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ERRATUM

Erratum to: Higher-Order Averaging, Formal Series and Numerical Integration II: The Quasi-Periodic Case

P. Chartier[1] · A. Murua[2] · J. M. Sanz-Serna[3]

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After formula (22), the tree u must have the label $\mathbf{l}$ at the root and the label $\mathbf{k}$ at the leaf. Furthermore, the condition $\mathbf{k} \neq \mathbf{0}$ must be imposed.

The formula at the second bullet point after (56) must be changed to $f_{\mathbf{k}_1 \cdots \mathbf{k}_r}(y) := \partial_y f_{\mathbf{k}_2 \cdots \mathbf{k}_r}(y) f_{\mathbf{k}_1}(y)$.

In the unnumbered displayed formula before (61), the left-hand side must be $H(y, \theta)$ rather than $\epsilon H(y, \theta)$. In formula (61), the lefft-hand side must be $\epsilon \bar{H}$ rather than $\bar{H}$.

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✉ J. M. Sanz-Serna
jmsanzserna@gmail.com

P. Chartier
Philippe.Chartier@inria.fr

A. Murua
Ander.Murua@ehu.es

1 INRIA Rennes and ENS Cachan Bretagne, Campus Ker-Lann, av. Robert Schumann, 35170 Bruz, France

2 Konputazio Zientziak eta A. A. Saila, Informatika Fakultatea, UPV/EHU, 20018 Donostia–San Sebastián, Spain

3 Departamento de Matemáticas, Universidad Carlos III de Madrid, E-28911 Leganés (Madrid), Spain

The unnumbered displayed formula before (64) may be simplified to read:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}Y = {} & \varepsilon \sum_{\mathbf{k}} \bar{\beta}_{\mathbf{k}} f_{\mathbf{k}} + \varepsilon^2 \sum_{\mathbf{k}>\mathbf{l}} \bar{\beta}_{\mathbf{lk}} [f_{\mathbf{l}}, f_{\mathbf{k}}] \\ & + \varepsilon^3 \left(\sum_{\mathbf{k}\neq\mathbf{l}} \bar{\beta}_{\mathbf{llk}} [f_{\mathbf{l}}, [f_{\mathbf{l}}, f_{\mathbf{k}}]] + \sum_{\mathbf{l}>\mathbf{k}<\mathbf{m},\ \mathbf{m}\neq\mathbf{l}} \bar{\beta}_{\mathbf{mlk}} [f_{\mathbf{m}}, [f_{\mathbf{l}}, f_{\mathbf{k}}]] \right) \\ & + \mathcal{O}(\varepsilon^4),\end{aligned}$$

where $<$ is some total ordering in the set of multi-indices $\mathbb{Z}^d$, such that $\mathbf{k} > \mathbf{0}$ for $\mathbf{k} \neq \mathbf{0}$. Formula (64) is wrong and must be replaced by

$$\frac{\mathrm{d}}{\mathrm{d}t}Y = \varepsilon f_{\mathbf{0}} + \varepsilon^2 F_2 + \varepsilon^3 F_3 + \mathcal{O}(\varepsilon^4),$$

where, with $<$ as before,

$$\begin{aligned}F_2 = {} & \sum_{\mathbf{k}>-\mathbf{k}} \frac{i}{\mathbf{k}\cdot\omega} ([f_{\mathbf{k}} - f_{-\mathbf{k}}, f_{\mathbf{0}}] + [f_{-\mathbf{k}}, f_{\mathbf{k}}]), \\ F_3 = {} & \sum_{\mathbf{k}\neq\mathbf{0}} \frac{1}{(\mathbf{k}\cdot\omega)^2} \Big([f_{\mathbf{0}}, [f_{\mathbf{0}}, f_{\mathbf{k}}]] + [f_{\mathbf{k}}, [f_{\mathbf{k}}, f_{-\mathbf{k}}]] \\ & - \frac{1}{2}[f_{\mathbf{k}}, [f_{\mathbf{k}}, f_{-2\mathbf{k}}]] + [f_{-\mathbf{k}}, [f_{\mathbf{k}}, f_{\mathbf{0}}]] \Big) \\ & + \sum_{\mathbf{0}\neq\mathbf{m}\neq-\mathbf{l}\neq\mathbf{0}} \frac{-1}{(\mathbf{l}\cdot\omega)((\mathbf{m}+\mathbf{l})\cdot\omega)} [f_{\mathbf{m}}, [f_{\mathbf{l}}, f_{\mathbf{0}}]] \\ & + \sum_{-\mathbf{l}>\mathbf{k}<\mathbf{l},\ \mathbf{k}\neq\mathbf{0}} \frac{1}{(\mathbf{k}\cdot\omega)(\mathbf{l}\cdot\omega)} [f_{-\mathbf{l}}, [f_{\mathbf{l}}, f_{\mathbf{k}}]] \\ & + \sum_{\substack{\mathbf{m}>\mathbf{k}<-\mathbf{k} \\ \mathbf{m}+\mathbf{k}\neq\mathbf{0}}} \frac{-1}{(\mathbf{k}\cdot\omega)(\mathbf{m}\cdot\omega)} [f_{\mathbf{m}}, [f_{-\mathbf{k}}, f_{\mathbf{k}}]] \\ & + \sum_{\substack{\mathbf{0}\neq\mathbf{m}\neq\pm\mathbf{l}\neq\mathbf{0} \\ \mathbf{m}>-\mathbf{m}-\mathbf{l}<\mathbf{l}}} \frac{-1}{(\mathbf{m}\cdot\omega)((\mathbf{m}+\mathbf{l})\cdot\omega)} [f_{\mathbf{m}}, [f_{\mathbf{l}}, f_{-\mathbf{m}-\mathbf{l}}]].\end{aligned}$$

In Theorem 5.7, the minus sign in the formula for $\beta_{\mathbf{k}}^{[j]}$ must be deleted.

For the results in Sect. 5.3 to hold, it is necessary to assume that the Hamiltonian functions $I_j(x)$, $j = 1, \ldots, d$, are in involution.

In formula (78), the minus sign after $=$ must be deleted.