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PREFACE

Preface

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The present special issue of Mathematical Programming, Series B, is published in close correspondence with the 11th Workshop on Well-Posedness of Optimization Problems and Related Topics, which was held in Alicante University (Alicante, Spain), the days 10–14 of September of 2007.

The 1st Workshop of this series was held in Milan, in 1987, and it consisted of a small meeting of some Bulgarian and Italian groups working on the subject. The subsequent meetings (each 2 years) took place in Sofia (1989), Santa Margherita Ligure (1991), Sozopol (1993), Marseille (1995), Sozopol (1997), Gargniano (1999), Warsaw (2001), Marseille (2003), and Borovets (2005). They quickly and substantially enlarged the audience, including participants from almost all the countries in Europe and North America, as well as the scope of the Workshop. The topics covered along the different editions of the Workshop ranged in a broad class of relevant mathematical problems, including the various approaches to well-posedness and stability of optimization models in different settings as calculus of variations, optimal control and mathematical programming (Hadamard, Tykhonov, and other types of well-posedness), topological aspects of well-posedness, game theory and equilibrium, variational principles, well-posedness concepts for vector optimization problems, stability

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in stochastic optimization, applications to the performance analysis of numerical optimization methods and their stable behavior under perturbations, critical point theory, etc.

The principal aim of this special issue has been to gather important recent contributions to this wide research field of well-posedness and stability in optimization, and their connections with different related topics in mathematics of increasing research interest. Next we provide a short description of the fourteen papers included in the issue, from which the reader may appreciate their main features, as well as their high scientific interest.

"Autoconjugate representers for linear monotone operators" (by H.H. Bauschke, X. Wang, and L. Yao)

A convex representer $F : X \times X^* \longrightarrow]-\infty, +\infty]$ of a monotone operator on a Banach space X is said to be autoconjugate if its Fenchel conjugate F^* satisfies $F^*(x^*, x) = F(x, x^*)$ for all $(x^*, x) \in X^* \times X$. In this paper it is proved that several known autoconjugate representers coincide in the case of monotone continuous linear operators. In spite of this result, the authors show that, for such operators, autoconjugate representers are not unique by providing a family of different autoconjugate representers for the identity operator on the real line.

"On totally Fenchel unstable functions in finite dimensional spaces" (by R.I. Boţ and A. Löhne)

The authors give a negative answer to the following question: "Do there exist a nonzero finite dimensional Banach space and a pair of extended real-valued, proper and convex functions, which is totally Fenchel unstable?" This problem was posed by Stephen Simons in his book "From Hahn–Banach to Monotonicity".

"Linear regularity, equirregularity, and intersection mappings for convex semi-infinite inequality systems" (by M.J. Cánovas, F.J. Gómez–Senent, and J. Parra)

The authors extend the classical concept of metric regularity to an arbitrary family of set-valued mappings (multifunctions). Consequently a relationship between the metric regularity moduli of the multifunctions in the family and the modulus of the associated intersection mapping is investigated. The results obtained are then applied to the stability analysis of the solution set of a system defined by infinitely many convex inequalities.

"A geometrical insight on pseudoconvexity and pseudomonotonicity" (by J.-P. Crouzeix, A. Eberhard, and D. Ralph)

The authors deal with the classical notion of pseudoconvexity for differentiable functions and propose an extension to the nondifferentiable case by considering geometrical properties of the lower level sets. For a special class of quasiconvex functions they introduce generators of the normal cones to the level sets, which provide a suitable substitute for the subdifferential. They also study pseudomonotone and quasimonotone operators and some variants of these concepts.

"Robust stochastic dominance and its application to risk-averse optimization" (by D. Dentcheva and A. Ruszczynski)

The authors introduce a preference relation in the space of random variables, called robust stochastic dominance, and they consider stochastic optimization problems for which the risk-aversion is expressed by a robust stochastic dominance constraint. This model gives rise to a composite semi-infinite optimization problem with constraints on compositions of measures of risk and utility functions. Optimality conditions are provided, requiring some additional smoothness assumptions in the non-convex case.

"Subdifferentials of value functions and optimality conditions for DC and bilevel infinite and semi-infinite programs" (by N. Dinh, B. Mordukhovich, and T.T.A. Nghia)

The value/marginal functions and their subdifferential estimates play a crucial role in many aspects of parametric optimization, including well-posedness and sensitivity. This paper is mainly devoted to establish verifiable conditions for the local Lipschitz continuity of the value function, yielding necessary optimality conditions for parametric DC infinite programs. The authors apply their techniques to the study of bilevel finite and infinite programs with convex data on both lower and upper level of the hierarchical optimization. The results given in the paper also admit semi-infinite counterparts.

"Newton's method for generalized equations: a sequential implicit function theorem" (by A. L. Dontchev and R. T. Rockafellar)

The authors present a version of Newton's method for parametric generalized equations of the form $f(p, x) + F(x) \ni 0$, with $f : P \times X \longrightarrow Y$ and $F : X \rightrightarrows Y$ (where P, X, and Y are Banach spaces), in which they linearize f but not F. They obtain an implicit function theorem suitable to the analysis of the dependence of this Newton's iteration on parameters, and also present a related inverse function theorem.

"A note on an approximate Lagrange multiplier rule" (by J. Dutta, S. R. Pattanaik, and M. Thera)

In this paper approximate calculus rules in nonsmooth analysis are discussed and applied to obtain an approximate Lagrange multiplier rule for a very general mathematical programming problem, namely, the minimization, over a closed subset of $\mathbb{R}^n$, of the sum of a locally Lipschitz function with the composition of a vector-valued locally Lipschitz function with a proper lower semicontinuous function.

"Critical angles in polyhedral convex cones: numerical and statistical considerations" (by D. Gourion and A. Seeger)

The paper is focused on the numerical computation of critical angles in polyhedral convex cones. The authors solve a series of generalized eigenvalue problems (involving the generators of the cone) to evaluate the set of proper critical angles. The expected numbers of critical angles in random polyhedral convex cones are estimated experimentally.

"Second order optimality conditions in $C^{1,0}$ multiobjective programming" (by C. Gutiérrez, B. Jiménez, and V. Novo)

The authors study a multiobjective optimization program with a feasible set defined by equality constraints and a generalized inequality constraint. Necessary and sufficient second order optimality conditions are derived without standard constraint qualifications and under minimal assumptions about differentiability of the functions involved. The basic tools employed in the analysis are the calculus of first and second order tangent sets, parabolic second order directional derivatives and Clarke's generalized gradients.

"Mixed semicontinuous perturbations of nonconvex sweeping processes" (by T. Haddad and L. Thibault)

The authors prove a theorem on the existence of a global solution of a differential inclusion governed by a class of nonconvex sweeping processes with unbounded pertubations. One relevant feature of this paper is that the considered perturbation is not required to be convex valued.

"Towards variational analysis in metric spaces: metric regularity and fixed points" (by A.D. Ioffe)

The author provides estimates for the surjection modulus of a "partial composition" of set-valued mappings between metric spaces (covering, as a particular case, a well-known Milyutin's theorem), and a "double fixed point" theorem for a couple of mappings, implying a fairly general version of the set-valued contraction mapping principle as well as a certain different version of the first theorem.

"Exact estimates of regularity modulus for infinite programming" (by Y. Sekiguchi)

An exact estimate on the modulus of metric regularity for linear systems is given in this paper. Then, the estimate is applied to obtain explicit forms of the modulus for linear conical systems and differentiable nonlinear systems on the space of continuous functions.

"Maximal monotonicity criteria for the composition and the sum under minimal interiority conditions" (by M.D. Voisei and C. Zalinescu)

The authors present some new sufficient conditions for the preservation of maximal monotonicity under compositions and sums. Their results are obtained by using the recently developed theory of convex representations of maximal monotone operators on Banach spaces, a theory that they extend to the setting of locally convex spaces. The paper also contains some subtle results on the continuity of the bilinear pairing between a space and its dual with respect to several topologies.