AN ADDENDUM TO OUR PAPER “FURTHER REMARKS ON δ - AND θ -MODIFICATIONS”

E. MAKAI, JR.*

Alfréd Rényi Mathematical Institute, H-1053 Budapest, Reáltanoda u. 13–15, Hungary
<http://www.renyi.hu/~makai>
e-mail: makai@renyi.hu

(Received August 12, 2009; accepted September 15, 2009)

Abstract. We show functoriality of γ^α in our joint paper with Á. Császár “Further remarks on δ - and θ -modifications”.

Let X, Y and f be like at Proposition 2.11 of [1], let $A \subset X$, let α be an ordinal and let γ^α be like before Theorem 4.5 of [1].

PROPOSITION. *If $f : X \rightarrow Y$ is (μ, ν) -continuous and (μ', ν') -continuous, then $f(\gamma^\alpha A) \subset \gamma^\alpha(f(A))$.*

PROOF. We begin with the case $\alpha = 1$. Let $x \in X$, $y := f(x)$ and $B := f(A)$. Let $y \notin \gamma(B)$, i.e., there exists $y \in N \in \nu$, such that $c_{\nu'}(N) \cap B = \emptyset$. Then $x \in f^{-1}(N) \in \mu$, and $f^{-1}(c_{\nu'}(N)) \cap f^{-1}(B) = \emptyset$. Here $f^{-1}(c_{\nu'}(N)) \supset f^{-1}(N)$ is μ' -closed, hence $c_{\mu'}f^{-1}(N) \subset f^{-1}(c_{\nu'}(N))$, and $A \subset f^{-1}(B)$. Hence $c_{\mu'}(f^{-1}(N)) \cap A = \emptyset$, showing $x \notin \gamma A$. This proves $f(\gamma A) \subset \gamma f(A)$.

Now we use transfinite induction, with induction base $\alpha = 0$, for which the statement is evident. For $\alpha = \beta + 1$ a successor ordinal we have $f(\gamma^\beta A) \subset \gamma^\beta(f(A))$, hence $f(\gamma^\alpha A) = f(\gamma\gamma^\beta A) \subset \gamma f(\gamma^\beta A) \subset \gamma\gamma^\beta f(A) = \gamma^\alpha f(A)$. For α a limit ordinal the induction step is evident. $\square$

References

- [1] Á. Császár and E. Makai, Jr., Further remarks on δ - and θ -modifications, *Acta Math. Hungar.*, **123** (2009), 223–228.

*Research (partially) supported by Hungarian National Foundation for Scientific Research, grant nos. K68398, K75016.

Key words and phrases: $\theta(\mu, \mu')$, (μ, ν) -continuous mapping, $\gamma^\alpha(\mu, \nu)$.

2000 Mathematics Subject Classification: 54A05.