Erratum to: Noncommutativity as a Colimit

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The main purpose of this erratum is to correct a claim we made in our paper “Non-commutativity as a colimit” (Applied categorical structures, Volume 20, Number 4, August 2012, pp. 393–414) at the bottom of page 411, in between Proposition 9 and Lemma 4. Fortunately, our mistake does not invalidate any of the results in the paper or make them less relevant.

Let us first define the categories **CstaredTopos** and **LocaledTopos**.

Definition 1 The category **CstaredTopos** has as objects pairs (T, A) of a topos T and an internal C^* -algebra A . A morphism $(T, A) \rightarrow (T', A')$ consists of a geometric morphism $F : T' \rightarrow T$ and an internal $*$ -morphism $\varphi : A' \rightarrow F^*(A)$ in T' .

The category **LocaledTopos** has as objects pairs (T, L) of a topos T and an internal locale object L . A morphism $(T, L) \rightarrow (T', L')$ consists of a geometric morphism $F : T' \rightarrow T$ and an internal locale morphism $\varphi : F^*(L) \rightarrow L'$ in T' .

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By Proposition 9, Bohrification determines a functor $\mathbf{PCStar}^{\text{op}} \rightarrow \mathbf{CstaredTopos}$. It is not immediately clear that composing this functor with taking internal spectra results in a functor $\mathbf{PCStar}^{\text{op}} \rightarrow \mathbf{LocaledTopos}$: that would require that taking the internal Gelfand spectrum of a commutative C^* -algebra commutes with inverse image functors of geometric morphisms. Contrary to what we state in the paper, these two constructions do indeed commute. As was pointed out to us by Steven Vickers, the description of the internal spectrum as a Grothendieck site is geometric (as is clear from [3]) and therefore preserved by inverse image functors. So Bohrification does yield a functor $\mathbf{PCStar}^{\text{op}} \rightarrow \mathbf{LocaledTopos}$ after all.

Nevertheless, there is no obvious functor $\mathbf{LocaledTopos} \rightarrow \mathbf{Loc}$ and therefore we still do not have a “spectrum functor” $\mathbf{PCStar}^{\text{op}} \rightarrow \mathbf{Loc}$. For that, it still seems necessary to restrict to a special class of morphisms in $\mathbf{PCStar}$ (like those that reflect commeasureability).

Additionally, let us also note that the clause $f(a + ib) = f(a) + if(b)$ in Definition 4 should be dropped: this condition only makes sense when $a \odot b$, in which case it is superfluous. Consequently, the proof of Theorem 5 can be considerably shortened by putting $m(a) = f_{A(a)}(a)$, making it completely analogous to the proof of Theorem 1.