it will be shown that quantised inertia can predict the new galaxy data presented by McGaugh et al. (2016) without any adjustable parameters, simply from the visible matter, the speed of light and the co-moving diameter of the observable universe. It is also shown that quantised inertia can be tested because it uniquely predicts a significant change in the galactic acceleration relation with redshift.

## 2 Method

To briefly recapitulate McCulloch (2012), we start with Newton's gravity and second laws, for a star of mass *m* orbiting a galaxy of mass *M* at radius *r* as follows

$$F = m_i a = \frac{GMm}{r^2} \tag{3}$$

Replacing *m<sub>i</sub>* using quantised inertia, Eq. (1), we get

$$\left(1 - \frac{2c^2}{|a|\Theta}\right) a = \frac{GM}{r^2} \tag{4}$$

Splitting the acceleration *|a|* up into a slowly varying (rotational) part *a = v<sup>2</sup>/r* and a variable part *a'* due to inhomogeneities in the matter distribution, gives

$$\left(|a| + |a'| - \frac{2c^2}{\Theta}\right) a = \frac{GM(|a| + |a'|)}{r^2} \tag{5}$$

At the edge of a galaxy, *|a|* becomes small, so the acceleration must be maintained above the minimum acceleration allowed in quantised inertia (McCulloch 2007) by the value of *a'*, and so *a' = 2c<sup>2</sup>/Θ*. Therefore the second and third terms cancel. In this way, McCulloch (2012) derived the following formula

$$a^2 = \frac{GM(|a| + |a'|)}{r^2} \tag{6}$$

The *a* on the left hand side can be called the predicted total acceleration *a<sub>pred</sub>*. The factor *GM/r<sup>2</sup>* and the *|a|* on the right hand side can be replaced with *a<sub>bar</sub>*: the baryonic, standard model, acceleration. Assuming, again, that at a galaxy's edge the residual acceleration *a' = 2c<sup>2</sup>/Θ* since accelerations cannot fall below this minimum in quantised inertia, then we get

$$a_{pred}^2 = a_{bar}^2 + a_{bar} \frac{2c^2}{\Theta} \tag{7}$$

which leads to the formula

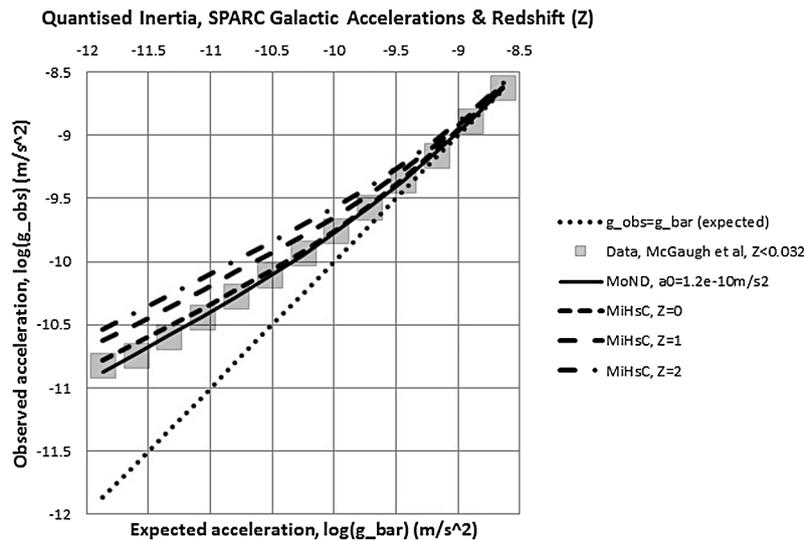
$$a_{pred} = a_{bar} \sqrt{1 + \frac{2c^2}{a_{bar}\Theta}} \tag{8}$$

In the next section this prediction is compared with the raw SPARC data collated by McGaugh et al. (2016).

## 3 Results

Figure 1 shows the log of the expected (Newtonian) stellar acceleration, predicted from the visible mass, on the horizontal axis and the log of the observed accelerations, observed from stellar motions, on the vertical axis. The expected result from standard physics is the diagonal dotted line. The grey squares show the observed accelerations from the binned data obtained from McGaugh et al. (2016) (pers. comm.). The size of the squares show the rms error in each bin. For very low accelerations (on the left) the data lifts above the dotted line, so that the observed accelerations are much higher than those expected from the standard Newtonian (or general relativistic) model. This is the well-known galaxy rotation problem.

The black line shows the prediction of one of the variations of MoND (Modified Newtonian Dynamics) of Milgrom (1983). MoND agrees with the data, but it has been



**Fig. 1** The x-axis shows the expected (Newtonian) acceleration of stars in a galaxy given the visible mass distribution. The y-axis shows the acceleration observed from the movements of its stars. The expected Newtonian result is shown by the *dotted line*. The data (binned using 3300 data points in 153 separate galaxies by McGaugh et al. 2016) is shown by the *grey squares* and the *error bars* are shown by the size of the squares. Both MoND (*black line*) and quantised inertia

(*dashed* for redshift  $Z = 0$ ) agree with the observations, but quantised inertia predicts the data just from the speed of light and the co-moving diameter of the cosmos (see Eq. (8)) whereas MoND and other solutions like dark matter need arbitrary ‘fitting’. Also shown are the predictions of quantised inertia for earlier galaxies with redshifts of  $Z = 1$  (*long-dashed*) and  $Z = 2$  (*dot-dashed*)

fitted to galaxy data using its adjustable parameter (the value used here was  $a_0 = 1.2 \times 10^{-10} m/s^2$ ) so the agreement is not so remarkable.

The prediction of quantised inertia for a redshift of zero, the present epoch, is shown by the dashed line, and it also fits the data for the present epoch within the error bars, and this agreement requires no adjustment at all. The curve is predicted, by Eq. (8) above, and therefore uses only the visible matter, the speed of light ( $c$ ) and the co-moving cosmic diameter ( $\Theta$ ), which is assumed to be 93 billion light years or  $\Theta = 8.8 \times 10^{26}$  m and is the cosmic diameter at the present time following Bars and Terning (2009) (see page 27). Note that this is different from the value of  $\Theta = 2.6 \times 10^{26}$  m used in McCulloch (2007, 2012) which represents the cosmic diameter at the epoch when the light from the galaxies was emitted.

#### 4 Discussion

It is always useful to suggest a test, by predicting something unique that has not yet been observed. Unlike MoND, which uses a constant and pre-set parameter  $a_0$ , quantised inertia relies on the value of  $2c^2/\Theta$ . This depends on the co-moving size of the cosmos  $\Theta$  which increases with time (or decreases with time into the past) and so quantised inertia predicts a change in galaxy rotation with time. This can be seen in Eq. (8) which depends on the cosmic diameter  $\Theta$  which was smaller in the distant past and, assuming a linear

expansion of the cosmos with time, depends on the redshift ( $Z$ ) as follows

$$\Theta_{atZ} = \frac{\Theta_{now}}{1+z} \quad (9)$$

Therefore, quantised inertia predicts that the acceleration relation in Fig. 1 should show a dependence on redshift. At higher  $z$  (further back in cosmic time) the galaxy rotation problem should be more obvious (everything else, such as galaxy evolution, being taken care of) so that equal-mass galaxies observed in the distant past should spin faster. This is illustrated in Fig. 1 with the dashed line which represents the prediction of quantised inertia for  $z = 0$ , the present epoch, the longer-dashed line which shows the prediction for  $z = 1$  (when the cosmos was half its present size) and the dot-dashed curve which shows the prediction for  $z = 2$ . This prediction is unique to quantised inertia, and the data is now becoming available to test this prediction. For example, Thomas et al. (2013) looked at galaxies at different redshifts from  $z = 0$  to  $z = 2$  from the Sloan Digital Sky Survey/Baryonic Oscillation Spectroscopic Survey SDSS/BOSS collaboration, and showed that higher redshift galaxies have higher velocity dispersions (see their Fig. 6) but better data is needed to confirm this.

A more common way of looking at this is to consider the mass required to produce a particular rotation speed. The required mass can be derived from quantised inertia as fol-

lows. At a galaxy’s edge, since the rotational acceleration ( $|a|$ ) is so slow, Eq. (6) can be rewritten as

$$a^2 = \frac{2GMc^2}{\Theta r^2} \tag{10}$$

and replacing  $a^2$  using  $v^4/r^2$  we get

$$v^4 = \frac{2GMc^2}{\Theta} \tag{11}$$

This is the Tully-Fisher relation predicted by quantised inertia, which now varies with time since  $\Theta$  was smaller in the past. Using Eq. (9) to take account of this evolution, we get

$$v^4 = \frac{2GMc^2(1 + Z)}{\Theta_{now}} \tag{12}$$

So that the amount of mass associated with a rotation speed of  $v$  is given by

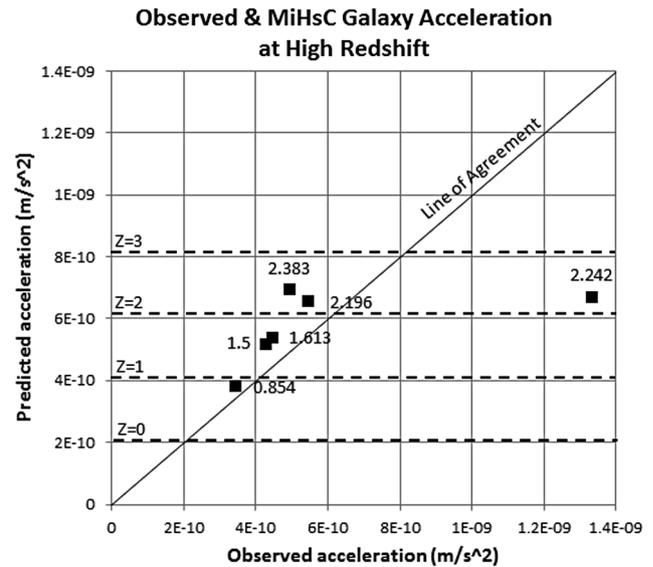
$$M = \frac{v^4 \Theta_{now}}{2Gc^2(1 + Z)} \tag{13}$$

Recently, Uebler et al. (2017) found that at redshifts of  $Z = 0.9$  the amount of mass associated with a specific rotation speed is reduced by between  $-0.38$  and  $-0.47$  dex ( $dex(x) = 10^x$ , see their Fig. 7) and at  $Z = 2.3$  the reduction in mass is between  $-0.2$  and  $-0.47$  dex. Quantised inertia (Eq. (13)) predicts a reduction in mass of  $-0.28$  and  $-0.52$  dex respectively.

Probably the best source of data on this to date is Genzel et al. (2017) who looked at six massive galaxies at high redshifts, between  $Z = 0.854$  and  $Z = 2.383$ , and also showed that their rotation speed increased at higher redshifts. Another way to model this with quantised inertia is to note that it precludes accelerations below  $2c^2/\Theta$ . Figure 2 shows along the x axis the observed acceleration of the galaxies (calculated from their half-light radii and their velocity dispersion) and along the y axis the minimum acceleration allowed by quantised inertia. The six black squares show the comparisons for the six galaxies, and the adjacent numbers indicate their redshifts. In both the observations and the predictions from quantised inertia, the galactic accelerations increase with redshift. The predictions show the same tendency as the observations (the squares are close to the line of agreement), but they are between 9% and 19% higher for the four lower redshift galaxies. This difference can be accounted for by uncertainties in the cosmic expansion model used to determine  $\Theta$ . Agreement is worse for the two highest redshift galaxies, which may be expected to have a larger uncertainty.

### 5 Conclusions

A new model for inertia (called quantised inertia or MiHsC) predicts the observed rotational accelerations of the 153 galaxies in the recent SPARC dataset simply from their visi-



**Fig. 2** The observed acceleration of the six galaxies (on the x axis) from Genzel et al. (2017) plotted against the minimum acceleration allowed by quantised inertia at that epoch on the y axis, which is  $2c^2/\Theta$  where  $\Theta = \Theta_{now}/1 + Z$ . The redshift,  $Z$ , is shown as a label against each data point and as the horizontal dashed lines. Both the observed and predicted accelerations rise with redshift. The predicted accelerations are close to those observed (close to the diagonal line) except for the two highest redshift galaxies

ble matter, the speed of light and the co-moving diameter of the cosmos (Fig. 1), without dark matter or any adjustable parameters.

As a test, quantised inertia uniquely predicts a significant increase in the galaxy rotation anomaly at higher redshifts and this is supported by recent data, at least up to a redshift of  $Z = 2.2$ .

**Acknowledgements** Many thanks to S.S. McGaugh for making available the binned SPARC data, and R. Ludwick, T. Short, Magnus Ihse Bursie and J.A.M. Lizcano and an anonymous reviewer for advice.

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