Addenda and Corrigenda: Picard Group of Rings of Coinvariants

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Published online: 20 January 2009
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Erratum to: *Algebr Represent Theor* (2008) 11: 25–42 DOI 10.1007/s10468-007-9073-2

We use the same notation and terminology from the paper quoted above. Let k be a field, A a k -algebra, C a coalgebra and H a Hopf algebra with bijective antipode S_H such that A is a right H -comodule algebra with the comodule structure given by $\rho_{A,H}(a) = a_{[0]} \otimes a_{[1]}$ and C is a right H -module coalgebra with the right action of H on C denoted by " $\leftarrow$ ". The triple (H, A, C) is called a right-right Doi Hopf datum and $\mathcal{M}(H)_A^C$ is the category of right-right Doi-Hopf modules. If C is a bialgebra then A is a right C -comodule with the comodule structure defined by $\rho_{A,C}(a) = a_{(0)} \otimes a_{(1)} = a_{[0]} \otimes (1_C \leftarrow a_{[1]})$. From now on we assume that C is a Hopf algebra with bijective antipode S_C . In the paper we considered four compatibility conditions (1), (2), (3) and (4). When C is commutative, the conditions (2), (3) and (4) are equivalent. The main results are obtained for C commutative and under the condition (2) or the conditions (1) + (2). Let us recall the conditions (1) and (3):

$$c' S_C(c \leftarrow h) = (c' \leftarrow S_H(h)) S_C(c); \quad \forall h \in H, c, c' \in C \quad (1)$$

$$(cc') \leftarrow h = c(c' \leftarrow h); \quad \forall h \in H, c, c' \in C \quad (3)$$

Note that for the definition of the condition (3), we can take C to be just a bialgebra. Note also that the condition (3) implies that

$$c \leftarrow h = c(1_C \leftarrow h); \quad \forall h \in H, c \in C \quad (3')$$

The online version of the original article can be found at
<http://dx.doi.org/10.1007/s10468-007-9073-2>.

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Lemma 1.1 Assume that the condition (3') is satisfied. Then

- (1) A is a right C -comodule algebra
- (2) the category $\mathcal{M}(H)_A^C$ coincides with the category $\mathcal{M}_A^C$ of right-right relative (A, C) -Hopf modules.

Proof

- (1) See pages 29–30.
- (2) Let $M \in \mathcal{M}(H)_A^C$. For every $m \in M$ and $a \in A$, we have

$$\begin{aligned}\rho_{M,C}(ma) &= m_{(0)}a_{[0]} \otimes (m_{(1)} \leftarrow a_{[1]}) \\ &= m_{(0)}a_{[0]} \otimes m_{(1)}(1_C \leftarrow a_{[1]}) \\ &= m_{(0)}a_{(0)} \otimes m_{(1)}a_{(1)}\end{aligned}$$

and so $M \in \mathcal{M}_A^C$. Conversely, let $M \in \mathcal{M}_A^C$. For every $m \in M$ and $a \in A$, we have

$$\begin{aligned}\rho_{M,C}(ma) &= m_{(0)}a_{(0)} \otimes m_{(1)}a_{(1)} \\ &= m_{(0)}a_{[0]} \otimes m_{(1)}(1_C \leftarrow a_{[1]}) \\ &= m_{(0)}a_{[0]} \otimes (m_{(1)} \leftarrow a_{[1]})\end{aligned}$$

So M is an object of $\mathcal{M}(H)_A^C$. □

Lemma 1.2 The condition (3') implies the condition (1).

Proof Assume Eq. 3'. For any $h \in H$, we have

$$\begin{aligned}1_C \epsilon_H(h) &= 1_C(\epsilon_C(1_C \leftarrow h)) \\ &= S_C((1_C \leftarrow h)_1)(1_C \leftarrow h)_2 = S_C(1_C \leftarrow h_1)(1_C \leftarrow h_2).\end{aligned}$$

Therefore

$$\begin{aligned}S_C(1_C \leftarrow h) &= S_C(1_C \leftarrow h_1)\epsilon_H(h_2) \\ &= S_C(1_C \leftarrow h_1) \leftarrow (h_2 S_H(h_3)) \\ &= (S_C(1_C \leftarrow h_1) \leftarrow h_2) \leftarrow S_H(h_3) \\ &= ((S_C(1_C \leftarrow h_1)(1_C \leftarrow h_2)) \leftarrow S_H(h_3)) \\ &= ((S_C(1_C \leftarrow h_{11})(1_C \leftarrow h_{12})) \leftarrow S_H(h_2)) \\ &= (1_C \epsilon_H(h_1)) \leftarrow S_H(h_2) = 1_C \leftarrow S_H(h).\end{aligned}$$

Thus for any $h \in H$ and $c, c' \in C$, we have

$$\begin{aligned}c' S_C(c \leftarrow h) &= c' S_C(c(1_C \leftarrow h)) \\ &= c' S_C(1_C \leftarrow h) S_C(c) = c'(1_C \leftarrow S_H(h)) S_C(c) \\ &= (c' \leftarrow S_H(h)) S_C(c)\end{aligned}$$

which is the condition (1). □

Note that in Lemma 1.1, we can assume that C is just a bialgebra.

What is behind this note is the following: while many results of the paper are established for the category of Doi-Hopf modules, the main results concern the category of relative Hopf modules.

Finally the following misprints have occurred in the paper:

Page 28: The comodule structure defined on A requires C to be a bialgebra.

Page 30: “Definition 2.2” should be “Definition 2.2 (see [9])”; that is “Definition 2.2 (see [9])” which appears in the statements of Definition 2.2 is not correct.

Page 33: In Lemma 3.2, “ $\mathcal{M}_A$ ” should be “ $\mathcal{M}_B$ ”.