ERRATUM TO: COMMON FIXED POINT THEOREMS FOR EXPANSION MAPPINGS IN VARIOUS SPACES

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Abstract. We prove Theorem 1.3 of [2] under a more general situation and rectify some typographic errors that appear in the paper.

In [2], the author proved some common fixed point theorems for expansion mappings in the framework of various spaces. The reader may consult [2] for terms not specifically defined here.

Wang et al. [3] initiated the study of fixed points of expansion mappings in metric spaces and proved the following theorem.

THEOREM 1. *If a self-mapping f of a complete metric space (X, d) satisfies the condition*

(i) $d(fx, fy) \geq qd(x, y)$ for each $x, y \in X$ and $q > 1$,
then f has a unique fixed point.

Kumar [2] proved the following theorem.

THEOREM 2 (Theorem 1.3 [2]). *Let (X, d) be a complete metric space. Let f and g be weakly compatible (whenever $fx = gx$ implies $fgx = gfx$, for all $x \in X$) self-mappings of X satisfying*

- (i) $g(X) \subseteq f(X)$;
- (ii) $d(fx, fy) \geq qd(gx, gy)$ for each $x, y \in X$ and $q > 1$.

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If one of the subspaces $g(X)$ or $f(X)$ is complete, then f and g have a unique common fixed point.

As an application of axiom of choice, Haghi et al. [1] proved a lemma and showed that some coincidence point or common fixed point abstractions in fixed point theory are not real abstractions as they could easily be obtained from the corresponding fixed point theorems. The lemma (below) also provides an interesting criterion for categorization of generalized common fixed point theorem into classes of well known fixed point theorem which are already known in the setting of metric spaces or more general metric spaces (for more details see [1]).

LEMMA 1 [1]. *Let X be a nonempty set and $f : X \rightarrow X$ a function. Then there exists a subset $E \subseteq X$ such that $f(E) = f(X)$ and $f : E \rightarrow X$ is one-to-one.*

REMARK 1. Using Lemma 1, we can further improve Theorem 2 in a more general setting.

THEOREM 3. *Let (X, d) be a metric space. Let f and g be weakly compatible self-mappings of X satisfying the following:*

- (i) $g(X) \subseteq f(X)$;
- (ii) $d(fx, fy) \geq qd(gx, gy)$ for each $x, y \in X$ and $q > 1$.

If one of the subspaces $g(X)$ or $f(X)$ is complete, then f and g have a unique common fixed point.

PROOF. By Lemma 1, there exists $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \rightarrow X$ is one-to-one. Now, define a map $h : g(E) \rightarrow g(E)$ by $h(gx) = fx$. Since g is one-to-one on E , h is well-defined. Note that, $d(h(gx), h(gy)) \geq qd(gx, gy)$ for all $gx, gy \in g(E)$. Since $g(E) = g(X)$ is complete, by using Theorem 1, there exists $x_0 \in X$ such that $h(gx_0) = gx_0$. Hence, $f(x_0) = g(x_0)$ and x_0 is a coincidence point of f and g . It is now clear that f and g have a unique common fixed point whenever f and g are weakly compatible. $\square$

REMARK 2. Similarly, other common fixed point theorems appeared in [2] can be improved in the same fashion.

Now we rectify some typographically errors of the paper.

REMARK 3. On p. 11, we note the followings:

- (i) Replace line 8:

$$d(STx, TSx) = \{x/(9-3x)\} - \{x/(9-x)\} = x^2/\{(9-x)(9-3x)\}$$

by

$$d(STx, TSx) = \{x/(9-3x)\} - \{x/(9-x)\} = 2x^2/\{(9-x)(9-3x)\}.$$

(ii) Replace in line 11: $STx = \{x/(9-3x)\} > \{x/(9-x)\}$ by

$$TSx = \{x/(9-3x)\} > \{x/(9-x)\}.$$

(iii) Replace in line 21: $\lim_{n \rightarrow \infty} d(SSx_n, TTx_n) = \lim_{n \rightarrow \infty} |n^8 - n^{18}| = 0$ by

$$\lim_{n \rightarrow \infty} d(SSx_n, TTx_n) = \lim_{n \rightarrow \infty} |n^8 - n^{18}| = \infty.$$

REMARK 4. On p. 12, in Example 1.3 (and similarly, on p. 13 in Example 1.6), we have $x_n = 5 + (1/n)$ for $n \geq 1$, then $Tx_n \rightarrow 2$ but $Bx_n \rightarrow 5 \neq 2$ as $n \rightarrow \infty$.

To rectify Example 1.3 (and also Example 1.6), we give the following example.

EXAMPLE 1. Let $X = [2, 20]$ and d be the usual metric on X . Define the mappings $B, T: X \rightarrow X$ by

$$B(x) = \begin{cases} 2 & \text{if } x = 2 \text{ or } > 5 \\ 6 & \text{if } 2 < x \leq 5, \end{cases} \quad T(x) = \begin{cases} 2 & \text{if } x = 2 \\ 12 & \text{if } 2 < x \leq 5 \\ x-3 & \text{if } x > 5. \end{cases}$$

Consider the sequence $\{x_n = 5 + (1/n), n \geq 1\}$. Then $Tx_n \rightarrow 2$, $Bx_n \rightarrow 2$, $TBx_n = 2$, $BTx_n = 6$. Clearly, the mappings B and T are weakly compatible or pointwise R-weakly commuting but not compatible, since for the sequence $\{x_n = 5 + (1/n), n \geq 1\}$, we have $d(Tx_n, Bx_n) \rightarrow 0$ but $d(BTx_n, TBx_n) \not\rightarrow 0$ as $n \rightarrow \infty$.

REMARK 5. (i) On p. 13, in Example 1.7, we have $x_n = 5 + (1/n)$ for $n \geq 1$, then $Tx_n \rightarrow 2$ but $Bx_n \rightarrow 5 \neq 2$ as $n \rightarrow \infty$. To rectify Example 1.7, we redefine $B: X \rightarrow X$, where $X = [2, 20]$ by

$$(1) \quad B(x) = \begin{cases} 2 & \text{if } x = 2 \text{ or } > 5 \\ 8 & \text{if } 2 < x \leq 5. \end{cases}$$

(ii) On p. 14, in Example 1.8, replace in line 28:

$$d(fx, fy) = (1/2)\{|x-y|\} \geq (q/6)\{d(gx, gy)\} \quad \text{for } 1 < q < 3$$

by

$$d(fx, fy) = (1/2)\{|x-y|\} \geq q\{d(gx, gy)\} \quad \text{for } 1 < q \leq 3.$$

(iii) On p. 16, in Example 2.2, replace in lines 29 and 30:

$$\begin{aligned} D(Sx, Sy, Sz) &= \{|x - y| + |y - z| + |z - x|\} \\ &\geq q(Tx, Ty, Tz) \quad \text{for } 1 < q < 2 \end{aligned}$$

by

$$\begin{aligned} D(Sx, Sy, Sz) &= \{|x - y| + |y - z| + |z - x|\} \\ &\geq qD(Tx, Ty, Tz) \quad \text{for } 1 < q \leq 2. \end{aligned}$$

(iv) On p. 17, in Example 2.3, replace in lines 27 and 28:

$$\begin{aligned} D(Sx, Sy, Sz) &= (1/2)\{|x - y| + |y - z| + |z - x|\} \\ &\geq (q/6)\{D(Tx, Ty, Tz)\} \quad \text{for } 1 < q < 3 \end{aligned}$$

by

$$\begin{aligned} D(Sx, Sy, Sz) &= (1/2)\{|x - y| + |y - z| + |z - x|\} \\ &\geq q\{D(Tx, Ty, Tz)\} \quad \text{for } 1 < q \leq 3. \end{aligned}$$

(v) On p. 22, replace in line 37: $\lim_{n \rightarrow \infty} M(fgx_n, fgx_n, t) = 1$ by

$$\lim_{n \rightarrow \infty} M(fgx_n, fgx_n, t) = 1.$$

(vi) On p. 25, in Example 4.3, we have $x_n = 5 + (1/n)$ for $n \geq 1$, then $Tx_n \rightarrow 2$ but $Bx_n \rightarrow 5 \neq 2$ as $n \rightarrow \infty$. To rectify Example 4.3, we redefine $B : X \rightarrow X$ by (1).

(vii) On p. 25, in Remark 4.1, replace $g(X) = [2, 7] \cup \{9\}$ and $\lim_{n \rightarrow \infty} fgx_n = 6$ by $g(X) = [2, 7] \cup \{12\}$ and $\lim_{n \rightarrow \infty} fgx_n = 4$, respectively.

(viii) On p. 26, in Proof of Theorem 4.1, replace in lines 27 and 28:

$$M(gz, gx_n, qt) \leq M(gz, gx_n, t)$$

by

$$M(gz, gx_n, qt) \leq M(fz, fx_n, t).$$

References

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