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ORIGINAL ARTICLE

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Splitting strength of beams loaded perpendicular to grain by dowel joints

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Abstract A linear elastic fracture mechanics model for calculation of the splitting strength of dowel-type fastener joints loaded perpendicular to grain (Van der Put/Leijten model) has previously been presented, and now forms the basis for design in Eurocode 5. The original Van der Put/Leijten model was derived using a number of simplifying assumptions, e.g., that the normal forces in the cracked parts of the beam can be ignored, leading to a solution that does not involve the effect of an initial crack. In the present article an extended version of the Van der Put/Leijten model is derived without any simplifying assumptions, and it is shown that the original Van der Put/Leijten model appears as a special case, namely by assuming that only contributions from shear deformations are significant. The model presented here involves the effect of an initial crack and may be characterized as a generalized linear elastic fracture mechanics model. Results of tests showing the influence of initial cracks of various lengths are presented and compared with the predictions.

Key words Dowel joints · Perpendicular to grain · Splitting · Fracture mechanics · Crack length

Introduction

A dowel-type fastener joint loading a beam perpendicular to grain may fail in a ductile manner, characterized by bending of the fastener and/or embedment of the fastener into the wood, or cause brittle failure characterized by splitting of the beam. The ductile failure modes are well understood and can be accurately predicted by the European yield model[1] (or extended theories), which now forms the basis of design of dowel-type fastener joints in major design codes.

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For brittle failure modes, no simple theory suitable for implementation in design codes has yet gained wide acceptance. Recently, however, a number of simple theoretical models based on fracture mechanics have been proposed.[2–6] A large number of test data compiled from the literature was analyzed,[4] and the models seem to be able to predict the splitting failure fairly well (at least if introducing appropriate empirically determined effectiveness factors). The Van der Put/Leijten model[2–4] now forms the basis of design in Eurocode 5.[7]

The original Van der Put/Leijten model was derived based on a number of simplifying assumptions, among others, the disregard of normal forces in the cracked part of the beam, leading to a very simple formula readily applicable to practical design. An extended version of the Van der Put/Leijten model was presented,[8] which takes into account the normal forces in the cracked parts of the beam, and does not resort to simplifying assumptions. The resulting formula for the splitting strength remains fairly simple, and the original Van der Put/Leijten solution appears as a special case if only the contribution from shear deformations is considered or if the crack length is assumed to be zero.

A basic limitation for application of linear elastic fracture mechanics (LEFM) is that the size of the fracture process zone must be small when compared with the length of the crack. This condition is not fulfilled in the Van der Put/Leijten model (at least for short cracks). A simple and widespread modification of LEFM to overcome this problem is to increase the assumed crack length.[9] Such a modification is easily accomplished using the model derived in the present article.

Strength analysis

Linear elastic fracture mechanics

For a linear elastic body loaded by a single load, P, the crack propagation energy release rate, $\mathcal{G}$, is given by[10]

$$\mathcal{G} = \frac{P^2}{2b}\frac{\mathrm{d}C}{\mathrm{d}a} \tag{1}$$

where b is the width of the body, a is crack length, and C is the compliance given by

$$C = \frac{\delta}{P} \tag{2}$$

where δ is the deflection of the loading point.

A crack starts propagating when the energy release rate reaches the critical value, $\mathcal{G}_c$. Assuming static or quasi static conditions and no energy dissipation outside the fracture region, $\mathcal{G}_c$ is equal to the fracture energy, $\mathcal{G}_f$, of the material.

The crack propagation load, P_c, of the body is thus given by

$$P_c = \sqrt{\frac{2b\mathcal{G}_f}{\frac{\mathrm{d}C}{\mathrm{d}a}}} \tag{3}$$

A simply supported beam loaded perpendicular to grain by a single joint is shown in Fig. 1. It is here assumed that the fastener is sufficiently stiff to ensure that the crack propagates along the grain simultaneously through the entire width of the beam.

The cracked beam is modeled using ordinary beam theory. The static model used is shown in Fig. 2 for a symmetrical crack of length $a = 2\beta h$, h being the beam depth. The span of the beam is $2L$, and the load applied at midspan is denoted P. The edge distance is denoted αh.

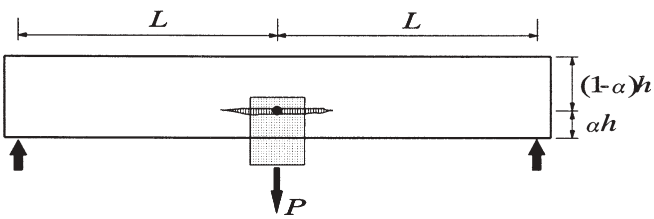

Fig. 1. Geometry, load, and support conditions

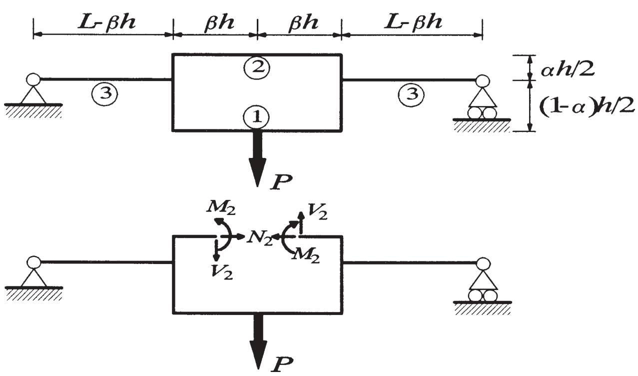

Fig. 2. Static model for cracked beam

Beam theory should formally not be expected to render useful solutions for short cracks (e.g., $2\beta h < \alpha h$), but is attempted here anyway. The beam structure shown in Fig. 2 is three times statically indeterminate. The unknown variables N_2, V_2, and M_2 are chosen as the sectional forces at the center of beam 2, and are found to be

$$N_2 = -\frac{3}{2}PL\frac{1}{h}\left(2 - \frac{\beta h}{L}\right)\alpha(1-\alpha),$$
$$M_2 = -\frac{h}{6}\frac{(1-\alpha)^2}{\alpha}N_2, \quad V_2 = 0 \tag{4}$$

The deflection at the loading point, δ, is given by

$$\delta = P\left[\frac{3}{5}\frac{1}{GA}\left(L + \beta h\frac{1-\alpha}{\alpha}\right) + \frac{1}{6EI}\left((L-\beta h)^3 + \frac{\beta h}{\alpha^3}\left[3L(L-\beta h) + (\beta h)^2 - \frac{3}{4}(2L-\beta h)^2(1-\alpha^3)\right]\right)\right] \tag{5}$$

where E is modulus of elasticity (MOE), G is shear modulus, A is cross-sectional area, and I is moment of inertia of the uncracked beam cross section (width b, depth h).

By means of Eqs. 3 and 5, P_c is found to be

$$P_c = 2b\sqrt{\frac{\alpha h G \mathcal{G}_f}{\frac{3}{5}(1-\alpha) + \frac{3}{2}\frac{G}{E}\left(\frac{\beta}{\alpha}\right)^2(1-\alpha^3)}} \tag{6}$$

It is noted that P_c is a decreasing function of β, i.e., stable crack growth is not possible, and the crack propagation load according to the present model is thus equal to the failure load causing catastrophic failure.

Zero crack length ($\beta \to 0$) or only considering the contribution to the deflection from the shear force ($E \to \infty$ or simply disregarding the second term in Eq. 5) leads to the failure load, P_{c0}, which is the original Van der Put/Leijten solution[2–4]

$$P_{c0} = 2b\sqrt{\frac{5}{3}G\mathcal{G}_f h\frac{\alpha}{1-\alpha}} \tag{7}$$

The extended Van der Put/Leijten solution as given by Eq. 6 may then also be expressed as

$$P_c = \mu P_{c0}, \quad \mu = \sqrt{\frac{1}{1 + \frac{5}{2}\frac{G}{E}\frac{1-\alpha^3}{1-\alpha}\left(\frac{\beta}{\alpha}\right)^2}} \tag{8}$$

A general LEFM solution for a beam with a crack, based on simple engineering beam theory, has previously been presented.[11] The left side of the crack (Figs. 1 and 2) is shown in Fig. 3. The sectional forces at the crack tip are given by Eq. 4 and by

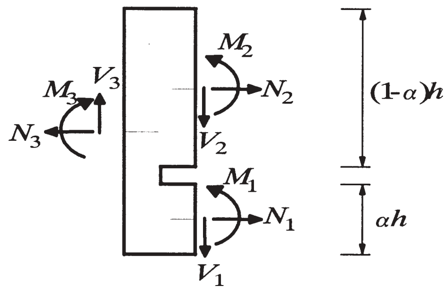

Fig. 3. Sectional forces at crack tip

$$N_1 = -N_2, \quad N_3 = 0$$
$$V_1 = \frac{1}{2}P, \quad V_3 = \frac{1}{2}P$$
$$M_1 = \frac{1}{2}P(L - \beta h) + \frac{1}{2}hN_2 - M_2, \tag{9}$$
$$M_3 = \frac{1}{2}P(L - \beta h)$$

The crack propagation load, P_c, is given by[11]

$$\frac{P_c}{P} = \sqrt{\frac{2b\mathcal{G}_f}{\frac{V_1^2}{\frac{5}{6}GA_1} + \frac{V_2^2}{\frac{5}{6}GA_2} - \frac{V_3^2}{\frac{5}{6}GA} + \frac{M_1^2}{EI_1} + \frac{M_2^2}{EI_2} - \frac{M_3^2}{EI} + \frac{N_1^2}{EA_1} + \frac{N_2^2}{EA_2} - \frac{N_3^2}{EA}}} \tag{10}$$

where $A = bh$, $A_1 = \alpha A$, $A_2 = (1 - \alpha)A$, $I = bh^3/12$, $I_1 = \alpha^3 I$, $I_2 = (1 - \alpha)^3 I$.

Inserting the sectional forces as given by Eqs. 4 and 9 in Eq. 10, P_c is found to be given by Eq. 6.

Generalized linear elastic fracture mechanics

A basic limitation for application of LEFM is that the size of the fracture process zone must be small as compared with the length of the crack. A simple modification of LEFM to overcome this problem is addition of a crack length, $\Delta\beta h$. An expression for estimation of such an additional crack length for mode I fracture of wood based on nonlinear fracture mechanics calculations using the finite element method was given by[9]

$$\Delta\beta h = \frac{0.2\mathcal{G}_f\sqrt{EG}}{f_t^2} \tag{11}$$

where f_t is the tensile strength perpendicular to grain.

The failure load of a structure is, according to Eq. 8, obtained for a minimum value of the crack length. The minimum crack length to be inserted in Eq. 8 may be estimated as $\beta h = \Delta\beta h + d/2$, d being the diameter of the dowel hole, leading to

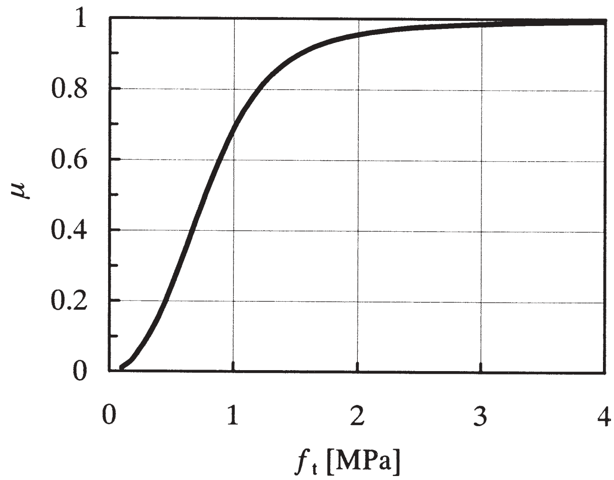

Fig. 4. Influence of perpendicular-to-grain tensile strength

$$P_c = \mu P_{c0},$$
$$\mu = \frac{1}{\sqrt{1 + \frac{5}{2}\frac{1-\alpha^3}{1-\alpha}\frac{G^2}{(\alpha h)^2}\left(\frac{0.2\mathcal{G}_f}{f_t^2} + \frac{d}{2\sqrt{EG}}\right)^2}} \tag{12}$$

According to Eq. 11, the tensile strength perpendicular to grain has a significant influence on the additional crack length. Figure 4 shows an example of the influence of the tensile strength on the failure load using $h = 300$ mm, $\alpha h = 80$ mm, $d = 20$ mm, $E = 12700$ MPa, $G = 870$ MPa, $\mathcal{G}_f = 0.25$ N/mm [material properties for laminated veneer lumber (LVL)[5]].

The perpendicular-to-grain tensile strength, which is seldom used and reported in the literature, is highly volume dependent, and an appropriate value for use in the present model is not obvious.

The splitting strength of LVL beams loaded perpendicular to grain by dowel joints has previously been tested.[6] The specimens used for determination of the perpendicular-to-grain tensile strength for LVL were $40 \times 70 \times 280\,\text{mm}^3$ (70 mm in parallel-to-grain direction, 280 mm perpendicular to the veneer plane), resulting in a reported tensile strength, $f_t = 0.89$ MPa.[5] The size of the tensile test specimens may be suspected of being somewhat too large for the present purpose, thus leading to a somewhat low tensile strength.

For glulam and solid timber, the original Van der Put/Leijten solution as given by Eq. 7 may be sufficiently accurate for practical use, but for materials with very low perpendicular-to-grain tensile strength it may significantly overestimate the failure load.

Experimental

The Van der Put/Leijten model (Eq. 7) has previously been compared with a considerable number of tests on glulam beams.[4,12] The tests presented in this article were conducted primarily to evaluate the appropriateness of the modification factor, μ, as given by Eq. 8.

Methods and materials

Test series 1

Glulam beams without finger joints were tested in three-point bending as shown in Fig. 1. One 14-mm dowel was used in a 15-mm hole. All beams had a span of 1200 mm, width b = 25 mm, and edge distance h_e = 40 mm. Two different beam depths, h = 100 mm (α = 0.4) and h = 200 mm (α = 0.2), were tested.

Initial crack lengths βh = 0, 20, 40, 80, and 160 mm (i.e., $\beta/\alpha = \beta h/h_e$ = 0, 0.5, 1, 2, and 4) were tested. Six specimens were tested for each condition.

The glulam was made of Japanese cedar (*Cryptomeria japonica*), lamella thickness 30 mm, and all lamellae were the same grade (machine graded). MOE was measured on the five glulam beams (100 × 200 mm^2 cross section, length 3000 mm) from which the specimens were cut by measuring the longitudinal vibration frequency, resulting in E = 7530 MPa. Moisture content (MC) at the time of testing was 10.5% and density at the given MC was 372 kg/m^3.

Furthermore, tests were conducted on plate-joint specimens (nine specimens were tested) as shown in Fig. 5 in order to derive the necessary material properties.

Test series 2

Test series 2 was conducted to evaluate the influence of the edge distance, h_e, of the plate joint on the fracture parameter as given by Eq. 13 (or the apparent fracture energy derived here from).

A total of 40 plate-joint specimens (dimensions as shown in Fig. 5) were cut from five glulam beams, which had a mean MOE of 5670 MPa. Three different edge distances were tested: h_e = 20 mm (10 specimens), h_e = 40 mm (20 specimens), and h_e = 60 mm (10 specimens).

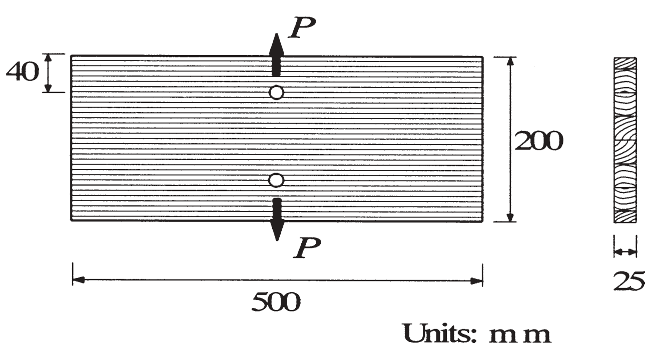

Fig. 5. Plate-joint specimen

Results and discussion

A so-called fracture parameter, C_1, was proposed and derived from plate-joint specimen tests[13]

$$C_1 = \sqrt{\frac{5}{3} G \mathcal{G}_f} = \frac{P_c}{2b\sqrt{h_e}} \tag{13}$$

where P_c is the recorded failure load. Equation 13 appears from Eq. 7 by assuming $h \rightarrow \infty$.

Test series 1

The plate-joint tests resulted in $C_1 = 9.38 \pm 1.40\,\mathrm{N/mm^{3/2}}$, or if calculating the fracture energy using $G = E/18$: $\mathcal{G}_f = 0.129 \pm 0.039\,\mathrm{N/mm}$.

Figures 6 and 7 show the modification factor, $\mu = P_c/P_{c0}$, as obtained from the beam tests and the theoretical solution

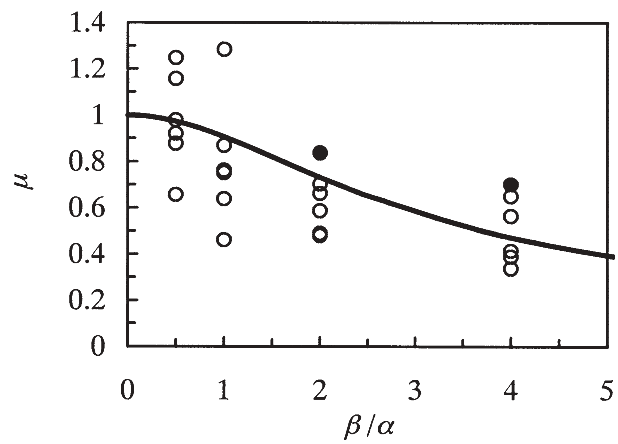

Fig. 6. Modification factor, μ, as function of normalized crack length for h = 100 mm. *Solid line*, Eq. 8; *filled circles*, knots in vicinity of both crack tips

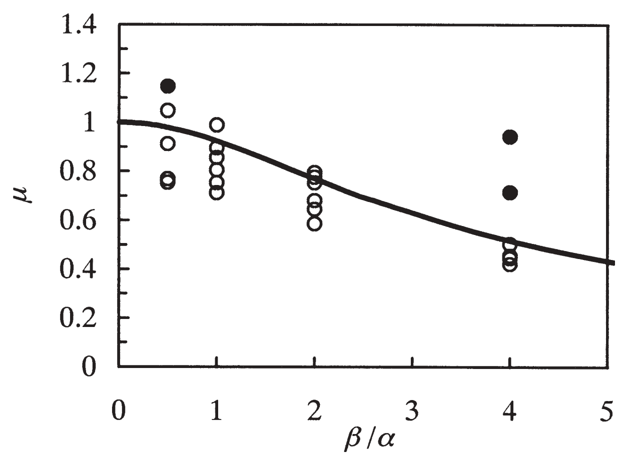

Fig. 7. Modification factor, μ, as function of normalized crack length for h = 200 mm. *Solid line*, Eq. 8; *filled circles*, knots in vicinity of both crack tips

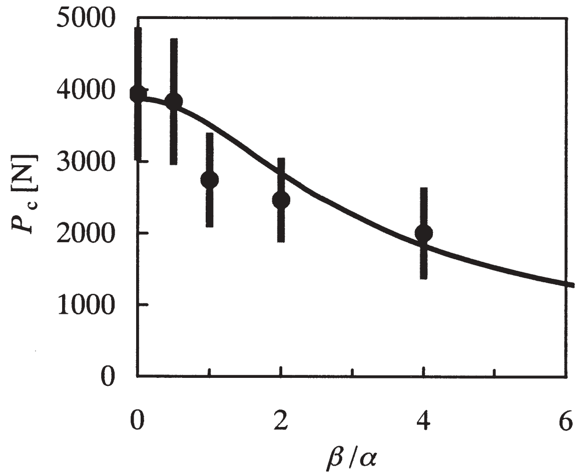

Fig. 8. Failure load as function of normalized crack length for h = 100 mm. *Solid line*, Eqs. 7 and 8; *filled circles*, tested mean value. *Bars* indicate standard deviation

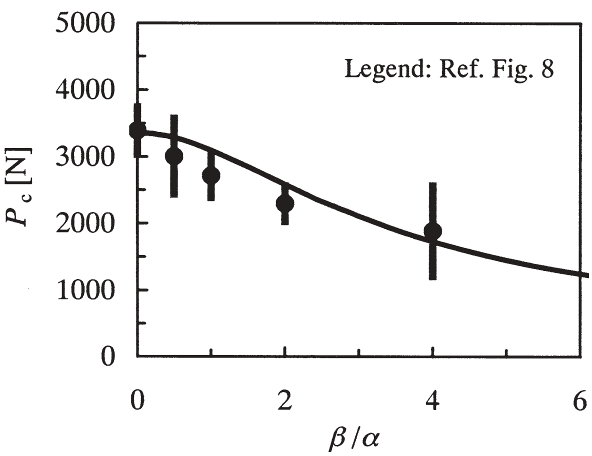

Fig. 9. Failure load as function of normalized crack length for h = 200 mm

(Eq. 8). For the test results, the mean value of the failure load obtained for $\beta h = 0$ (or in reality $\beta h = 7.5$ mm due to the dowel hole) has been used for P_{c0}. (Note: for $h = 100$ mm as well as for $h = 200$ mm, one excessively high failure load has been discarded in calculation of P_{c0}). Note that the only material property involved in μ is the ratio G/E.

In Figs. 8 and 9, the tested and calculated (Eqs. 7 and 8) failure loads are shown. Figures 6 and 7 indicate that the modification factor, μ, as given by Eq. 8 predicts an influence of the crack length in good agreement with test results. Figures 8 and 9 indicate that the original Van der Put/Leijten formula as given by Eq. 7 is in good agreement with test results if the so-called fracture parameter[13] (or apparent fracture energy) is determined by means of plate-joint specimens (Fig. 5).

According to Eq. 6, stable crack growth is not possible; catastrophic failure is deemed to occur once the crack starts propagating. This prediction is, however, not in accordance with the experiments, in which cases of crack growth were observed during increasing load. The loads given in the figures are the catastrophic failure loads; no attempts were made to measure the crack lengths at failure.

Table 1. Fracture properties determined by plate-joint test

h_e (mm)	C_1 (N/mm$^{3/2}$)	$\mathcal{G}_f$ (N/mm)
20	9.68 ± 1.29	0.181 ± 0.048
40	9.95 ± 1.56	0.193 ± 0.062
60	9.31 ± 1.85	0.171 ± 0.071
Mean ± sd	9.72 ± 1.56	0.185 ± 0.060

$\mathcal{G}_f$ based on $G = E/18$

Test series 2

It may be suspected that the good agreement between predicted and tested failure loads as shown in Figs. 8 and 9 obtained by use of the plate-joint specimens is due to the fact that the same constant edge distance ($h_e = 40$ mm) was used in all plate-joint specimens and beam specimens. Test series 2 was conducted in order to evaluate the influence of h_e on the fracture parameter, C_1.

The plate-joint specimens resulted in the values of the fracture parameter, C_1, and corresponding apparent fracture energy, $\mathcal{G}_f$, as given in Table 1. Table 1 shows that the fracture properties derived from plate-joint tests using Eq. 13 are constant in the tested range of edge distances.

It may be worthy of notice that the fracture parameter, C_1, obtained for the two test series ($E = 7530$ MPa and $E = 5670$ MPa) only varies insignificantly. However, if G is assumed to be a fixed ratio of E, Eq. 13 leads to significantly different values of the fracture energy for different E. Thus, if as usual a constant value of the fracture energy is used regardless of the MOE, the model presented here (and the Van der Put/Leijten model) may give somewhat poor predictions for some values of the MOE.

Such large variation in fracture energy due to variations in MOE as reported here is not in accordance with experiences using double cantilever beam (DCB) specimens for determination of the fracture energy. It is unclear at present whether the observed difference in fracture energy is due to an improper estimation of G as a constant fraction of E, or due to improper model assumptions, although the latter is the most likely. At any rate, use of the fracture parameter, C_1, determined from plate-joint tests using Eq. 13 seems to make theoretical predictions as given by Eq. 6 in good agreement with test results.

Although it would be more satisfactory to be able to use common values of shear modulus and fracture energy rather than resorting to introducing a new property like C_1, which needs to be determined by some special tests, this procedure is by no means unique. For instance in the treatment of glued lap joints,[14] glued-in steel rods,[15] and glued-in hardwood dowels,[16] it is commonly accepted that special tests are needed for determination of the bond line parameters, the values of which may be orders of magnitude different from those determined directly on the adhesive.

Testing standards are available and commonly used for testing of material properties (embedding strength and foundation moduli) needed as input in calculation models for prediction of ductile failure modes in dowel-type fastener connections. However, no testing standards seem available for testing of properties relevant to brittle splitting failure modes. It seems reasonable to suggest such standard tests, and because the plate-joint specimens are very easy to make and the tests very easy to conduct, this specimen is recommended as a standard test.

Conclusions

A linear elastic fracture mechanics model for calculation of the splitting failure load of beams loaded perpendicular to grain by dowel-type fastener connections was presented. The new model is based on an approach similar to the previous Van der Put/Leijten model,[2] but includes normal forces in the cracked parts of the beam. The Van der Put/Leijten solution appears as a special case, namely for zero crack length or by only considering contributions from shear deformations.

Like the Van der Put/Leijten model, the presented model is two-dimensional, i.e., it assumes that the fasteners are sufficiently stiff to ensure that crack propagation along the grain occurs simultaneously through the entire width of the beam. Especially for joints with large relative edge distances (α values), this condition may not always be fulfilled because the dowels are designed sufficiently slender so as to enable ductile bending at the ultimate limit state.

Because the new model includes the effect of crack length, it is easily extended from LEFM to generalized linear elastic fracture mechanics by adding the finite size of the fracture process region to the crack length. Tests were conducted to evaluate the influence of crack length, and good agreement was found between theoretical predictions and test results. Furthermore, tests were conducted to derive appropriate material properties needed as input parameters. It is strongly recommended that the so-called plate-joint specimen be used as a standard test specimen for determination of relevant fracture properties.

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