On the Locus of Curves with Automorphisms

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As pointed out to me by Giancarlo A. Urzúa, to whom I am very grateful, the statement of Theorem 1 on page 141 of the paper in question contains an error. To correct it, replace point iii) on lines 15–16 with

- (iii) $g' = 0$, $n = 4$, $a_4 = p - 1$ (and hence $a_2 + a_3 = p$); σ acts on $\{q_1, q_2, q_3, q_4\}$ as the product of two disjoint transpositions.

The mistake in the original proof occurs on line 30 of page 142, where it is said that “The other cases are similar”. Similar indeed they are, but the conclusions are not. Here is a correct argument. Suppose σ interchanges q_i and q_j . Then Lemma 1 says that there is an integer b , with $1 \leq b < p$, such that $a_j \equiv ba_i$ and $a_i \equiv ba_j$ modulo p . Thus $b^2 \equiv 1$ modulo p , and hence b equals 1 or $p - 1$. As $1 = a_1 \leq a_2 \leq a_3 \leq a_4 < p$, and the a_i add up to a multiple of p , if σ interchanges q_1 with q_2 the only possibility is that $a_1 = a_2 = 1$ and $a_3 = a_4 = p - 1$, as correctly stated in the original proof. The same happens if σ interchanges q_1 with q_3 . If instead σ interchanges q_1 with q_4 , all one can conclude is that $a_4 = p - 1$ and $a_2 + a_3 = p$; the second equality is anyway implied by the first and by the fact that the sum of the a_i is divisible by p .

There remains to show that the latter case does indeed occur. This is proved exactly as on lines 27–31 of page 141 of the paper. Namely, suppose that $1 = a_1 \leq a_2 \leq a_3 \leq a_4 = p - 1$ and that $a_2 + a_3 = p$. Normalize things so that $q_1 = 0$, $q_2 = 1$, $q_3 = \zeta$, and $q_4 = \infty$, where ζ is a complex number different from 0 and 1. If we let σ denote the linear fractional transformation $z \mapsto \zeta/z$, then σ acts on $\{q_1, \dots, q_4\}$ as required. Moreover, if we set

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$b = p - 1$, then $ba_1 \equiv a_4$, $ba_4 \equiv a_1$, $ba_2 \equiv a_3$, $ba_3 \equiv a_2$ modulo p ; hence σ lifts to an automorphism of C by Lemma 1.

The error in Theorem 1 causes an error to occur also in Corollary 1 of the paper. To correct this, one merely has to replace line 4 on page 147 with

$$(iii) \quad g' = 0, n = 4, a_4 = p - 1.$$

The proof is unchanged.

The tables appended to the paper, which were calculated using Corollary 1, are also partially incorrect. To correct the first table all one has to do is delete all subvarieties $S(p, 0; 1, a_2, a_3, p - 1)$. To correct the second table one needs to modify the number of 1-dimensional components of the singular locus of M_g to $g(g + 2)/24$ if $g + 1$ is a prime, and correct the total number of components accordingly.