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ORIGINAL ARTICLE

Comparative study on measurement of elastic constants of wood-based panels using modal testing: choice of boundary conditions and calculation methods

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Abstract Modal testing based on the theory of transverse vibration of orthotropic plate has shown great potentials in measuring elastic constants of panel products. Boundary condition (BC) and corresponding calculation method are key in affecting its practical application in terms of setup implementation, frequency identification, accuracy and calculation efforts. To evaluate different BCs for non-destructive testing of wood-based panels, three BCs with corresponding calculation methods were investigated for measuring their elastic constants, namely in-plane elastic moduli (E_x, E_y) and shear modulus (G_{xy}). As a demonstration of the concept, the products used in this study were oriented strand board (OSB) and medium density fiberboard (MDF). The BCs and corresponding calculated methods investigated were, (a) all sides free (FFFF) with one-term Rayleigh frequency equation and finite element modeling, (b) one side simply supported and the other three free (SFFF) with one-term Rayleigh frequency equation, (c) a pair of opposite sides along minor strength direction simply supported and the other pair along major strength direction free (SFSF) with improved three-term Rayleigh frequency equation. Differences between modal and static results for different BCs were analyzed for each case. Results showed that all three modal testing approaches could be applied for evaluation of the elastic constants of wood-based panels with different accuracy levels compared with standard static test methods. Modal testing on full-size panels is recommended for developing design properties of structural panels as it can provide global properties.

Keywords Elastic properties · Wood panels · Non-destructive technique · Modal testing

Introduction

Wood-based panel products are used for both structural and non-structural applications. Engineered wood-based panels such as oriented strand board (OSB) are even more widely used in modern wood constructions, especially in light frame wood constructions. Elastic constants are critical mechanical properties for structural design, which are also the key quality control parameters. Research studies of evaluating the elastic constants of wood-based panels by use of modal testing could be traced back to the 1980s. Different boundary conditions (BCs) with corresponding calculation methods have been adopted for measuring the elastic constants, namely the modulus of elasticity (E) and shear modulus (G), of panel type products such as solid wood panels, particleboard, OSB, plywood and medium density fibreboard (MDF) and cross-laminated timber (CLT).

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Boundary condition with all four sides free (FFFF) has been mostly used among the studies done for modal testing of panel-shaped wood products because it requires the least efforts for implementation. However, there is no analytical solution for FFFF BC. The one-term Rayleigh frequency solution was frequently applied for the calculation of elastic constants due to its simple and straight-forward formula [1–5]. The natural frequency of torsional mode was used for measuring the in-plane shear modulus of wood-based panels by Nakao and Okano [1]. The method appeared to be much simpler than static plate-twist shear tests. Coppens [2] measured the elastic constants (E_x, E_y and G_{xy}) of particleboard by modal testing in the laboratory of individual company for quality control purposes. Sobue and Kitazumi [3] applied the same vibration technique for measuring elastic constants of wood panels (western red cedar, hemlock, buna and keyaki). The results were verified with static test results of beam specimens. Carfagni and Mannucci [4] simplified the method in identifying modal shapes based on assessing whether the response and excitation were in or out of phase. The number of impact points was reduced to six for rectangular wood panels. Bos and Casagrande [5] presented the E_x and E_y values of selected eight OSB panels, 260 plywood panels, one MDF panels tested by an on-line non-destructive evaluation system called VibraPann, which utilized the measurement of the first bending modes in two strength directions. The results showed an absolute difference within 15% of E_x and E_y values for plywood compared with static test values. The spatial variability of elastic properties within a panel was also reported by testing of small panels cutting from a full-size panel.

Besides Rayleigh frequency solution, finite element modeling (FEM) was often used for the determination of elastic constants combined with modal testing [6–8]. The elastic constants were estimated by minimizing the difference between the experimental frequencies and FE modeled values using an iterative process. Full-size MDF and OSB panels, modeled as thin orthotropic plates under FFFF BC, were tested by Larsson [6, 7] using modal testing. The proposed method was proved to be accurate because of the good agreement between measured and calculated natural frequencies (up to the 7th mode) within 1–5%, though the average differences between dynamic and static bending data of E_x and E_y were 14.1 and 31.0%, respectively. A similar method was adopted to study the effects of moisture content on the in-plane elastic constants of wooden boards used in musical instruments [8]. It was found that, with the moisture content ranging from 0 to 25%, the E values in radial and longitudinal directions and G of longitudinal and radial plane changed approximately 88, 51 and 47%, respectively. Gsell et al. [9] measured the natural frequencies and mode shapes of a rectangular CLT specimen. An analytical model based on Reddy's higher order plate theory [10] was applied to calculate natural frequencies and mode shapes numerically. All three G and the two in-plane E values were identified by minimizing the difference between measured and estimated natural frequencies based on the least-squares method. Gülzow [11] further studied the modal testing method to evaluate the elastic properties of CLT panels with different layups and characteristics.

FFFF, however, is not the best BC for large size panels, especially in the production environment. Other BCs such as one side simply supported and the other three sides free (SFFF) and one side clamped and the other three sides free (CFFF) were also used for the determination of elastic constants of full-size structural panels for the purpose of quality control in production. A simultaneous determination of orthotropic elastic constants of standard full-size plywood by vibration method was conducted with SFFF BC [12]. The results showed an agreement to within 10% of E and G values measured using static bending and plate torsional tests, respectively. Particleboard and MDF panels of full-size dimensions were tested using a vibration technique in both vertical and horizontal cantilever (CFFF) arrangements [13]. It was found that there was no significant difference between measured frequencies from the vertical and horizontal position, which indicated that the deflection caused by self-weight under horizontal position had no effect on measured frequencies. The absolute values of the dynamic E values were about 20–25% higher than the static values, while MDF had a better correlation and smaller difference between dynamic and static results than particleboard.

Currently, boundary condition of a pair of opposite sides along minor strength direction simply supported and the other pair free (SFSF) was adopted with improved approximate natural frequency expressions for measuring elastic constants for full-size wood-based panels including CLT, OSB and MDF panels [14]. The difference between dynamic and static test results was about 10% or less except for E_y of OSB. The reason was thought to be the inappropriate strip specimen size for static bending test, which could not well represent the E_y of full-size OSB panels. The method with SFSF BC has great potential for further implementation in on-line evaluation of full-size wood-based panels.

The study described in this paper was conducted to compare three methods of measuring elastic constants of wood-based panel products with different BCs (FFFF, SFFF and SFSF) with corresponding calculation procedures. Standard static tests were performed to provide reference values for comparison. The objective of the study was to develop a better understanding on how the accuracy of measured elastic constants are affected by the BC chosen for modal testing and data analysis procedure. The ultimate goal is to contribute to the development of standard modal testing method for measuring the elastic constants of wood-based panels as well as potential development of on-line quality control techniques.

Theoretical background

In the application of the three methods, the following assumptions were made:

(a) the material has a uniform mass and in-plane elastic property distribution;
(b) the effects of transverse shear deformation and rotatory inertia are negligible.

Forward problem

The governing differential equation for the transverse vibration of a thin rectangular orthotropic plate based on the above assumptions is expressed as follows [15],

$$D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 y^2} + \rho h \frac{\partial^4 w}{\partial t^2} = 0, \quad (1)$$

where $D_x = \frac{E_x h^3}{12(1-v_{xy}v_{yx})}$, $D_y = \frac{E_y h^3}{12(1-v_{xy}v_{yx})}$, $D_1 = D_x v_{xy} = D_y v_{yx}$, $D_{xy} = \frac{G_{xy}h^3}{12}$, E_x = modulus of elasticity in length (x)/major strength direction, E_y = modulus of elasticity in width (y)/minor strength direction, G_{xy} = in-plane shear modulus, v_{xy} and v_{yx} = Poisson's ratios, $(1 - v_{xy}v_{yx}) \approx 0.99$ for most wood materials [16], a = length of the plate, b = width of the plate, h = thickness of the plate, and ρ = mass density.

For the cases considered in this study, the aspect ratio ($r = a/b$) of the test specimens were greater than 1.

With the input of four elastic constants, dimensional information and density, all the natural frequencies and corresponding mode shapes can be calculated under different BCs as a forward problem. However, due to the complexity of boundary condition, the analytical solution of the forward problem cannot be simply generated from the governing differential equation. Therefore, numerical methods such as Rayleigh method and FEM have been applied for solving the forward problem. In this study, the forward problem solutions for FFFF and SFFF BCs were both generated by Rayleigh method with one-term deformation expression [3, 5]. These frequency equations are explicit and in closed form, which need less computation efforts compared with the analytical method and FEM. The frequency equation can be expressed as,

$$f_{(m,n)} = \frac{1}{2\pi} \sqrt{\frac{1}{\rho h}} \sqrt{D_x \frac{\alpha_{1(m,n)}}{a^4} + D_y \frac{\alpha_{2(m,n)}}{b^4} + 2D_1 \frac{\alpha_{3(m,n)}}{a^2 b^2} + 4D_{xy} \frac{\alpha_{4(m,n)}}{a^2 b^2}}. \quad (2)$$

For SFSF BC, a closed-form approximate frequency equation by Rayleigh method with three-term deformation expression was adopted from Ref. [17],

$$f_{(m,n)} = \frac{ab}{\pi^2} \sqrt{\frac{\rho h}{H}} \sqrt{\frac{C_{ij} + c^2 C_{\dot{i}\dot{n}} + d^2 C_{\dot{m}j} - 2cE_{ij} - 2dE_{ji} + 2cdF}{1 + c^2 + d^2}}, \quad (3)$$

where $f_{(m,n)}$ = natural frequency of mode (m, n), m and n = mode indices, the number of node lines including the simply supported sides in y and x directions, respectively, and $H = D_1 + 2D_{xy}$.

In Eq. (2), $\alpha_{1(m,n)}$, $\alpha_{2(m,n)}$, $\alpha_{3(m,n)}$ and $\alpha_{4(m,n)}$ are the coefficients for mode (m, n), which can be pre-determined for different boundary conditions [3, 5]. In Eq. (3), (m, n) is equivalent to (i, j) as in Ref. [17], $\dot{m}$ and $\dot{n}$ are equal to m and n in the reference, respectively. The expressions of the other terms including $C_{ij}, C_{\dot{i}\dot{n}}, C_{\dot{m}j}, E_{ij}, E_{ji}, c, d, F$ can be found in the same reference as well.

Inverse problem

Theoretically, with a proper forward solution, density, dimensions and any four measured frequencies, the four elastic constants (E_x, E_y, G_{xy} and v_{xy}) can be calculated through an inverse process, known as an inverse problem. However, the sensitivity of each natural frequency to elastic constants is different. Only the sensitive frequencies result in accurate determination of the appropriate elastic constants. Sensitivity analysis is always required in order to identify the most sensitive natural frequencies for calculation of each elastic constant [18].

To exclude the difference among different Rayleigh frequency solutions for different BCs, FEM was employed for sensitivity analysis by changing with ±10% of the mean of each elastic constant. FEM was performed in ABAQUS finite element software ver. 6.12-3 (ABAQUS, MA, USA) with initial elastic constants and geometry information listed in Table 1 [14, 19, 20]. OSB and MDF panels were modeled as a 3D deformable shell using shell element S4R (ABAQUS, MA, USA) with a global mesh size of 0.02. For FFFF BC, no constrains were added to the plate, and for SFFF and SFSF BCs, the simply supported edge or edges were constrained in three translational directions. The natural frequencies of up to 20 modes were computed with embedded 'Lanczos eigensolver'. The ratio

Table 1 Material properties for sensitivity analysis by finite element modeling

Material	E_x (MPa)	E_y (MPa)	v_{xy}	G_{xy} (MPa)	G_{xz} (MPa)	G_{yz} (MPa)	Density (kg/m^3)	Dimension ($a \times b \times h$ mm)
OSB	6400	2700	0.23	2500	770	750	614	1210 × 610 × 11.1
MDF	3100	3300	0.33	1500	120	120	697	1220 × 620 × 15.7

OSB and MDF are short for oriented strand board, and medium density fiberboard, respectively

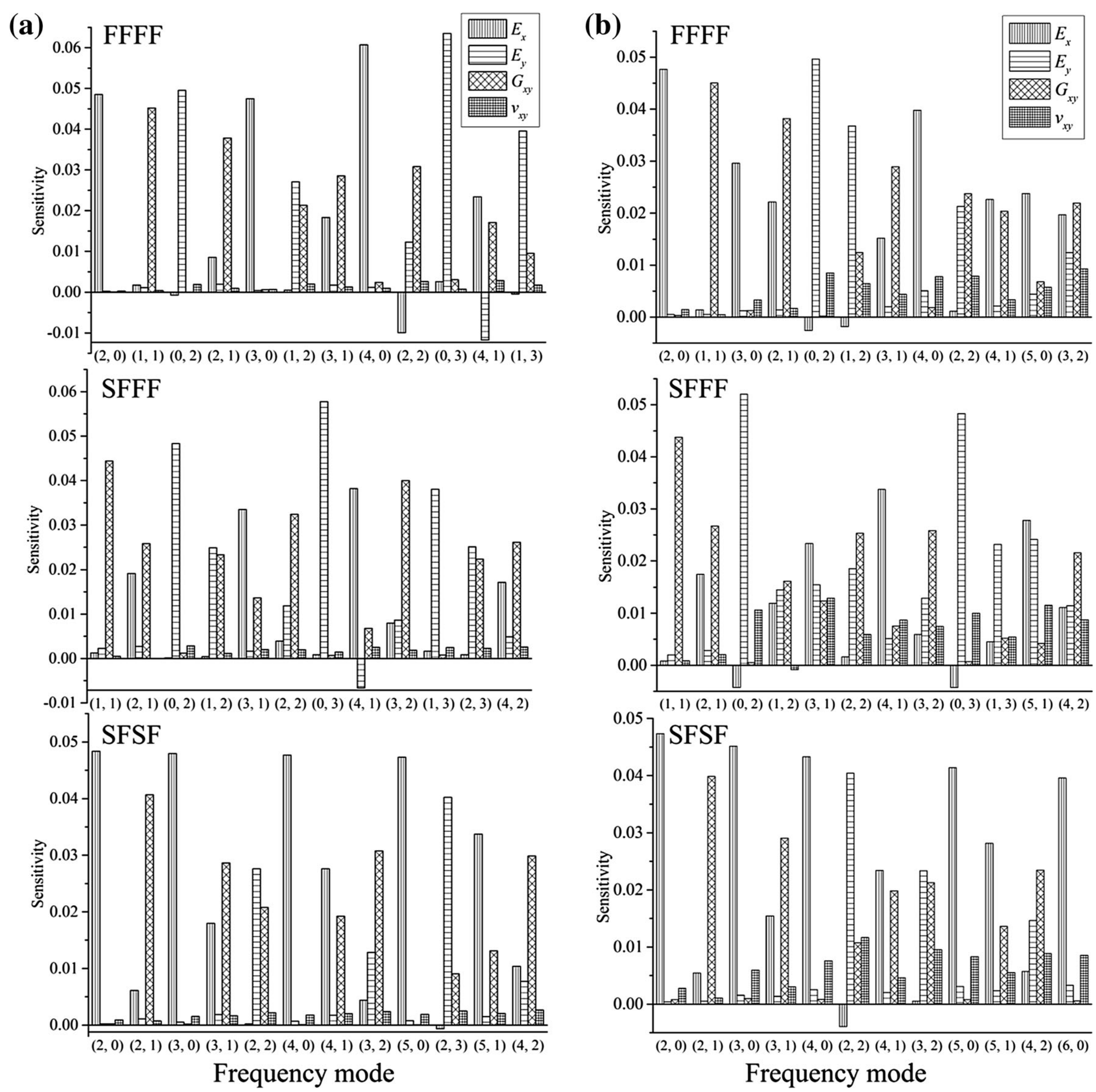

Fig. 1 Sensitivies of each frequency mode to elastic constants under different BCs for **a** a OSB panel and **b** a MDF panel

of frequency difference of each mode to corresponding frequency obtained from initial elastic constants is defined as the sensitivity of each mode to the elastic constants.

FEM sensitivity analysis results of a OSB and a MDF panel are shown in Fig. 1. It can be seen that for all three BCs, Poisson's ratio is almost not sensitive to any frequency modes, therefore Poisson's ratio cannot be properly determined. As reported in previous research [2], Poisson's ratio might be determined unless the plate has a certain aspect ratio of $\sqrt[4]{E_x/E_y}$. The sensitive modes for E_x, E_y and G_{xy} are (2, 0), (0, 2) and (1, 1) with FFFF BC, respectively, and are ($m \geq 2$, 1), (0, 2) and (1, 1) with SFFF BC, respectively. The sensitivity of mode ($m \geq 2$, 1) to E_x increases with the increase of m. A desirable sensitivity can be found with m equal to 3 or 4 depending on the aspect ratio (a/b) and elastic constant ratio (E_x/E_y) of the panel. Natural frequency of mode (3, 1) was used in this study. For SFSF BC, the sensitive frequency modes for E_x, E_y and G_{xy} are (2, 0), (2, $n \geq 2$) and (2, 1), respectively. The sensitivity of mode (2, $n \geq 2$) to E_x increases with the increase of n. In most cases, the frequency of mode (2, 2) or (2, 3) is sensitive enough for calculating E_y. For all three BCs, the sensitive modes for E_x and E_y are those bending modes in x and y axis, and sensitive mode for G_{xy} is the first torsional mode shown in Fig. 2. If there is no constrain at the edge along the minor stiffness axis, the sensitivity of a low bending mode with only one half sine wave is sufficient for E_x or E_y. Otherwise, the sensitivity of a higher bending mode with two or three half sine waves is required. For highly coupled modes with comparable equal m and n, each elastic constant contributes more evenly to the frequency value than modes with $m >> n$ or $m << n$. If such mode is used for calculation, the coupled effect of all elastic constants should be included in the calculation.

From Eq. (2), for FFFF BC, the elastic constants can be calculated using the following formulas [1, 5],

$$E_x = \frac{48\pi^2 \rho a^4 f_{(2,0)}^2 (1 - v_{xy} v_{yx})}{500.6h^2}, \tag{4}$$

$$E_y = \frac{48\pi^2 \rho b^4 f_{(0,2)}^2 (1 - v_{xy} v_{yx})}{500.6h^2}, \tag{5}$$

$$G_{xy} = 0.9\rho \left(\frac{ab}{h} f_{(1,1)} \right)^2. \tag{6}$$

Furthermore, the calculated values can be used as initial input values for FEM updating. First the difference Δf_i between sensitive FEM frequencies ($f_{\mathrm{FEM}i}$) and experimental frequencies ($f_{\exp i}$) will be calculated.

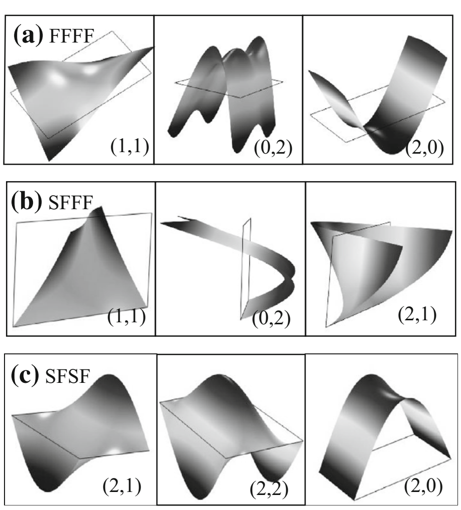

Fig. 2 Illustration of mode shapes of sensitive frequency modes under different BCs

$$\Delta f_i = (f_{\mathrm{FEM}i} - f_{\exp i})/f_{\exp i}, \tag{7}$$

where $i =$ to 1, 2, 3, and corresponds to (2, 0), (0, 2), (1, 1).

If any relative frequency difference $|\Delta f_i|$ is larger than 0.01, then:

$$X = X_0 \times (1 - \Delta f_i)^2, \tag{8}$$

where $X =$ elastic constant (E_x, E_y, G_{xy}) to be updated and $X_0 =$ the corresponding initial value from Eqs. (4), (5) or (6).

The iteration process stops when all $|\Delta f_i|$ are smaller than 0.01 and outputs from the last iteration will be the calculated elastic constants. Experience has shown that less than five iterations are required to achieve convergence.

For SFFF BC, the elastic constants can be calculated using the following formulas [12],

$$E_x = \frac{12\pi^2 \rho a^4 (1 - v_{xy} v_{yx})(4f_{(3,1)}^2 - 36.27 f_{(1,1)}^2)}{3805.04h^2}, \tag{9}$$

or

$$E_x = \frac{12\pi^2 \rho a^4 (1 - v_{xy} v_{yx})(4f_{(2,1)}^2 - 16.49 f_{(1,1)}^2)}{500.6h^2} \tag{10}$$

$$E_y = \frac{48\pi^2 \rho b^4 f_{(0,2)}^2 (1 - v_{xy} v_{yx})}{237.86h^2} \tag{11}$$

$$G_{xy} = \frac{\pi^2 \rho a^2 b^2 f_{(1,1)}^2}{3h^2}, \tag{12}$$

For SFSF BC, a calculation method was developed using the improved frequency equation, Eq. (3), based on an iteration process. The initial value of E_x is first calculated using the fundamental frequency, $f_{(2,0)}$. The other initial values are set as the ratios with E_x based on reported reference value or theoretical prediction. The iteration stops when the total difference between measured and calculated frequencies is less than 1%. Details about the calculation method can be found in Ref. [14].

To summarize, the BCs and corresponding calculation methods to be investigated are listed in Table 2.

Materials and methods

Materials

Five full-size 11.1 mm thick OSB panels of dimensions 2.44 m × 1.22 m and five full-size 15.7 mm thick MDF panels of dimensions 2.46 m × 1.24 m were purchased from a building supplies store. The average moisture contents and densities of OSB and MDF panels were about 4 and 5%, 614 and 697 kg/m^3, respectively. Each full-size panel was cut into four panels of dimensions 1.21 m × 0.60 m. In total, twenty panels were obtained

Table 2 Selected boundary conditions and corresponding calculation method

Boundary condition	Calculation method	Sensitive modes	Note
FFFF	Closed-form frequency equation by Rayleigh method with one-term deformation expression	(2, 0), (1, 1), (0, 2)	Can be used as initial values for finite element modeling
	Finite element modeling updating by ABAQUS with S4R shell element		S4R shell element includes the effect of transverse shear deformation
SFFF	Closed-form frequency equation by Rayleigh method with one-term deformation expression	($m \geq 2$, 1), (1, 1), (0, 2)	$f_{(3,\ 1)}$ is used for calculating E_x
SFSF	Improved frequency equation by Rayleigh method with three-term deformation expression	(2, 0), (2, 1), (2, $n \geq 2$)	MATLAB-based iteration process

FFFF represents the boundary condition of all sides free, SFFF represents the boundary condition of one side simply supported and the other three free and SFSF represents the boundary condition of a pair of opposite sides along minor strength direction simply supported and the other pair along major strength direction free

from each type of panel for modal testing. Then two panels with the closest masses were selected from four panels of the same full-size panel to glue a double-thick panel using a two-component structural polyurethane adhesive. Five panels were prepared from each type of panel, respectively, for investigating the effect of thickness on the accuracy of modal tests. The average thicknesses of double-thick OSB (DOSB) and MDF (DMDF) panels were 22.1 and 31.2 mm, respectively. The remaining ten panels of each type were cut into square panels of dimension 0.60 m × 0.60 m for in-plane shear tests. Then three strips were cut from each strength direction from a square panel for bending tests. For the double-thick panels, they were cut into square panels for in-plane shear tests and panel bending tests as well. The cutting scheme is shown in Fig. 3.

Modal tests

The impact vibration tests were conducted on the specimens with three different BCs for both OSB and MDF panels. Only modal tests with FFFF and SFSF BCs were conducted for DOSB and DMDF panels, because SFFF BC could not be achieved easily in practice as the other two BCs for thick panels. The BCs were realized using ropes and steel pipes in the lab. The panel was suspended with a pair of ropes on a steel frame as shown in Fig. 4a to simulate FFFF BC. A pair of steel pipes were used to clamp one side of the panel to simulate simple support. As shown in Fig. 4b and c, the panels were clamped with proper pressure at one side parallel to major strength direction or a pair of two opposite sides parallel to minor strength direction to achieve SFFF or SFSF BC, respectively. For SFFF BC, one edge along the length direction of the panel was supported, which should not touch the base.

For FFFF and SFFF BCs, the accelerometer was attached at the top left corner of the panel, while for SFSF it was attached at 7/12 length of one free edge. The locations selected were not on the nodal lines of first several modes up to the first 15 modes including the sensitive natural frequencies. The impact and acceleration time domain signals were recorded by a data acquisition device (LDS Dactron, Brüel & Kjær) and the frequency response function (FRF) was calculated from the time domain signals using a data analysis software (RT Pro 6.33, Brüel & Kjær). The frequency spectra were post-processed by MATLAB software ver. 2014a (MathWorks, CA, USA) for frequency identification and calculation of the elastic constants.

Identification of sensitive frequencies

Mathematically, for a given plate, natural frequency increases nonlinearly with the increase of mode indices (m, n). From the sensitivity analysis, it can be seen that for all three BCs the sensitive frequencies have small mode indices with either m or n less than 3 for the material considered in this study. Low mode frequencies are easier to be detected than high mode frequencies. Normally, for 2D and 3D structures, it is necessary to conduct modal test on the whole surface of a structure with a grid to obtain the experimental mode shapes for frequency identification. However, for simple structures such as plates with given BCs and approximate material properties, it is possible to identify the frequency modes with a few impacts at specific locations, based on modal displacements at those locations. Modal displacements are generally estimated from the imaginary part of the FRF as shown in Fig. 5.

For FFFF BC, the frequencies of modes (2, 0) and (1, 1) are the first two in a frequency spectrum because (2, 0) or (1, 1) is the mode indices giving the starting frequency value. Modes (m, n) with either m or n being an odd number have a node at the center of a plate. Therefore, modes (2, 0) and (0, 2) are the first two modes that would appear and mode (1, 1) is the first mode that would vanish

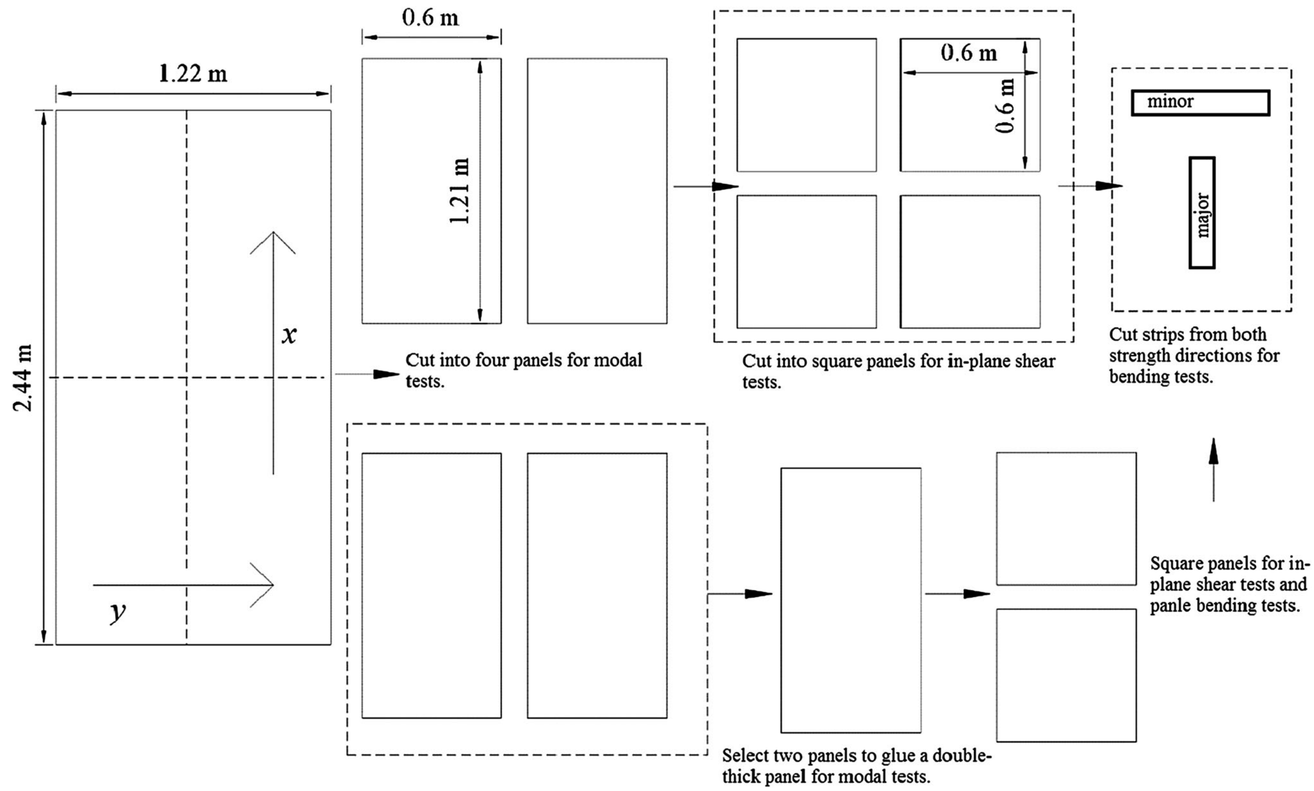

Fig. 3 Cutting scheme for different tests

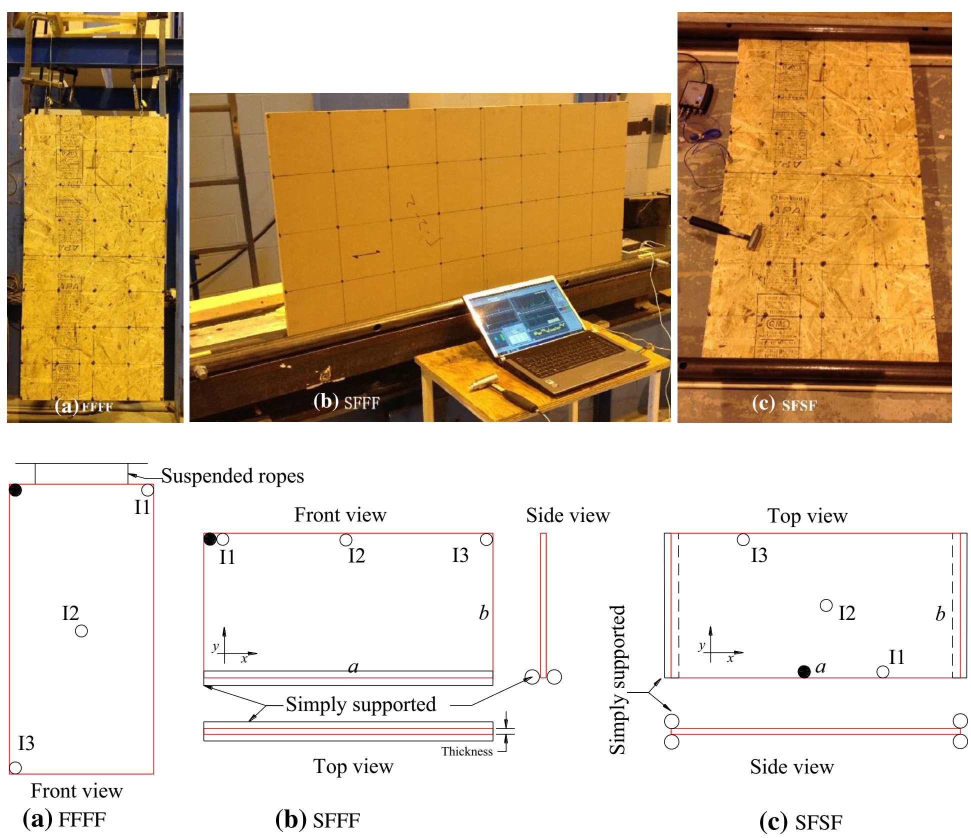

Fig. 4 Test setups for modal tests under different boundary conditions (*solid circle* refers to the location of accelerometer and *blank circle* (I1–3) refers to the impact location)

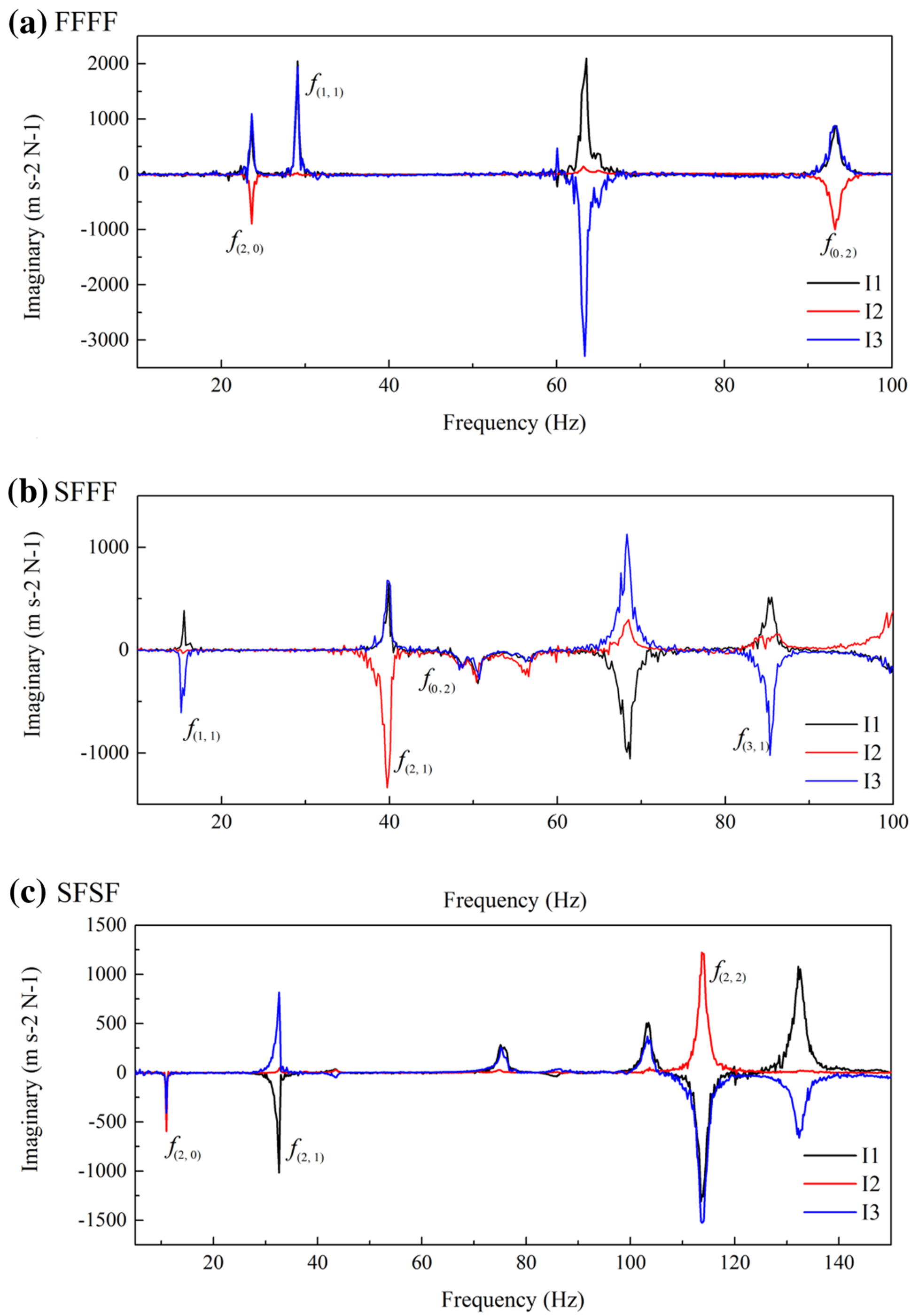

Fig. 5 Selected plots of imaginary part of FRF for sensitive frequency identification at three impact locations under the three BCs

when impacted at the center of the plate. Thus, with the accelerometer located at the left right corner, only three spectra with impacts at the center (I2) and a pair of diagonal corners (I1 and I3) are sufficient for sensitive frequency identification as shown in Fig. 5a. Mathematically, frequency of mode (0, 2) or any (0, *n*) bending mode in *y* direction decreases with any increase of E_x/E_y and decrease of *a*/*b*. Slender plate with similar E_x and E_y (i.e., that approaching an isotropic plate) would result in a very high mode (0, 2), which is difficult to be detected. However, for isotropic material, there is no need to identify mode (0, 2), as modes (2, 0) and (1, 1) are sufficient for calculating *E* and *G*. For nearly isotropic material like MDF, plate of aspect ratio *a*/*b* greater than 3 is not recommended.

For SFFF BC, modes (1, 1) and (1, 2) are the first two modes in a frequency spectrum for the materials considered in this study. Frequency of mode (0, 2) decreases with the

Table 3 Dimensions of specimens for different static tests

Material	Strip bending test (length × width)				In-plane shear test or/and panel bending test (mm^2)
	Major (mm^2)	Span (mm)	Minor (mm^2)	Span (mm)	
OSB	600 × 50	540	450 × 50	270	600 × 600
MDF	600 × 50	570	450 × 50	380	600 × 600
DOSB	600 × 50	540	450 × 50	400	600 × 600
DMDF	600 × 50	570	450 × 50	400	600 × 600

OSB and MDF are short for oriented strand board, and medium density fiberboard, respectively. DOSB and DMDF are short for double-thick oriented strand board, and double-thick medium density fiberboard, respectively

increase of E_x/E_y and the decrease of a/b, which behaves similarly with mode (0, 2) with FFFF BC. With the accelerometer located at the left right corner, frequency spectra from three impacts at the middle (I2) and two ends (I1 and I3) of the top edge are helpful for frequency identification as is shown in Fig. 5b. Modes (*m*, *n*) with *m* being an odd number have out-of-phase modal displacements (i.e., movement is in opposite direction) when impacted at the two ends and vanish when impacted in the middle. Mode (1, 1) is the first of such modes and mode (3, 1) is the second one. While modes (*m*, *n*) with *m* being an even number have in-phase modal displacement (i.e., movement is in the same direction) when impacted at the two ends but out-of-phase modal displacement when impacted in the middle. Mode (2, 1) is the first of such modes. Modes (0, *n*) have in-phase modal displacements when impacted at all three locations (I1, I2 and I3), and mode (0, 2) is the first of such modes.

For SFSF BC, as was discussed in previous research [14, 21], three spectra with impacts at the center (I2) and two locations from the two opposite free edges (I1 and I3) can help identify the sensitive frequency modes needed for calculation. Modes (2, 0) and (2, 1) are the first two modes that appear in the frequency spectra. Mode (2, 2) is the first mode that has out-of-phase modal displacement to mode (2, 0) while mode (2, 1) vanishes when impacted at the center, Fig. 5c. Similar to mode (0, 2) in FFFF BC, frequency of mode (2, 2) decreases with the increase of E_x/E_y and the decrease of a/b. For some wood-based products, E_x/E_y can be close to 1 for MDF, 1–10 for OSB or laminated wood products, and about 20 for solid wood. The effort for identifying mode (2, 2) depends on the material property and specimen aspect ratio.

Static tests

Static tests were conducted as a reference for comparison with dynamic tests. The elastic moduli and shear modulus of OSB and MDF panels were obtained from static center-point flexure tests according to Ref. [22] and shear tests according to Ref. [23], respectively. A total of twelve strips along each strength direction were cut from full-size panel and they were tested for *E* values. A total of four square panels were cut from each panel and tested for G_{xy} values. For the double-thick panels (DOSB and DMDF), two square panels from one DOSB or DMDF specimen are used for both in-plane shear tests and panel bending tests. Then a total of six strips along each strength direction of DOSB and DMDF panels were cut for center-point flexure tests as well. The dimensions of specimens for different static tests are given in Table 3.

Results and discussion

Mean value comparison

The mean elastic constants of OSB and MDF panels measured by dynamic methods with different BCs and static methods are listed in Table 4. It can be seen that, for all three BCs, dynamic *E* values of OSB panels are larger than their static counterparts, while dynamic G_{xy} values are smaller than static G_{xy} value. The differences between dynamic and static E_x values of OSB panels are 16.9, 2.5 and 9.4% for FFFF, SFFF and SFSF BCs, respectively. The differences between dynamic and static E_y values of OSB panels are 39.9, 29.0 and 22.5% for FFFF, SFFF and SFSF BCs, respectively. The differences between dynamic and static G_{xy} values of OSB panels are −27.5, −22.6 and −16.6% for FFFF, SFFF and SFSF BCs, respectively. Among the three BCs, the three elastic constants of OSB panels obtained from FFFF BC exhibited the largest difference from the corresponding static values. The difference between dynamic and static E_y values of OSB has been discussed in previous research [14]. For commercial OSB panels, around 50% of the strands are oriented within 20° from the major strength axis and thus stiffness distribution varies a lot spatially [24, 25]. However, the width of the strips for bending tests was 50 mm, which is much smaller than the length of a single strand, 150 mm. Therefore, the static data are lower than those obtained by modal tests of full-size panels.

Table 4 Elastic constants of OSB and MDF measured by dynamic methods with different boundary conditions and static methods

Panel #	Elastic constants measured by dynamic methods (MPa)									Elastic constants measured by static methods (MPa)		
	FFFF[a]			SFFF			SFSF					
	E_x	E_y	G_{xy}	E_x	E_y	G_{xy}	E_x	E_y	G_{xy}	E_x	E_y	G_{xy}
OSB1	7769	3534	1790	6533	3168	2092	7496	3177	2045	6424	2654	2504
OSB2	7740	3530	1820	6818	3060	1929	7334	3071	2162	6732	2520	2589
OSB3	8231	3848	1935	6994	3434	2119	7542	3319	2243	6376	2627	2450
OSB4	7678	3483	1768	6820	3225	1897	6921	3049	1959	6808	2574	2488
OSB5	7782	3700	1872	7223	3789	1763	7386	3222	2155	6694	2555	2637
Mean	7840	3619	1837	6878	3335	1960	7336	3167	2113	6607	2586	2534
COV (%)	6.4	6.5	7.1	6.5	9.7	9.9	6.3	8.1	8.8	10.7	14.9	7.5
Diff. (%)	16.9	39.9	−27.5	2.5	29.0	−22.6	9.4	22.5	−16.6	–	–	–
MDF1	3323	3282	1394	3273	2818	1324	3297	3216	1341	3073	3162	1626
MDF2	3290	3188	1377	3233	2938	1319	3210	2906	1411	2929	3078	1450
MDF3	3460	3423	1459	3276	3072	1406	3473	3185	1492	3371	3453	1568
MDF4	3221	3182	1353	3050	2848	1359	3055	3017	1280	3041	3231	1492
MDF5	3440	3310	1446	3280	3031	1434	3403	3200	1526	3078	3251	1477
Mean	3347	3277	1406	3222	2942	1368	3288	3105	1410	3098	3235	1522
COV (%)	4.0	3.6	3.9	5.3	4.5	4.9	5.4	7.3	7.3	6.7	6.0	8.1
Diff. (%)	8.0	1.3	−7.6	4.0	−9.1	−10.1	6.1	−4.0	−7.7	–	–	–

COV is short for the coefficient of variation. Diff. refers to the difference in percentage based on corresponding static value. The dynamic elastic constants of four panel specimens cut from the same full-size panel specimen were averaged as the representatives of each full-size panel specimen. OSB and MDF are short for oriented strand board, and medium density fiberboard, respectively. FFFF represents the boundary condition of all sides free, SFFF represents the boundary condition of one side simply supported and the other three free and SFSF represents the boundary condition of a pair of opposite sides along minor strength direction simply supported and the other pair along major strength direction free

[a] Results of FEM updating

For MDF panels, the differences between dynamic and static values are much smaller than those for OSB panels. The differences between dynamic and static E_x values of MDF panels are 8.0, 4.0 and 6.1% for FFFF, SFFF and SFSF BCs, respectively. The differences between dynamic and static E_y values of MDF panels are 1.3, −9.1 and −4.0% for FFFF, SFFF and SFSF BCs, respectively. The differences between dynamic and static G_{xy} values of MDF panels are −7.6, −10.1 and −7.7% for FFFF, SFFF and SFSF BCs, respectively. There are no significant differences between the values measured using the three BCs for a specific elastic constant.

Generally, dynamic E_x values from all three BCs are larger than static values, and dynamic G_{xy} values from all three BCs are smaller than static values. Dynamic E_y values of MDF from SFFF and SFSF BCs are slightly smaller than static E_y values, while dynamic E_y values of MDF from FFFF BC is slightly larger than static E_y values. From the comparisons between mean values by dynamic and static methods, it can be seen that all three dynamic methods show the same trends of measured values, though the differences with static values varied.

Correlation between dynamic and static results

To better compare the dynamic methods with static methods, the dynamic values from each BC were compared with static values through paired-sample t tests. As shown in Table 5, most of the paired groups have a p value less than 0.05 at the 95% confidence level except paired group 'SFFF & static' of E_x for OSB panels and paired groups 'FFFF & static' and 'SFSF & static' of E_y for MDF panels. Generally, the elastic values by dynamic methods exhibit a significant difference with the elastic values by static methods at the 95% confidence level. Thus the linear correlation of each elastic constant between dynamic and static method are not as good as most reported correlation between dynamic and static values of beam-like specimens [26].

Figures 6 and 7 illustrate the differences in percentage (Diff.) between each dynamic and static elastic constant of all panel specimens with different BCs. It can be seen that in Fig. 6, the difference between dynamic and static E_x of each individual OSB panel ranges from −1 to 37% with most of them around −16% for FFFF BC, from −11 to

Table 5 Paired-samples t test results of each elastic constant between dynamic and static test values

Material	Elastic constants	Paired group	Correlation	t	df	p (2-tailed)
OSB	E_x	FFFF & static	0.174	7.879	19	0.000
		SFFF & static	0.459	1.559	19	0.**136**
		SFSF & static	0.099	4.371	19	0.000
	E_y	FFFF & static	−0.428	11.242	19	0.000
		SFFF & static	−0.280	7.220	19	0.000
		SFSF & static	−0.332	6.284	19	0.000
	G_{xy}	FFFF & static	0.220	−14.722	19	0.000
		SFFF & static	−0.097	−8.744	19	0.000
		SFSF & static	0.220	−14.722	19	0.000
MDF	E_x	FFFF & static	0.351	5.482	19	0.000
		SFFF & static	0.059	2.107	19	0.049
		SFSF & static	0.467	4.196	19	0.000
	E_y	FFFF & static	0.**520**	1.147	19	0.**266**
		SFFF & static	0.431	−7.245	19	0.000
		SFSF & static	0.132	−2.068	19	0.**053**
	G_{xy}	FFFF & static	−0.065	−3.664	19	0.002
		SFFF & static	−0.050	−4.678	19	0.000
		SFSF & static	−0.128	−3.003	19	0.007

df is short for degree of freedom. Sig. refers to the significance of paired-samples *t* test of each group. If sig. <0.05, there is significant difference between paired group at the 95% confidence level. The tests were performed using software SPSS 19.0. OSB and MDF are short for oriented strand board, and medium density fiberboard, respectively. FFFF represents the boundary condition of all sides free, SFFF represents the boundary condition of one side simply supported and the other three free and SFSF represents the boundary condition of a pair of opposite sides along minor strength direction simply supported and the other pair along major strength direction free

Bold values are > 0.05

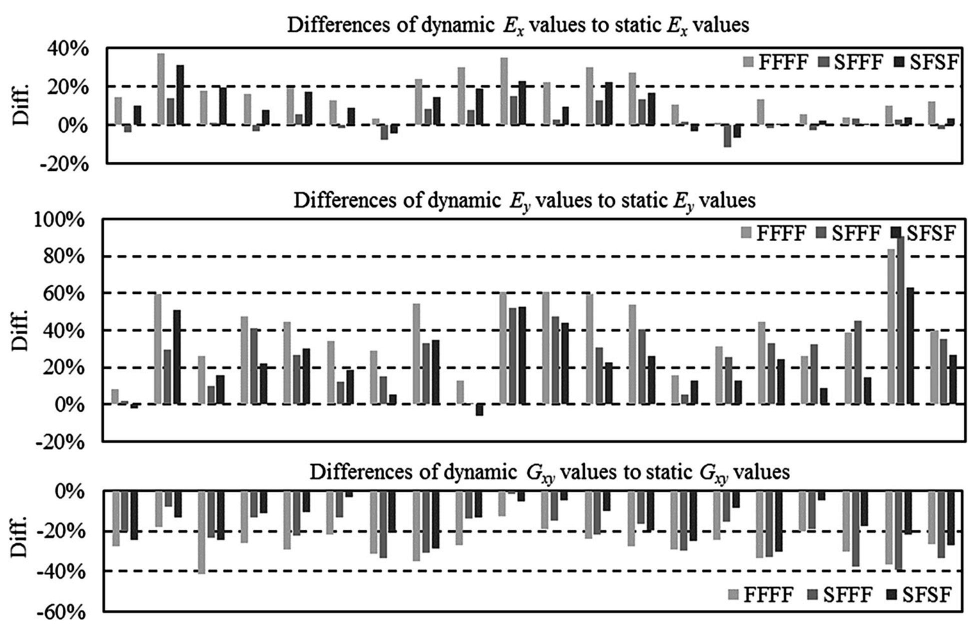

Fig. 6 Differences of dynamic elastic constants from different BCs to corresponding static values of OSB

16% with most of them around −3% for SFFF BC, from −6 to 31% with most of them around −9% for SFSF BC, respectively. The difference between dynamic and static E_y and G_{xy} values of each individual OSB panel is within 60% (except for one panel) and −40%, respectively. Most of the differences are distributed around their averaged differences for each BC. The exceptions happen when the static values are either too large or too small. However,

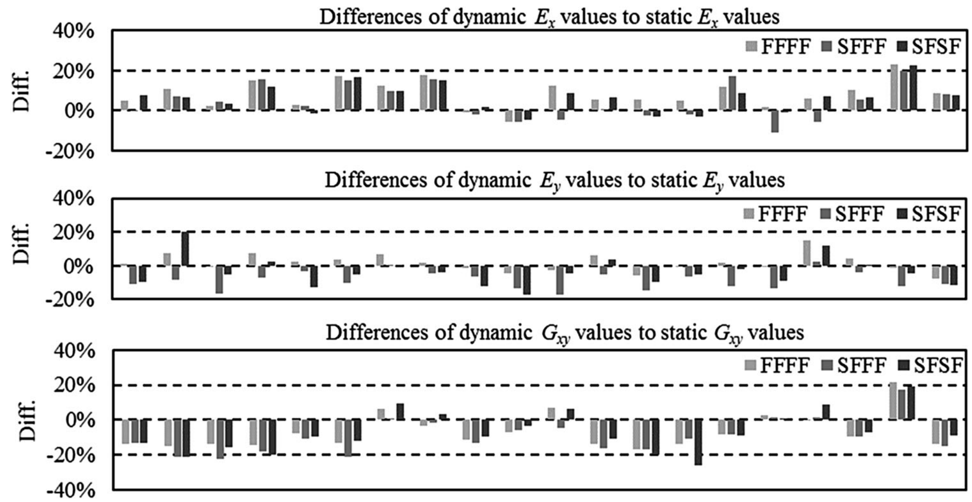

Fig. 7 Differences of dynamic elastic constants from different BCs to corresponding static values of MDF

corresponding dynamic values from the three BCs are consistent with each other, indicating their reliability. Compared with OSB panel results, MDF panel results show better uniformity in differences distributions for three elastic constants within an absolute difference of 20%.

The differences between dynamic and static values can be mainly explained by the material structure of the panels and the nature of the test methods. Dynamic values by modal tests of panels are always considered to be the general elastic constants as representative of the whole panel, while the static values are the localized elastic constants. Nakao and Okano [1] reported differences between dynamic and static G_{xy} values for particleboard and fiberboard such as hardboard and MDF panels of −35 to 18%, while the difference for plywood was −8 to 14%. Larson [6, 7] also reported an average difference between dynamic and static E_x and E_y of 14 and 31% for OSB panels, respectively. The results from the static tests on small strip specimens are questionable for some particle-based wood panel products because the relative size of the specimen and wood elements in the panel [27].

Accuracy analysis of dynamic test methods

The differences between each BC are primarily caused by the influence of BCs in practice, the accuracy of chosen forward problem solutions and sensitivity level of selected vibration modes. The influence of implementing BC in practice is not easy to be assessed. FFFF BC is the one requiring the least efforts and free from added constraints among three BCs. SFFF and SFSF BCs require partial clamping to stabilize the test panel. Aside from the influences of implementation of BC, the chosen forward solutions affect the results to different extents for OSB and MDF panels. As shown in Fig. 8, the differences between values obtained from Eqs. (4)–(6) and FEM updating are different for different elastic constants and materials. There is virtually no difference for E_x and E_y of OSB panels from both calculations, while there is an average difference of about 5% for G_{xy}. Similarly, no difference was found for E_x value of MDF from both calculations, but there are differences of about 5 and 10% for E_y and G_{xy} value of MDF from both calculations, respectively. For both MDF and OSB panels, E_y was obtained from $f_{(0,2)}$, where the effect of transverse shear may contribute if the transverse shear moduli are small or the wavelength to depth ratio become small for high modes. MDF has a much smaller transverse shear modulus than OSB. FEM updating in this study employed a shell element that included this effect, while Eqs. (4)–(6) do not. In Eq. (6), a factor of 0.9 is used based on previous research [1], but current results shown here suggest a 5 and 10% increase for OSB and MDF panels, respectively.

Sensitivity level of selected frequencies has an effect on calculated values. For instance, the E_x value can be obtained from Eq. (9) or (10) for SFFF BC. However, selected frequencies with different sensitivity will result in different calculated values. As shown in Fig. 9, the differences of E_x values of OSB and MDF panels from the two equations can vary from 10 to 40% because of the lower sensitivity to mode (2,1) than to mode (3, 1). Sobue and Katoh [12], who first adopted SFFF BC for modal testing of wood-based panel material, used different combinations of frequency equations to calculate the elastic constants. It is an alternative method but ignored the effect of nonlinear distribution of sensitivity. Also, in the case of calculating E_y value of MDF using frequency of mode (2, 2) under SFSF BC, the coupled effect of E_y and G_{xy} was included in the iteration of frequency of mode (2, 2) as both elastic constants contribute evenly.

Other influences may include width to thickness ratio and transverse shear rigidity. FEM was performed using

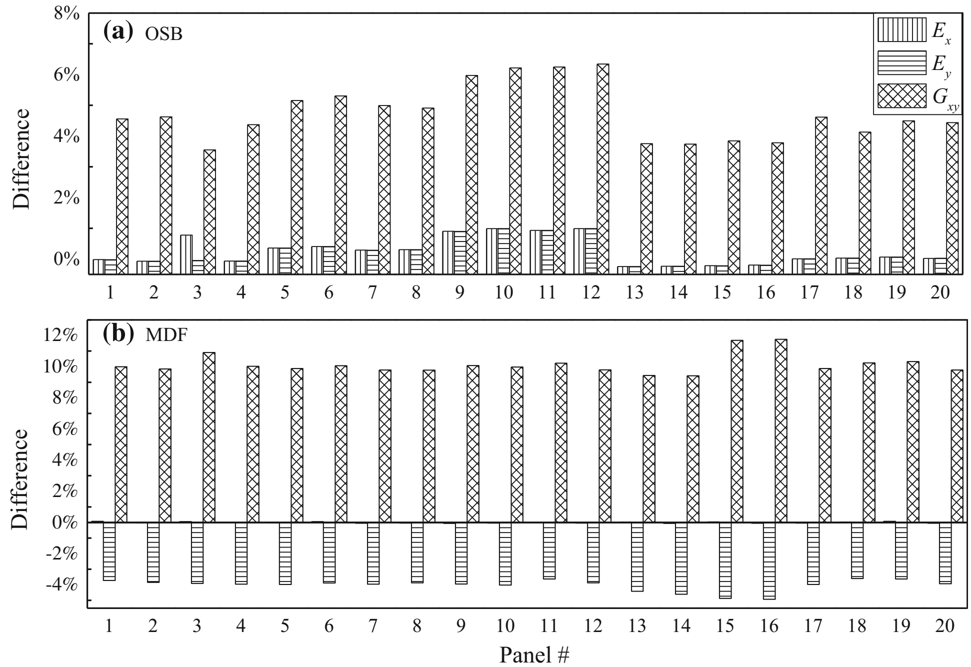

Fig. 8 Influence of forward solution on elastic constants by dynamic method under FFFF BC for **a** OSB and **b** MDF panels

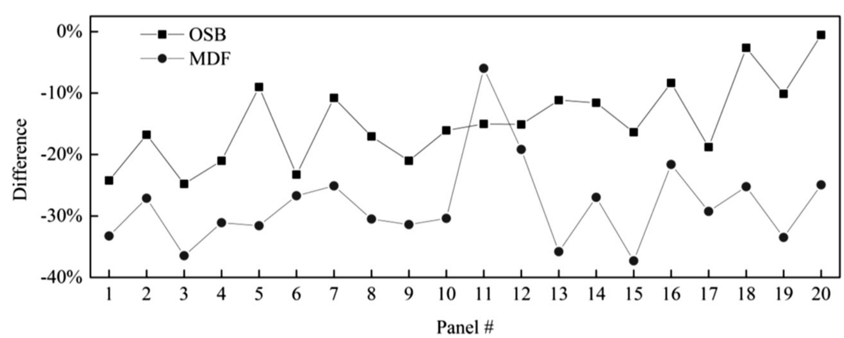

Fig. 9 Differences of calculated E_x values using frequencies of different sensitivities under SFFF BC

material properties in Table 1 and two types of shell elements, S4R and STRI3. STRI3 ignores the effect of transverse shear deformation, while S4R considers it. MDF has much lower transverse shear modulus than OSB. The theoretical effect of transverse shear deformation on natural frequency increases with an increase in thickness is shown in Table 6. As expected the effects are different under FFFF and SFSF BCs. The differences are almost doubled under SFSF BC compared with FFFF BC, for natural frequencies related to E_y and G_{xy}.

Commercial OSB and MDF panels, due to the saddle-shaped vertical density profile, can be regarded as three-layer composites. DOSB and DMDF panels become five-layer composites after gluing, which are expected to have slightly different elastic properties to the component OSB and MDF due to lamination. As shown in Table 7, compared with panel static test results, dynamic results of both DOSB and DMDF panels from SFSF BC seems to be much closer to panel static test results than those from FFFF BC. This may be explained by the same degree of effect of transverse shear deformation on vibration and deflection under static load of the panel for SFSF BC with the increase of thickness. For DOSB panels, the differences between dynamic results from FFFF BC and panel static test results are 8.3, 7.7 and 4.9% for E_x, E_y and G_{xy}, respectively, which are just a little higher than the differences between dynamic results from SFSF BC and panel static test results. However, the corresponding differences between dynamic and static tests for DMDF panels are much higher with FFFF BC than those with SFSF BC. As shown in Table 6, with the increase of thickness, the effect of transverse shear on the selective frequencies are two times smaller with FFFF BC than with SFSF BC. In addition, the transverse shear modulus of MDF are much smaller than OSB. Thus, the dynamic results of DMDF from FFFF BC are least affected with the increase of thickness and are much larger than those with SFSF BC.

In addition, the increase in thickness has a decreasing effect on measured G_{xy} values due to increasing transverse shear deflection. Yoshihara and Sawamura [28] found that

Table 6 Theoretical effects of thickness and transverse shear deformation on selected sensitive natural frequencies

Material	FFFF			SFSF		
	$f_{(2,0)}$ (%)	$f_{(1,1)}$ (%)	$f_{(0,2)}$ (%)	$f_{(2,0)}$ (%)	$f_{(2,1)}$ (%)	$f_{(2,2)}$ (%)
OSB	0.08	−0.74	0.39	−0.02	−1.47	−0.83
DOSB	−0.06	−1.81	0.16	−0.15	−3.43	−2.55
MDF	−0.32	−3.01	−1.06	−0.47	−5.45	−3.45
DMDF	−1.17	−6.52	−4.28	−1.33	−11.44	−9.07

Finite element modeling was performed using material properties presented in Table 1 for DOSB and DMDF. OSB and MDF are short for oriented strand board, and medium density fiberboard, respectively. DOSB and DMDF are short for double-thick oriented strand board, and double-thick medium density fiberboard, respectively. FFFF represents the boundary condition of all sides free, and SFSF represents the boundary condition of a pair of opposite sides along minor strength direction simply supported and the other pair along major strength direction free

Table 7 Elastic constants of DOSB and DMDF measured by modal methods with different boundary conditions and static methods

Panel #	Elastic constants measured by dynamic method (MPa)						Elastic constants measured by static methods (MPa)				
	FFFF			SFSF			Panel bending		Strip bending		G_{xy}
	E_x	E_y	G_{xy}	E_x	E_y	G_{xy}	E_x	E_y	E_x	E_y	
DOSB1	6438	4200	1760	6194	3909	1638	6141	4099	4785	2970	1716
DOSB2	6664	4701	1867	6325	4066	1557	6151	4430	4681	2867	1804
DOSB3	6768	4562	1843	6291	3734	1522	5908	3960	5253	3111	1672
DOSB4	6269	4158	1739	5514	4147	1782	5618	3877	4499	2646	1530
DOSB5	6418	4312	1743	6520	3715	1779	6231	4008	5073	2897	1814
Mean	6511	4387	1790	6169	3914	1656	6010	4075	4858	2898	1707
Diff. (%)	8.3	7.7	4.9	2.6	−3.9	−3.0	–	–	−19.2	−28.9	–
DMDF1	3207	3282	1343	2750	2812	1148	2772	2670	2599	2297	1128
DMDF2	2854	2790	1188	2611	2321	1046	2434	2440	2331	2022	1062
DMDF3	2715	2766	1180	2252	2305	1040	2392	2468	2259	2072	1050
DMDF4	2636	2564	1100	2420	2662	970	2208	2430	2170	1862	1107
DMDF5	2865	2796	1171	2456	2663	969	2394	2405	2279	2112	1069
Mean	2855	2840	1196	2498	2553	1035	2440	2483	2328	2073	1083
Diff. (%)	17.0	14.4	10.5	2.4	2.8	−4.5	–	–	−4.6	−16.5	–

Diff. refers to the difference in percentage based on corresponding static value. DOSB and DMDF are short for double-thick oriented strand board, and double-thick medium density fiberboard, respectively. FFFF represents the boundary condition of all sides free, and SFSF represents the boundary condition of a pair of opposite sides along minor strength direction simply supported and the other pair along major strength direction free

in-plane shear modulus of western hemlock solid wood plates measured by static square-plate twist method increased from 0.5 to 1.0 GPa with an increase in length or width to thickness ratio from 14 to 60.

Panel versus beam bending tests

In Table 7, it can be observed that the differences in E_x and E_y values measured using panel and strip bending tests are −19.2 and −28.9% for DOSB, and −4.6 and −16.5% for DMDF, respectively. For DOSB, the difference is due to the inappropriate size (width) of strip specimens from two strength directions. McNatt [27] once tested bending properties of structural wood-based panels of large panel size [2.44 (length) × 1.22 (width) m^2] and small strip size specified in ASTM D1037 [29]. The results indicated that for OSB, waferboard and flakeboard panels, the E values were not affected much by reducing panel size from 2.44 × 1.22 to 0.61 × 0.30 m^2. The panel bending test values of E_x and E_y were about 23 and 15% larger than corresponding strip bending test values for OSB, respectively. This was likely caused by the reduction in the strand length when strip specimens were cut which reduced the lap lengths of the adhesive bond between strands. He suggested that large panel test should be used when developing design properties for structural panels.

For DMDF, a fiber-based material, which is an almost isotropic material and also more uniform than DOSB, the difference between panel and strip bending test E_x values is small. The difference between E_x and E_y by strip bending test is likely caused by a shorter span-to-depth of the strips along the width direction (18.3 for strips in the length direction and 12.8 for strips in the width direction). The smaller span-to-depth ratio and transverse shear modulus of the material are the reasons for the smaller E_x and E_y values of DMDF panel and strip specimens than the corresponding MDF specimens. It can be concluded that static panel test results are closer to dynamic test results than strip bending test results.

Conclusions

Through this study it has been shown that different accuracy levels are achieved with the three modal testing approaches, which incorporate different boundary conditions and calculation procedures. The influences of different aspects on accuracy have been also discussed. Modal test methods can be an option for measuring elastic constants of engineered wood-based panels due to its non-destructive nature and fast testing time. For orthotropic wood-based panel products, modal testing is recommended as it can account for the influence of coupling between elastic constants and is less tedious to conduct compared with static testing approaches. The elastic constants obtained are the general properties of the panel products, which are comparable to the static test of the whole panel. It is recommended for property evaluation of panel products, especially those intended for structural application.

All three BCs with corresponding calculation methods can be applied in the laboratory environment. FFFF is the easiest BC to be replicated in a testing environment and can be applied for panels of small to moderate dimensions, but advanced forward problem solution such as FEM is needed. Simple frequency solution can give appropriate initial guess of elastic constants. SFFF is not recommended for large and thick panels as the support condition is practically unstable which requires some efforts in restraining the specimen in a vertical position. SFSF BC with the proposed calculation method shows great potential for laboratory and on-line application, especially for massive panels with large dimensions. Proper selection of BC and corresponding calculation method is important for characterizing the material of interest.

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