On Fractional Power Moments of L -functions Associated with Certain Cusp Forms

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Published online: 24 January 2009
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Erratum to: Acta Appl Math
DOI [10.1007/s10440-008-9338-8](https://doi.org/10.1007/s10440-008-9338-8)

In Lemma 2.10 of [1], we used Lemma 2.4 and Lemma 2.7 to prove the upper bound of $I_k(T, \sigma)$ under the analogue of the Riemann hypothesis for $\phi(s, f)$. But in the proof of Lemma 2.4 there was a mistake, so Lemma 2.4 was incorrect as stated. Now we give corrections to Lemma 2.4 as follows.

Lemma 2.4' Let $\kappa/2 \leq \sigma \leq (2\kappa + 1)/4$ and $T \geq 2$. Then

$$J(\kappa/2, T, f) \ll T^{k(2\sigma - \kappa)} J(\sigma, T, f) + e^{-ckT^2}.$$

Proof See Lemma 4 of [2]. □

We give some corresponding changes to Lemma 2.7 of [1]. Put $\tilde{g}(s, f) = \phi(s, f)^k - S_N(s)$, and

$$\tilde{K}(\sigma, T, f) = \int_{-\infty}^{+\infty} |\tilde{g}(\sigma + it, f)|^2 w(t, T) dt.$$

If the analogue of the Riemann hypothesis holds for $\phi(s, f)$, the function $\phi(s, f)^k$ is defined in the half-plane $\sigma > \kappa/2$, cut along the line $\sigma = (\kappa + 1)/2$, $t \leq 0$.

This work is supported by the National Natural Science Foundation of China (Grant No. 10701048).

The online version of the original article can be found under doi:[10.1007/s10440-008-9338-8](https://doi.org/10.1007/s10440-008-9338-8).

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Lemma 2.7' Assume the analogue of the Riemann hypothesis for $\phi(s, f)$. Let $\kappa/2 \leq \sigma \leq (2\kappa + 1)/4$ and $T \geq 2$. Then

$$\begin{aligned} \tilde{K}(\sigma, T, f) &\ll \tilde{K}(\kappa/2, T, f)^{(2\kappa+3-4\sigma)/3} (TN^{-3/2})^{(4\sigma-2\kappa)/3} \\ &\quad + \tilde{K}(\kappa/2, T, f)^{(2\kappa+3-4\sigma)/3} N^{(4\sigma-2\kappa)/3} e^{-c_5 T^2 (4\sigma-2\kappa)}. \end{aligned}$$

Proof We prove the lemma as we did in Lemma 2.6 of [1], just substitute k and 1 for p and q in Lemma 2.6 of [1] respectively. $\square$

Now using Lemma 2.4' and Lemma 2.7' we get a general result of Lemma 2.10 of [1].

Lemma 2.10' Assume the analogue of the Riemann hypothesis for $\phi(s, f)$. Let $0 < k \leq \frac{1}{2}$ be a positive real number. Then

$$J(\kappa/2, T, f) \ll_k T (\log T)^{k^2}.$$

Note that in Lemma 2.10' we enlarge the range of k from $k = \frac{p}{q} \leq \frac{1}{2}$ to $0 < k \leq \frac{1}{2}$. Hence we improve the theorem of [1] as follows.

Theorem 1.1' Let $k = \frac{p}{q} \leq \frac{1}{2}$, where p, q are positive integers. Then, as $T \rightarrow \infty$,

$$I_k(T, \kappa/2) \gg_k T (\log T)^{k^2}.$$

Assume that the analogue of the Riemann hypothesis for $\phi(s, f)$ is true and $0 < k \leq \frac{1}{2}$ is a positive real number. Then

$$I_k(T, \kappa/2) \ll_k T (\log T)^{k^2}.$$

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