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Testing and modeling of the behavior of wooden stairs and stair joints

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Abstract Joints in traditional wooden stairs can be constructed in many different ways. Connections between tread and string are commonly made with tenons and screw joints. It is often hard to know which joint rigidity should be used in calculations. To study this type of stair, a total of 12 small straight stairs with three steps each were tested. The stairs were loaded and the deformation was measured. Modeling of the stairs with a finite element program was also performed, using linear elastic models in the analysis. The conclusion derived from the study is that the tested tread-to-string connections are not very rigid. Tolerances and accuracy of manufacture of the stair parts are important for the behavior of the joints. Comprehensive knowledge about stair joints can be used for development of new cost-effective and reliable design tools for stairs. In a computer program for stair design based on beam elements, the treads in this type of staircase should, according to this study, preferably be calculated as being hinged to the string, or else the rigidity of the joint should be a function of joint parameters.

Key words Joint · Wooden stair · FE modeling · Tenon

Introduction

Most wooden stairs are manufactured according to traditional design with dimensions and mechanical properties of connections based on experience. Stair design and the dimensions of steps and balustrades are often determined from an aesthetic point of view. Simple beam calculations are occasionally used, and special computer programs based on beam calculations can also be used for entire stairs.

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Mechanical properties of the joints are very important for the structural behavior of a stair. Joints in traditional wooden stairs can be designed in several ways, as discussed by Kress.[1] Connections between tread and string are usually made with different types of tenons and screw joints. The connections are commonly not glued. There are many variables that can be changed in this type of joint; the thickness of tread and tenon, the length of the tenon, the gap between the tread and the routed hole in the string, the friction, the stiffness, and other properties of the wood at the connection. The joints can be assumed to behave as semirigid connections. Much research has been conducted on semirigid joints, but not particularly on stair joints. Finite element (FE) calculations are often used to study the behavior of semirigid joints of steel, concrete, and wood. Some results are presented in the European project C1.[2]

The aim of this investigation was to extend knowledge of how connections between tread and string in wooden stairs behave structurally and to give some guidelines on what degree of restraint should be used in calculations.

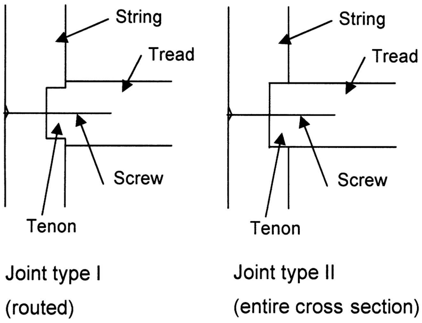
Materials and methods

This study used small straight stairs with strings and three treads without risers. The steps were numbered 1–3 from the bottom of the stair. A total of 12 stairs with a height of 800 mm and a width of 900 mm were tested in the laboratory. The stairs were made of pine (*Pinus sylvestris*), beech (*Fagus sylvatica*), or oak (*Quercus robur*) and had tread thicknesses of 40 or 42 mm. Tread 1 and 2 had a width of 275–282 mm and tread 3 had a width of 250 or 282 mm. Three different stair manufacturers produced the stairs according to their normal procedure, using joinery timber that had not been strength graded, but rather was classified according to appearance quality and sorted by color. The timber is usually acquired as room-dry wood with a moisture content of 8%–10%, which corresponds to the equilibrium moisture content of wood in houses with permanent heating. The treads and strings were made of laminae of

Table 1. Measured material properties

Wood species	No. of treads	Modulus of elasticity (MPa)		Density (kg/m^3)		Moisture content (%)	
		Mean	SD	Mean	SD	Mean	SD
Pine	27	10151	996	491	31	9.0	0.65
Beech	9	13302	1230	701	24	8.4	0.68
Oak	9	11412	1115	725	54	9.3	0.60

SD, standard deviation



Joint type I (routed)

Joint type II (entire cross section)

Fig. 1. Connection of tread to string. *Left*, type I; *right*, type II

40-mm-wide planed wood glued with polyvinyl acetate adhesive. The density of the treads was calculated from the weight and volume of the treads. The moisture content was measured with a resistance meter. The modulus of elasticity was determined from three-point bending tests of the treads with a span of 0.77 m. The measured values in Table 1 are in agreement with values in the literature, e.g., Boutelje and Rydell,[3] for the respective wood species.

The stairs tested had the treads fastened to the inner side of the strings. The end of the tread formed a tenon that was inserted into a routed hole in the string (see Fig. 1). The lengths of the tenons were 12, 15, or 18 mm. Two stair manufacturers designed the treads with a routed tenon (type I), while one used tenons of the entire tread cross section (type II). The wood parts of the stairs were planed and for stairs with tenon type II were also varnished. The treads were screwed firmly from the outside of the string with two screws at each end in predrilled holes in the string. The screws had dimensions 6 × 70 mm or 5 × 70 mm.

The first test was a cantilever test performed with the upper tread fastened to the left string according to Fig. 2. The string was fixed with two clamps against a steel column in the laboratory. The tread was loaded with a point load placed centrically sideways 300 mm from the inner side of the string. The tread was loaded to a displacement of approximately 20 mm by a hydraulic cylinder and a load cell. The contact surface of the point load was 40 × 40 mm. The time to maximum load was about 1–2 min. A strain gauge displacement sensor measured the displacement on the underside of the tread beneath the point load.

The complete, mounted stairs were tested in a steel test rig with the bottom of the strings attached to supporting

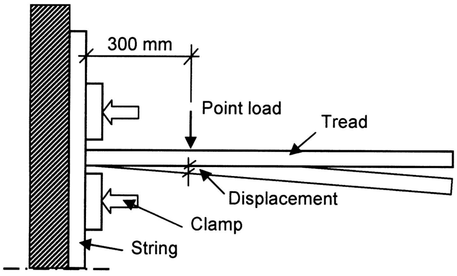


Fig. 2. Cantilever test of tread with point load

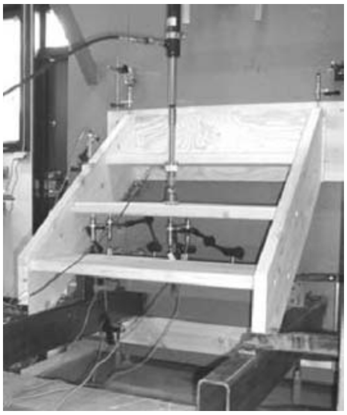


Fig. 3. Testing of stair with point load on tread 2

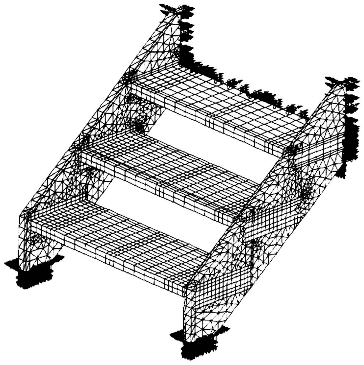


Fig. 4. Finite element model of staircase

steel beams (see Fig. 3). A 12-mm plywood sheet was screwed to the upper ends of the stair and was fixed to a stud on the test rig wall. Tread 2 was loaded centrically in the middle with a point load. Displacement was measured in four places on the underside of the tread. Furthermore, vertical and horizontal displacement at the middle and on the upper edge of the left string was measured (Fig. 3).

The stairs were analyzed with three-dimensional (3D) solid models in the FE software I-DEAS Master Series.[4] The element types were brick elements and tetrahedral elements, and the tread and the string were connected with linear contact elements (see Fig. 4). Linear elastic calculations were performed. The program used detects surface-to-surface contact from the element free faces. If a gap

Table 2. Material properties in calculations

Timber	Material Property					
	E_z	E_x, E_y	G_{zx}, G_{zy}	G_{xy}	ν_{zx}, ν_{zy}	ν_{xy}
Pine	10200	340	680	68	0.025	0.4
Beech	13300	930	880	88	0.025	0.4
Oak	11400	750	700	70	0.025	0.4

All data given in units of MPa
E, Young's modulus; *G*, shear modulus; *ν*, Poisson's ratio; *z*, in the fiber direction; *x,y,* perpendicular to the fiber direction

between contacting faces is zero, the contact element is closed and contact pressures are generated. The initial distance between tenon and string was set to 0.01 mm. The coefficient of friction in the contact analysis was assumed to be 0.2. The two screws in the joint were modeled as springs between two nodes, with a translational stiffness set to 150 MN/m, corresponding to a 5-mm steel bar. This value was used in all analyses, although the withdrawal resistance of a wood screw can differ for different wood species. In the cantilever calculations, the string was supported in three directions over and under the joint. In the staircase calculations the lower and upper end of the strings, and the back of the third tread all had displacement restrained in three directions. Tenon type, tenon length, and material properties were varied in the FE models. The treads and strings of the FE models had a thickness of 40 mm. Tread 1 and 2 had widths of 280 mm, tread 3 a width of 250 mm and the strings had a width of 248 mm. The material properties in Table 2 were used in the model. They were chosen to correspond to the measured Young's modulus in the fiber direction for the treads. The Young's modulus perpendicular to the laminations and the shear moduli were calculated according to the relationships in ENV 1995–2:1997.[5] The Young's modulus perpendicular to the fiber direction had the same value in two directions. The radial and tangential directions were not considered because the laminae were placed at many different angles. The Poisson's ratios were chosen according to Bodig and Jayne.[6]

Results

The measured forces from the cantilever tests are shown as mean values for the different species in Fig. 5. Treads made of pine allowed the lowest loads, which agreed with the lowest modulus of elasticity for pine. The mean values for the beech treads were larger than for the oak treads, although the modulus of elasticity of oak was higher. This probably is a result of large variation in the test specimens and the fact that few treads were tested. The cantilever tests gave large deformations, which are not achieved in normal stairs. The remaining deformation after unloading was approximately 8–13 mm, and the tread was loose. The screw head had been pressed into the string, by about 0.5–0.7 mm. The measured forces that were needed to give 10-mm displacement of the treads in the cantilever tests are given in Table 3. Calculated values of the FE models are also presented and compared with the measured values.

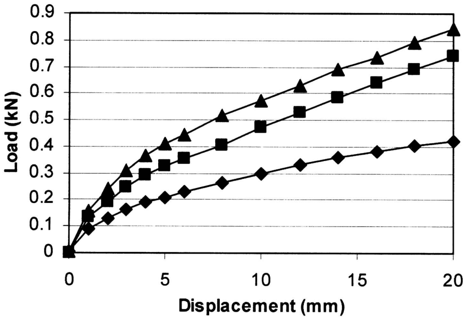


Fig. 5. Mean values of measured loads from cantilever tests. *Diamonds*, pine; *squares*, beech; *triangles*, oak

Measured values and calculation results from FE analysis for stairs made of pine with tenon length of 12 mm are presented in Table 4. The remaining displacements were 0.02–0.10 mm for this load case.

Discussion

The measured figures were quite different from the analytical results. One of the reasons could be that ordinary stairs were used for the tests, and consequently the variations in the test specimens were large. Inaccurate manufacturing with different tolerances between stair parts could influence the measured results. In addition, the wood material has variation in material properties between different parts in a piece of wood. To obtain some information about the material properties to use in the calculations, the modulus of elasticity of the treads was measured by three-point bending. The measured values were for an entire tread, without consideration of variations within the tread or the laminations of the tread. The analyses were made with uniform material properties in all parts, and constant tolerances for all stairs.

In the load case with a tread as a cantilever, the rigidity of the joint was larger for small loads but was then reduced. Most of the maximum displacements probably depended on deformation of the screwed joint and especially pressing of the screw head into the string. There was also an indentation in both string and tread at the lower edge after unloading, which means that the wood fibers were compressed perpendicular to grain in this area. After unloading, the tread had a remaining displacement, and this confirms that the joint was deformed. There was a difference in the force required to cause a deformation of 20 mm for the different materials; that is, beech and oak were different from pine. There was no statistically significant difference between the average remaining deformations for different materials or joint types or lengths of tenons.

In Table 3 the measured loads represent one tread each. Different moduli of elasticity of the treads can be one rea-

Table 3. Load required for 10-mm displacement of tread as cantilever

Wood species	Manufacturer	Tenon type	Tenon length (mm)	Measured tolerance (mm)	Measured load (kN)	Calculated load (kN)	Calc/meas
Pine	1	I	12	0	0.433	0.429	0.99
Pine	1	I	18	0.81	0.327	0.737	2.25
Pine	2	I	12	0.38	0.151	0.429	2.84
Pine	2	I	15	0.53	0.156	0.550	3.52
Pine	3	II	12	0	0.339	0.416	1.23
Pine	3	II	18	0	0.381	0.726	1.91
Beech	1	I	12	0	0.559	0.778	1.39
Beech	2	I	15	0.78	0.262	1.026	3.92
Beech	3	II	12	0	0.593	0.756	1.27
Oak	1	I	12	0.31	0.831	0.631	0.76
Oak	2	I	15	0.95	0.581	0.824	1.42
Oak	3	II	12	0.46	0.315	0.613	1.95

Table 4. Maximum displacements for pine staircases with tenon length of 12 mm and loaded with point load 3 kN in the middle of tread 2

Wood species	Manufacturer	Tenon type	Measured displacement (mm)	Calculated displacement (mm)	Calc/meas
Pine	1	I	3.96	2.43	0.61
Pine	3	II	1.97	2.51	1.27
Beech	1	I	1.35	1.83	1.35
Beech	3	II	1.58	1.92	1.22
Oak	1	I	3.23	2.17	0.67
Oak	3	II	1.92	2.25	1.17

son for the large variation in the test results. For tenon type I (routed) there is also a difference between the two manufacturers, which may be caused by different design, workmanship, and accuracy of manufacture. The dimensional accuracy varied more for tenon type I than for tenon type II. Treads with tenon type II fitted more precisely in the holes in the strings. Tolerances in height between tenon and hole are presented in Table 3, but they cannot explain all the differences.

The FE calculations were linear and the models had a fairly crude mesh. However, the results can be used to compare the joints (see Fig. 6). The FE calculations showed that for tenon type I the lower edge of the tread was pressed against the string, while for tenon type II the tread could slide inward in the string and the back edge underwent a contact pressure toward the string. The largest contact pressure for tenon type I (routed tenon) was in the lower front verge of the hole in the string, which is the support for the tread. According to Table 3, the calculations showed only small differences between type I and type II. A difference between the joint types is that the bearing length was shorter for the joint with a routed tenon. The indication in the calculations was that a longer tenon increased the rigidity of the joint. The reason for this could be a longer bearing length for the tenon.

In the finite element analysis of the entire stairs, the treads were deformed almost like simply supported beams. Figure 7 shows the average measured displacements of treads in stairs of pine adjusted with the displacements of the string. The adjusted measured displacements were on average 1.4 times larger than the calculated displacements for tenon type I (routed tenon). By way of contrast, for tenon type II (entire tread cross section as tenon) the measured displacements were on average 0.9 times the calculated displacements. For comparison, the displacement was also calculated with another FE model with the treads fixed to the string and the result was 1.58 mm. A calculation for one tread that was simply supported at the ends resulted in

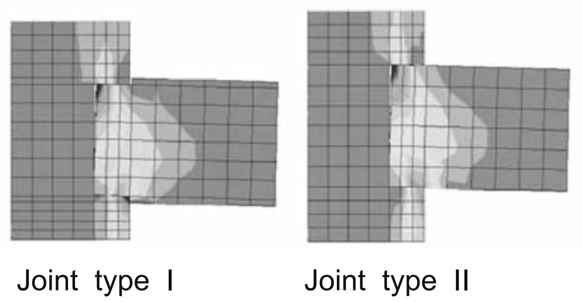


Fig. 6. Vertical stresses at joint with tread loaded with 0.3 kN. For joint type I (*left*), the highest compressive stresses are 3.3 MPa at the upper corner of the tread and in the lower front verge of the hole in the string. For joint type II (*right*), the highest compressive stress is 2.8 MPa at the upper corner of the tread

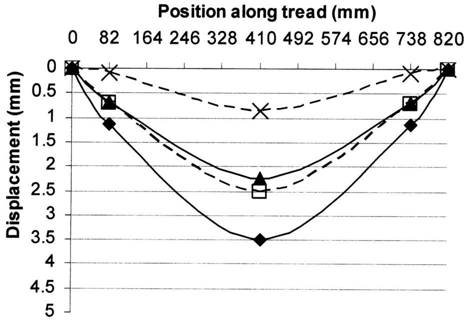


Fig. 7. Measured and calculated displacements of treads (thickness 40–42 mm) in stairs loaded with a point load of 3 kN on the middle step. *Diamonds*, measured values for tenon type I; *triangles*, measured values for tenon type II; *squares*, calculated values for simply supported beam; *crosses*, calculated values for fixed-end beam

a displacement of 2.52 mm, while calculation for one tread with fixed ends gave a displacement of 0.85 mm.

The conclusion from this study is that the tested types of tread connections to strings are not very rigid. In a program for stair design based on beam models, the treads in this type of staircase should preferably be calculated as hinged to the string, or else the rigidity of the joint should be a function of joint parameters. More detailed studies are needed to construct a more accurate model of the joint behavior and how the joints depend on different parameters.

References

1. Kress F (1988) Der Treppen-und Geländerbauer. Th. Schäfer, Hannover, pp 35–64
2. Cost C1 (1998) Numerical simulation for behaviour analysis. In: Proceedings of the international conference on control of the semi-rigid behaviour of civil engineering structural connections, Liège, 17–19 September, pp 467–536
3. Boutelje JB, Rydell R (1995) Träfakta, 44 träslag i ord och bild. Trätek, Stockholm, pp 19–23, 70–71
4. Anon. (1999) I-DEAS Master series 6, student guide. Structural Dynamics Research Corporation, Milford, OH, pp 1–476
5. Anon. (1997) ENV 1995-2:1997, Eurocode 5 – design of timber structures – part 2: bridges. CEN European Committee for Standardization, Brussels, pp 1–45
6. Bodig J, Jayne BA (1982) Mechanics of wood and wood composites. Van Nostrand Reinhold, New York, pp 87–126