The Stone–Čech Compactification and the Cozero Lattice in Pointfree Topology

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In Section 2, after the account of various properties of the cozero map, it is claimed that

$$\text{coz}((\alpha - r\beta)^+) \ll \text{coz}(\alpha)$$

for $r > 0$ in $\mathbf{Q}$ and arbitrary $\alpha, \beta \geq 0$ in $\mathcal{RL}$. This is clearly false (although it does hold in the particular situation considered later on, where $\text{coz}(\alpha) \vee \text{coz}(\beta) = e$), and hence the proof of Lemma 2 as given is incomplete. It should be replaced by the following.

Proof For any $\alpha, \beta \geq 0 \in \mathcal{RL}$ such that $\text{coz}(\alpha) \vee \text{coz}(\beta) = e$, we show

- (i) $\text{coz}((\alpha - \beta)^+) \vee \text{coz}(\beta) = e$, and
- (ii) $\text{coz}((\alpha - \beta)^+) \ll \text{coz}(\alpha)$.

The following calculations specifically use the rules for $\vee$ and $\wedge$ involving coz listed above as well as the fact that $\text{coz}(s\gamma) = \text{coz}(t\gamma)$ for any $\gamma \in \mathcal{RL}$ and $s, t > 0$ in $\mathbf{Q}$.

Now, regarding (i)

$$\begin{aligned} \text{coz}((\alpha - \beta)^+) \vee \text{coz}(\beta) &= \text{coz}((\alpha - \beta)^+ + \beta) \\ &= \text{coz}(\alpha \vee \beta) = \text{coz}(\alpha) \vee \text{coz}(\beta) = e \end{aligned}$$

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as desired. For (ii) we show

$$\text{coz}((\alpha - s\beta)^+) \prec \text{coz}((\alpha - r\beta)^+)$$

for any $r, s \in \mathbf{Q}$ such that $0 \leq r < s \leq 1$. If $r = 0$ this follows since

$$\begin{aligned} \text{coz}(\alpha) \vee \text{coz}((s\beta - \alpha)^+) &= \text{coz}(\alpha + (s\beta - \alpha)^+) \\ &= \text{coz}(s\beta \vee \alpha) = \text{coz}(\beta) \vee \text{coz}(\alpha) = e \end{aligned}$$

and $(\alpha - s\beta)^+ \wedge (s\beta - \alpha)^+ = 0$.

For $r > 0$ the corresponding calculation is somewhat more elaborate:

$$\begin{aligned} \text{coz}((\alpha - r\beta)^+) \vee \text{coz}((s\beta - \alpha)^+) &= \text{coz}((\alpha - r\beta)^+ + (s\beta - \alpha)^+) \\ &= \text{coz}((s - r)\beta \vee (\alpha - r\beta) \vee (s\beta - \alpha) \vee 0) \\ &\geq \text{coz}((s - r)\beta \vee ((\alpha - r\beta) \vee 0)) \\ &= \text{coz}((s - r)\beta) \vee \text{coz}((\alpha - r\beta)^+) \\ &= \text{coz}(r\beta) \vee \text{coz}((\alpha - r\beta)^+) \\ &= \text{coz}(r\beta + (\alpha - r\beta)^+) = \text{coz}(\alpha \vee r\beta) \\ &= \text{coz}(\alpha) \vee \text{coz}(\beta) = e, \end{aligned}$$

the third step since coz is order preserving on functions ≥ 0 and the fifth because $0 < r < s$ here. $\square$

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