Optimum heating of cylindrical pressure vessels

P. Dzierwa¹ · D. Taler¹ · J. Taler²

Received: 4 November 2014 / Published online: 15 March 2016
© The Author(s) 2016. This article is published with open access at Springerlink.com

Abstract A method for determining time-optimum fluid temperature changes during heating of the thick walled cylinder was presented. Optimum fluid temperature changes were determined both for the cylindrical pressure vessels without holes. Heating of the hollow cylinder will be carried out in such a way that the circumferential thermal stress at the inner surface is equal to the allowable stress value. Optimum fluid temperature changes were assumed in the form of simple time functions containing unknown parameters. The unknown parameters were determined from the condition that the circumferential thermal stress at the inner surface of the hollow cylinder without holes is equal to the allowable stress at given time points. An over-determined system of nonlinear algebraic equations was solved for unknown parameters using the least squares method. At first, the thermal stress was calculated using the discrete form of the Duhamel integral. The Finite Element Method (FEM) was used to determine the circumferential thermal stress in the second method. For practical reasons the optimum temperature in the ramp form is preferred. It is possible to increase the fluid temperature stepwise at the beginning of the heating process and then increase the fluid temperature

with the constant rate. Because of the possibility of practical implementation a more appropriate is the ramp function for approximating optimum fluid temperature changes.

Optimierte Erwärmung zylindrischer Druckbehälter

Zusammenfassung Ein Verfahren zum Bestimmen der zeitoptimalen Fluid Temperaturänderungen während der Aufheizung des dickwandigen Zylinders wurde dargestellt. Optimale Fluidtemperatur – Änderungen wurden für den zylindrischen Druckbehälter ohne Löcher bestimmt. Das Erhitzen des Hohlzylinders erfolgt in der Weise, dass die thermische Umfangsspannung an der inneren Oberfläche gleich dem zulässigen Wert ist. Optimale Fluidtemperatur – Änderungen wurden in der Form von einfachen Zeitfunktionen mit unbekannten Parametern angenommen. Die unbekannten Parameter wurden aus der Bedingung, daß die thermische Umfangsspannung an der inneren Zylinderoberfläche bei gegebenen Zeitpunkten gleich der zulässigen Spannung ist. Ein überbestimmtes System von nicht-linearen algebraischen Gleichungen wurde für unbekannte Parameter mit Hilfe der Methode der kleinsten Quadrate gelöst. Zuerst wurde die thermische Spannung anhand der diskreten Form des Duhamelschen Integrals berechnet. In dem zweiten Verfahren wurde die Finite-Elemente-Methode (FEM) verwendet, um die tangentielle thermische Spannung zu bestimmen. Aus praktischen Gründen ist der rampenförmige optimale Temperaturverlauf bevorzugt. Es ist möglich zu Beginn des Aufheizvorgangs die Fluidtemperatur stufenweise zu erhöhen und dann die Temperatur der Flüssigkeit mit der konstanten Geschwindigkeit zu erheben. Aufgrund der Möglichkeit der praktischen Umsetzung ist eine rampenförmige Funktion zur Annäherung der optimalen Fluid – Temperaturänderungen sehr geeignet.

The research presented in this paper has been partially funded by the National Science Centre—Project No. 3239/B/T02/2011/40.

✉ J. Taler
taler@mech.pk.edu.pl

¹ Institute of Thermal Power Engineering,
Faculty of Mechanical Engineering,
Cracow University of Technology,
Al. Jana Pawła II 37, 31-864 Cracow, Poland

² Institute of Thermal Engineering and Air Protection,
Faculty of Environmental Engineering,
Cracow University of Technology,
ul. Warszawska 24, 31-155 Cracow, Poland

List of symbols

$a = \lambda / (c\rho)$	Thermal diffusivity, m ² /s
$Bi = \alpha r_{in} / \lambda$	Biot number
c	Specific heat capacity, J/(kgK)
E	Modulus of elasticity, MPa
f	Excess fluid temperature above the initial temperature, °C
$Fo = at / r_{in}^2$	Fourier number
$k_o = r_o / r_{in}$	Ratio of outer to inner radius
p	Absolute pressure, MPa
p_o	Atmospheric pressure, MPa
r	Radius, m
r_o, r_{in}	Outer and inner radius of the vessel, m
r, φ, z	Cylindrical coordinates
$\mathbf{r}_p$	Position vector of a point P on the edge of the hole
$R = r/r_{in}$	Dimensionless radius
s	Wall thickness, m
t	Time, s
T	Temperature, °C
T_f	Fluid temperature, °C
$\bar{T}$	Mean temperature over the wall thickness, °C
u_T	Influence function for temperature
u_N	Influence function for circumferential stress, MPa/K
v_T	Rate of temperature changes, K/s
α	Heat transfer coefficient, W/(m ² K)
α_m	Stress concentration factor for circumferential stress caused by pressure at point P on the edge of the hole
β	Linear thermal expansion coefficient, 1/K
Δt	Time step, s
λ	Thermal conductivity, W/(mK)
μ_n	nth root of characteristic equation
ν	Poisson's ratio
ρ	Density, kg/m ³
σ	Normal stress, MPa
σ_a	Allowable stress, MPa
σ_e	Equivalent stress, MPa
$\sigma_{p,r}, \sigma_{p,\varphi}, \sigma_{p,z}$	Radial, circumferential and axial component of the stress due to pressure, MPa
$\sigma_{T,r}, \sigma_{T,\varphi}, \sigma_{T,z}$	Radial, circumferential and axial thermal stress component, MPa

1 Introduction

Optimization of heating and cooling of thick boiler components is the subject of many studies [1–6], since too rapid heating or cooling element causes high thermal stresses.

There is a great need for reducing start-up and shut-down costs in thermal power plants. The main motivation is the new demand caused by a deregulated electricity market

where more frequent and shorter start-ups and shut-downs are necessary in order to realize the short-term power requests from the electrical load dispatcher [4]. Fast start-up and shutdown of the power units are desired to reduce fuel oil consumption during start-up and to decrease emissions of pollutants into the atmosphere.

The major limiting factor relevant to fast power plant start-ups is maximum allowable thermal stress for thick-walled components such as headers of superheaters and reheaters, boiler drums and steam turbine rotors. Any exceeding of the stress limit reduces lifetime of these components. The new control system proposed by Krüger et al. [4] aims at improving of the start-up procedures of boilers and explicitly takes the thermal stress values of critical components into account.

Taking into account the increasing share of wind farms in the production of electricity, combined—cycle gas and steam power plants must be quickly activated to supply the missing electrical energy into the power system when the wind velocity drops. The heat recovery steam generator (HRSG) is a critical element limiting fast start-up of the gas and steam power units. During the start-up period, HRSG components, especially the drum, are subject to high thermal stresses, which are caused by the non-uniform temperature distribution over the component wall thickness. To avoid excessive thermal stresses in the boiler drum, the flue gas exiting a gas turbine flows directly to the chimney bypassing the HRSG [6]. It makes possible to deliver electricity to the grid at the expense of low efficiency of the gas and steam unit. The gas bypass is open during the initial stage of the start-up. Estimation of the maximum thermal stress enables the optimization of the bypass mode [6]. In both papers drum is considered as a thick-walled cylinder in which the temperature distribution is a function of radius and time [4–5].

Allowable heating rates of thick-walled boiler pressure components are calculated in engineering practice using European Standard [7] which is based on the quasi-steady state temperature distribution inside the pressure element. Quasi steady state temperature distribution is formed in the thick-walled element after heating it at a constant rate for a long time [8].

Haneke and Speitkamp [9] determined approximate optimum fluid temperature changes and the optimum thickness of the thick-walled pressure vessels. They considered thick-walled pressure elements with openings. The optimum changes of the fluid temperature was approximated by a piecewise linear function.

A method of determining the optimum time changes of fluid temperature during heating process of boiler structural pressure elements was proposed in [10]. The optimization procedure is based on the discrete form of the Duhamel's integral.

Another method for optimum heating of thick walled pressure components was presented in [11, 12]. Allowing stepwise fluid temperature increase at the beginning of heating, the allowable heating rates were calculated according to the EN 12952-3 European Standard: v_{T1} for the initial pressure p_1 and v_{T2} for the final pressure p_2 .

Optimum heating of the thick walled plate is the subject of the paper by Taler [13]. A linear inverse heat conduction problem was solved using various methods when linear time temperature variation is prescribed at the insulated rear surface of the plate. Physical properties of plate material were assumed to be constant.

The paper presents a new method of determining the optimum fluid temperature changes during heating of thick walled hollow cylinders. Optimum temperature curve for a hollow cylinder without holes will be determined from the condition that the von Mises equivalent stress at the cylinder inner surface is equal to the permissible stress.

2 Optimum heating of cylindrical vessel

Optimum heating of a long hollow cylinder with free ends will be analyzed. Three thermal stress components, radial $\sigma_{T,r}$, circumferential $\sigma_{T,\varphi}$, and axial $\sigma_{T,z}$ are given by [14, 15]

$$\sigma_{T,r} = \frac{E\beta}{2(1-\nu)} \left(1 - \frac{r_{in}^2}{r^2} \right) [T_m(t) - T_{r,m}(r, t)] \quad (1)$$

$$\sigma_{T,\varphi} = \frac{E\beta}{2(1-\nu)} \left[\left(1 + \frac{r_{in}^2}{r^2} \right) T_m(t) + \left(1 - \frac{r_{in}^2}{r^2} \right) T_{r,m}(r, t) - 2T(r, t) \right] \quad (2)$$

$$\sigma_{T,z} = \frac{E\beta}{1-\nu} [T_m(t) - T_{r,m}(r, t)] \quad (3)$$

where the mean temperatures $T_{r,m}(r, t)$ and $T_m(t)$ are given by

$$T_{r,m}(r, t) = \frac{2}{r^2 - r_{in}^2} \int_{r_{in}}^r T r \, dr \quad (4)$$

$$T_m(t) = \frac{2}{r_o^2 - r_{in}^2} \int_{r_{in}}^{r_o} T r \, dr \quad (5)$$

The stress components in a thick walled cylinder under internal pressure loading are [14, 18]

$$\sigma_{p,r} = -(p - p_o) \frac{\left(\frac{r_o}{r} \right)^2 - 1}{\left(\frac{r_o}{r_{in}} \right)^2 - 1} \quad (6)$$

$$\sigma_{p,\varphi} = (p - p_o) \frac{\left(\frac{r_o}{r} \right)^2 + 1}{\left(\frac{r_o}{r_{in}} \right)^2 - 1} \quad (7)$$

$$\sigma_{p,z} = (p - p_o) \frac{1}{\left(\frac{r_o}{r_{in}} \right)^2 - 1} \quad (8)$$

Equations (1–8) were derived assuming the elastic state of stress and constant physical properties which are independent of the position and temperature. The cylinder wall temperature varies only in the radial direction while in the axial and circumferential directions remains constant. For the flow of steam, hot water or other fluids changes of temperature in the axial direction are small compared to the radial direction. Non-uniformity of the temperature on the perimeter of the cylindrical vessel occurs only for steam condensation or stratified flow in the horizontal vessels. The equivalent stress at the inner surface of the cylinder is calculated according to the Maxwell–Huber–Hencky–von Mises theory as follows

$$\sigma_e = \sigma_{\max} - \sigma_{\min} = \left(\sigma_{T,r}|_{r=r_{in}} + \sigma_{p,r}|_{r=r_{in}} \right) - \left(\sigma_{T,\varphi}|_{r=r_{in}} + \sigma_{p,\varphi}|_{r=r_{in}} \right) \quad (9)$$

Substituting Equations: (1), (6), (2), and (7) for $r=r_{in}$ into (9) gives

$$\sigma_e = \sigma_{\max} - \sigma_{\min} = -(p - p_o) \left(1 + \frac{r_o^2 + r_{in}^2}{r_o^2 - r_{in}^2} \right) - \frac{E\beta}{1-\nu} (T_m - T|_{r=r_{in}}) \quad (10)$$

If fluid temperature $f(t)$ is time dependent, the temperature $T(r_{in}, t)$ and the circumferential stress $\sigma_{T,\varphi}(r_{in}, t)$ at the cylinder inner surface will be calculated using the Duhamel integral [16, 17]

$$T(r_{in}, t) = \int_0^t f(\theta) \frac{\partial u_T(r_{in}, t - \theta)}{\partial t} d\theta \quad (11)$$

$$\sigma_{T,\varphi}(r_{in}, t) = \int_0^t f(\theta) \frac{\partial u_S(r_{in}, t - \theta)}{\partial t} d\theta \quad (12)$$

where $f(t) = T_f(t) - T_o$ designates time dependent fluid excess temperature. The initial temperature T_o is assumed to be uniform.

Equations (11) and (12) are based on the superposition principle, which in turn can be used for solving linear problems. This means that the material properties must be independent of temperature.

The function $u_T(r_{in}, t)$ is a solution of the initial boundary problem at the surface $r=r_{in}$, for a unit step function, $f(t)=1$, $t>0$. This function is also called an influence function. The function $u_S(r_{in}, t)$ represents time changes of the circumferential thermal stress component at the inner surface $r=r_{in}$ caused by the unit stepwise increase in the fluid temperature for $t>0$.

Time changes in the fluid temperature are approximated by a stepwise function (Fig. 1)

$$\begin{aligned} f_1 &= f(\theta_1/2) & 0 \leq t \leq \theta_1, \\ f_2 &= f[\theta_1 + (\theta_2 - \theta_1)/2] & \theta_1 \leq t \leq \theta_2, \\ &\vdots \\ f_i &= f[\theta_{i-1} + (\theta_i - \theta_{i-1})/2] & \theta_{i-1} \leq t \leq \theta_i, \\ &\vdots \\ f_M &= f[\theta_{M-1} + (\theta_M - \theta_{M-1})/2] & \theta_{M-1} \leq t \leq \theta_M. \end{aligned} \quad (13)$$

Coordinates of the points in time θ_i are given by

$$\theta_i = \theta_{i-1} + \Delta\theta_i, i=1, \dots, M \quad (14)$$

where $\theta_0=0$.

If time steps $\Delta\theta_i$ are constant and equal $\Delta\theta = t_M / M$ then

$$\theta_i = i\Delta\theta, i=0, \dots, M \quad (15)$$

Using the method of rectangles, the integral in the formula (1) for temperature at time t_M gives

$$\begin{aligned} T(r_{in}, t_M) &= \sum_{i=1}^M f_i \frac{u_T(r_{in}, t_M - \theta_{i-1}) - u_T(r_{in}, t_M - \theta_i)}{\Delta t_i} \Delta\theta_i \\ &= \sum_{i=1}^M f_i [u_T(r_{in}, t_M - \theta_{i-1}) - u_T(r_{in}, t_M - \theta_i)] = \\ &= f_1 [u_T(r_{in}, t_M - \theta_0) - u_T(r_{in}, t_M - \theta_1)] + \\ &\quad f_2 [u_T(r_{in}, t_M - \theta_1) - u_T(r_{in}, t_M - \theta_2)] + \\ &\quad \vdots \\ &\quad f_M [u_T(r_{in}, t_M - \theta_{M-1}) - u_T(r_{in}, t_M - \theta_M)] \end{aligned} \quad (16)$$

where $\theta_0 = 0$ and $u(r_{in}, t_M - \theta_M) = u(r_{in}, 0) = 0$.

The symbol $T(r_{in}, t_M)$ designates the wall temperature excess above the initial temperature T_0 which is constant through the thickness of the cylinder wall. Circumferential stresses $\sigma_{T,\varphi}(r_{in}, t_M)$ are calculated in the same way as the temperature $T(r_{in}, t_M)$. To determine the influence function $u_S(r_{in}, t_M)$ for circumferential thermal stresses, the influence function $u_T(r_{in}, t_M)$ for the wall temperature, must be found at first. If we assume that the cylinder outer surface is thermally insulated (Fig. 2), then the influence function $u_T(r, t)$ is given by:

Fig. 1 Stepwise approximation of fluid temperature changes

Fig. 2 Infinite hollow cylinder with insulated outer surface and convective heat transfer at the inner surface

$$\frac{\partial u_T}{\partial Fo} = \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial u_T}{\partial R} \right), \quad 1 \leq R \leq k_o \quad (17)$$

for the following boundary and initial conditions

$$-\frac{\partial u_T}{\partial R}\bigg|_{R=1} = Bi(1 - u_T|_{R=1}) \quad (18)$$

$$\frac{\partial u_T}{\partial R}\bigg|_{R=k} = 0 \quad (19)$$

$$u_T|_{Fo=0} = 0 \quad (20)$$

Here $Bi = \alpha r_{in} / \lambda$ denotes the Biot number, $k_o = r_o / r_{in}$ is the ratio of the outer to inner radius, $Fo = at / r_{in}^2$ the Fourier number, and $R = r / r_{in}$.

The solution of the initial-boundary problem (17–20) is [16–19]

$$u_T = 1 - \sum_{n=1}^{\infty} D_n P_n \exp(-\mu_n^2 Fo) \quad (21)$$

where

$$D_n = \frac{\pi \frac{Bi}{\mu_n} \left[J_1(\mu_n) + \frac{Bi}{\mu_n} J_0(\mu_n) \right]}{\left[J_1(\mu_n) + \frac{Bi}{\mu_n} J_0(\mu_n) \right]^2 - \left[1 + \left(\frac{Bi}{\mu_n} \right)^2 \right] J_1(k_o \mu_n)} \quad (22)$$

$$P_n = J_1(k_o \mu_n) Y_0(R \mu_n) - Y_1(k_o \mu_n) J_0(R \mu_n) \quad (23)$$

The roots μ_n are determined from the solution of the following characteristic equation

$$\frac{Y_1(k_o \mu)}{J_1(k_o \mu)} \cdot \frac{J_1(\mu) + \frac{Bi}{\mu} J_0(\mu)}{Y_1(\mu) + \frac{Bi}{\mu} Y_0(\mu)} = 1 \quad (24)$$

$$J_0(\mu) Y_1(k_o \mu) - J_1(k_o \mu) Y_0(\mu) = 0, \text{ for } Bi \rightarrow \infty \quad (25)$$

where J_0 and J_1 are the zero and first order Bessel functions of the first kind, Y_0 and Y_1 are the zero and first order Bessel functions of the second kind. The first ten roots of the characteristic equations (24) and (25) for selected values of the ratio k_o are shown in Tables 1 and 2 respectively. The roots listed in Tables 1 and 2 were obtained by solving transcendental equations by the Müller root-finding algorithm [20, 21]. The circumferential stress in the hollow cylinder with free ends due to unit stepwise increase in fluid temperature is given by

$$u_N = u_N(R, Fo) = \frac{E\beta}{1-\nu} [\bar{u}_T - u_T] \quad (26)$$

where $\bar{u}_T$ is the average temperature over the wall thickness which is defined as

$$\bar{u}_T = \frac{2}{k_o^2 - 1} \int_1^{k_o} R u_T dR \quad (27)$$

Substituting expression (21) into equations (27) and (26) gives

$$u_N = \frac{E\beta}{1-\nu} \sum_{n=1}^{\infty} D_n \left\{ \frac{2}{(k_o^2 - 1) \mu_n} [J_1(\mu_n) Y_1(k_o \mu_n) - J_1(k_o \mu_n) Y_1(\mu_n)] - P_n \right\} \exp(-\mu_n^2 Fo) \quad (28)$$

If the ends of the hollow cylinder are free, axial and circumferential stresses at the inner as well as at the outer surface are equal. The previous optimization studies show [10, 11] that the optimum fluid temperature changes $T_f(t)$ obtained from the solution of the Volterra integral equation of the first kind, can be well approximated

$$T_f = T_0 + a + bt + c/t \quad (29)$$

where: T_0 —initial fluid temperature, a , b , c —constants, t —time.

At first, the optimum fluid temperature changes are approximated by the ramp function $T_f(t)$ (Fig. 2)

$$T_f = T_0 + a + bt \quad (30)$$

which can easily be carried out in practice. The symbols in Eq. (30) stand for: a —initial stepwise temperature increase, b —constant rate of fluid temperature changes. The optimum values of parameters a , b and c appearing in the function (29) or the parameters a and b in the function (30) will be determined from the condition

$$\sigma_e(r_{in}, t_i) \equiv \sigma_a, \quad i = 1, \dots, n_t \quad (31)$$

where: $\sigma_e(r_{in}, t_i)$ —the equivalent von Mises stress at the inner surface, r_{in} —inner radius of the cylinder, t_i — i -th time point, n_t —number of time points in the analyzed period of time, σ_a —allowable stress determined using current standards.

The overdetermined system of nonlinear algebraic equations (31) was solved using the least squares method. Optimum values of searched parameters a and b or a , b , and c were determined from the condition:

$$\sum_{i=1}^{n_t} [\sigma_e(r_{in}, t_i) - \sigma_a]^2 = \min \quad (32)$$

by the Levenberg Marquardt method [21].

If the pressure p inside the cylinder is equal to the ambient pressure p_o , condition (9) simplifies to the form

Table 1 Roots μ_n of characteristic equation (24)

μ_i	$Bi = \frac{\alpha r_{in}}{\lambda}$	$k_o = r_o/r_{in}$					
		1.1	1.2	1.4	1.6	1.8	2.0
μ_1	5	6.37638	4.10599	2.47154	1.76594	1.36587	1.10791
	10	8.39980	5.14618	2.91462	2.01227	1.52320	1.21751
	20	10.52048	6.05539	3.23569	2.17535	1.62209	1.28414
	40	12.36636	6.70752	3.43415	2.27039	1.67798	1.32110
	60	13.20543	6.96755	3.50715	2.30438	1.69768	1.33401
	80	13.68337	7.10697	3.54502	2.32182	1.70774	1.34058
μ_2	5	32.92922	17.13285	9.10328	6.33122	4.89375	4.00241
	10	34.25400	18.20306	9.80180	6.80585	5.23144	4.25264
	20	36.41490	19.63103	10.50287	7.20663	5.48700	4.42866
	40	39.30119	21.06000	11.02707	7.46956	5.65192	4.53201
	60	41.04864	21.73637	11.23432	7.56607	5.70012	4.56900
	80	42.18820	22.12187	11.34427	7.61815	5.72920	4.58783
μ_3	5	63.62090	32.19391	16.45439	11.17975	8.52063	6.90908
	10	64.37606	32.89418	17.03102	11.64286	8.89144	7.20806
	20	65.78187	34.06045	17.79229	12.14851	9.24238	7.46245
	40	68.12635	35.61035	18.50987	12.54142	9.48571	7.62667
	60	69.89947	36.49580	18.82794	12.69782	9.57758	7.68687
	80	71.23045	37.05379	19.00356	12.78057	9.62537	7.71789
μ_4	5	94.77802	47.65324	24.08237	16.21607	12.27295	9.89978
	10	95.29529	48.15190	24.53275	16.61106	12.61516	10.19375
	20	96.29689	49.05829	25.23730	17.13050	13.00537	10.49233
	40	98.11500	50.46198	26.02296	17.60819	13.31589	10.70771
	60	99.65708	51.41329	26.42221	17.81495	13.44008	10.79000
	80	100.93400	52.06524	26.65265	17.92723	13.50574	10.83289
μ_5	5	126.06280	63.23115	31.81291	21.33624	16.09407	12.94375
	10	126.45600	63.61534	32.17395	21.66819	16.39434	13.21489
	20	127.22460	64.34005	32.78292	22.16086	16.78864	13.53039
	40	128.67680	65.56509	33.58286	22.68388	17.14638	13.78623
	60	129.98400	66.49017	34.03271	22.93186	17.29913	13.88890
	80	131.13370	67.17609	34.30704	23.07070	17.38158	13.94317
μ_6	5	157.39910	78.86008	39.59018	26.49795	19.94999	16.01900
	10	157.71370	79.17117	39.88872	26.78024	20.21297	16.26187
	20	158.33610	79.77008	40.50014	27.23302	20.59345	16.57964
	40	159.53700	80.83500	41.18840	27.77437	20.98098	16.86550
	60	160.64770	81.69999	41.66551	28.05286	21.15813	16.98655
	80	161.67060	82.37418	41.97190	28.21477	21.25608	17.05163
μ_7	5	188.76210	94.51521	47.39200	31.68345	23.82800	19.11330
	10	189.02550	94.77634	47.64553	31.90219	24.05884	19.33084
	20	189.54830	95.28437	48.11044	32.33905	24.41809	19.64000
	40	190.56510	96.21706	48.83464	32.87909	24.82126	19.94693
	60	191.53080	97.01323	49.32102	33.18002	25.01856	20.08419
	80	192.43330	97.67191	49.64924	33.36110	25.13056	20.15941
μ_8	5	220.14010	110.18510	55.20803	36.88237	27.71896	22.21967
	10	220.36640	110.41000	55.42818	37.09546	27.92383	22.41529
	20	220.81630	110.85030	55.83986	37.46998	28.25860	22.71200
	40	221.70000	111.67570	56.51490	37.99789	28.66784	23.03019
	60	222.54500	112.40500	56.99776	38.31281	28.88121	23.18247
	80	223.34980	113.03060	57.34000	38.51061	29.00571	23.26704
μ_9	5	251.52800	125.86500	63.03399	42.08998	31.61800	25.33410
	10	251.72600	126.06200	63.22772	42.27941	31.80141	25.51088
	20	252.12000	126.45000	63.59597	42.62099	32.11295	25.79197
	40	252.89410	127.19100	64.22296	43.13015	32.52084	26.11896
	60	253.65050	127.85630	64.69708	43.44986	32.74650	26.28177
	80	254.37380	128.44580	65.04505	43.66381	32.88197	26.37499

Table 1 (continuation)

μ_{10}	5	282.92100	141.55010	70.86611	47.30404	35.52296	28.45403
	10	283.09800	141.72600	71.03886	47.47403	35.68903	28.61505
	20	283.89300	142.07280	71.37132	47.78694	35.97911	28.88111
	40	284.14280	142.73980	71.95402	48.27415	36.38008	29.21006
	60	284.87300	143.34260	72.41243	48.60157	36.61469	29.38206
	80	285.47530	143.90650	72.76199	48.82077	36.75959	29.48341

Table 2 Roots μ_n of characteristic equation (25)

k_o	1.1	1.105882	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	3.0	4.0
n													
1	15.4060	14.5335	7.5663	4.9619	3.6645	2.8899	2.3761	2.0110	1.7388	1.5282	1.3608	0.6256	0.3935
2	47.0248	44.4033	23.4688	15.6198	11.6972	9.3448	7.7775	6.6583	5.8194	5.1673	4.6459	2.3040	1.5266
3	78.4791	74.1265	39.2138	26.1272	19.5847	15.6601	13.0441	11.1761	9.7750	8.6855	7.8141	3.8954	2.5908
4	109.9132	103.7941	54.9381	36.6126	27.4532	21.9570	18.2932	15.6765	13.7140	12.1880	10.9671	5.4751	3.6456
5	141.3353	133.4832	70.6553	47.0947	35.3152	28.2472	23.5365	20.1711	17.6479	15.6850	14.1151	7.0509	4.6970
6	172.7474	163.1677	86.3708	57.5720	43.1742	34.5348	28.7772	24.6639	21.5792	19.1802	17.2607	8.6249	5.7470
7	204.1718	192.8262	102.0802	68.0516	51.0315	40.8223	34.1063	29.1549	25.5093	22.6731	20.4051	10.1980	6.7961
8	235.5981	222.5045	117.7868	78.5218	58.8864	47.1079	39.2547	33.6453	29.4383	26.1662	23.5487	11.7704	7.8447
9	267.0048	252.1756	133.5009	88.9947	66.7440	53.3914	44.4925	38.1350	33.3671	29.6586	26.6918	13.3424	8.8930
10	298.4339	281.8564	149.2068	99.4693	74.5968	59.6776	49.7298	42.6254	37.2957	33.1505	29.8347	14.9144	9.9411

$$-\sigma_{T,\varphi}(r_{in}, t_i) \cong \sigma_a, \quad i = 1, \dots, n_t \quad (33)$$

Optimum fluid temperature changes during heating of the hollow cylinder will be calculated for $p = p_o$ and $p > p_o$.

The solution steps are depicted in the flow chart (Fig. 3).

When considering a vessel with holes, the method proposed in the paper may also be used for determining the optimum temperature of the fluid. In that case, the sum of squared differences of the calculated circumferential stress: $\sigma_\varphi = \sigma_{T,\varphi} + \sigma_p$ and allowable stress σ_a for the selected n_t time points at the point P of the maximum stress concentration on the hole edge should be minimum

$$\sum_{i=1}^{n_t} \left[\int_0^{t_i} f(\theta) \frac{\partial u(r_p, t - \theta)}{\partial t} d\theta + \alpha_m (p - p_o) \frac{2r_{in} + s}{2s} - \sigma_a \right]^2 = \min \quad (34)$$

Fluid excess temperature $f(\theta) = T_f - T_o$ in the sum (34) can be assumed a function (29) or (30).

3 Test calculations

To illustrate the effectiveness of the proposed method test calculations have been carried out for a water separator of the supercritical boiler and for a drum installed in the boiler with natural circulation.

The following data were adopted for the water separator: $r_{in} = 0.255$ m, $r_o = 0.315$ m, $\lambda = 29.45$ W/(mK); $c = 513.75$ J/(kgK); $\rho = 7782.5$ kg/m³; $E = 2.07 \cdot 10^{11}$ N/m²; $\beta = 1.13 \cdot 10^{-5}$ 1/K, and $\nu = 0.3$. The heat transfer coefficient on the inner surface of the water separator was assumed

to be: $\alpha = 2000$ W/(m²K). The pressure inside the vessel is equal to atmospheric pressure i.e. $p = p_o$. The first 12 roots of the characteristic equation (24) are: $\mu_1 = 5.12868$, $\mu_2 = 16.71547$, $\mu_3 = 28.98302$, $\mu_4 = 41.72671$, $\mu_5 = 54.71176$, $\mu_6 = 67.82378$, $\mu_7 = 81.00823$, $\mu_8 = 94.23722$, $\mu_9 = 107.49529$, $\mu_{10} = 120.77334$, $\mu_{11} = 134.06564$, $\mu_{12} = 147.36846$. A comparison of the influence function calculated analytically and using the FEM for the heat transfer coefficient equal to $\alpha = 2000$ W/(m²K) is shown in Fig. 4. The compatibility of the results is very good. Small differences between the results may be observed at the beginning of heating. This difference is mainly due to insufficient accuracy of the analytical solution (28) for small values of the time, in spite of the 12 terms in the series.

In order to assess the accuracy of the method, first the circumferential stress on the inner surface of the water separator was calculated using the FEM. The fluid temperature T_f is given by the following formula

$$T_f = 80 + 0.2t \quad (35)$$

The calculated circumferential stress is shown in Fig. 5. Thus this circumferential stress is considered as a time dependent allowable stress σ_a . The allowable stress $\sigma_a(t_i)$ in (33) was given at 50 time points: $t_i = 4 + 40.61(i-1)$, $i = 1, \dots, 50$. The allowable stresses are indicated in Fig. 5 by means of hollow circles. The optimum temperature determined from the condition (32) is illustrated in Fig. 5. The optimum fluid temperature

$$T_f = 79.9029 + 0.1999t \quad (36)$$

Fig. 3 Flow chart illustrating determination of the optimum fluid temperature, defined by the function (29)

coincides with the temperature (35) adopted for the solution of the direct problem (Fig. 5). In the next test calculation the optimum fluid temperature was determined from the condition (32) approximating fluid temperature changes by the function (29) or (30). In addition, the optimum fluid temperature was determined from the solution of the integral equation

$$\sigma_{T,\phi} = \sigma_a \quad (37)$$

Fig. 4 Comparison of influence function for water separator calculated by analytical method and FEM for $\alpha = 2000 \text{ W/(m}^2\text{K)}$

Fig. 5 Allowable stress and optimum fluid temperature for water separator

where the circumferential stress is given by the Duhamel integral (12). The method of rectangles was used to solve sequentially (37) [11]. The allowable stress $\sigma_a = -253.3 \text{ MPa}$ was determined using the European Standard EN 12952-3 assuming 2000 start-ups of the boiler from the cold state. The comparison of the results illustrates Fig. 6. Larger differences between the optimum temperature runs occur only at the beginning of the heating process.

Next, optimum fluid temperature changes were estimated for a thick walled boiler drum. The dimensions of the

Fig. 6 Optimum fluid temperature during heating water separator for allowable circumferential stress equal to -253.3 MPa

Fig. 7 Comparison of influence function u_N for boiler drum calculated by analytical method and FEM

Table 3 Roots of characteristic equation (24) for the hollow cylinder for $k_o = r_o/r_m = 1.10588$

μ_i	Bi=5	Bi=10	Bi=20	Bi=30	Bi=80
μ_1	6.16106	8.08822	10.08103	11.13402	12.98685
μ_2	31.17888	32.48773	34.59562	36.16588	40.03724
μ_3	60.12975	60.88211	62.27421	63.50099	67.51989
μ_4	89.54222	90.05928	91.05506	91.98782	95.57679
μ_5	119.08148	119.47360	120.24079	120.97858	124.08235
μ_6	148.67273	148.98807	149.60978	150.21570	152.90117
μ_7	178.29025	178.55378	179.07559	179.58799	181.93378
μ_8	207.92287	208.14914	208.59835	209.04154	211.11394
μ_9	237.56494	237.76315	238.15733	238.54742	240.39827
μ_{10}	267.21334	267.38966	267.74072	268.08890	269.75803
μ_{11}	296.86616	297.02494	297.34134	297.65563	299.17378
μ_{12}	326.52222	326.66666	326.95455	327.24089	328.63202

drum are: inner radius $r_{in}=850$ mm and wall thickness $s=90$ mm. The following properties of steel were adopted for the calculation: $\lambda=47.3$ W/(mK); $c=511$ J/(kgK); $\rho=7775$ kg/m³; $E=2.07 \cdot 10^{11}$ N/m²; $\beta=1.13 \cdot 10^{-5}$ 1/K, and $\nu=0.3$. Allowable stress is: $\sigma_a=-68$ MPa. To calculate the influence function $u_N(t)$ for various Biot numbers the first twelve roots of (24) were determined (Table 3). Time changes of the influence function for various Biot numbers obtained using the analytical solution (26) and the Finite Element Method (FEM) are compared in Fig. 7. The heat transfer coefficient on the inner surface of the drum was assumed to be: $\alpha=1000$ W/(m²K). First, optimum fluid temperature were estimated by solving overdetermined system of non-linear algebraic equations (33). At every time step, the

Table 4 Optimum temperature changes during heating of the hollow cylinder

Bi		$T_f = T_0 + a + bt + c/t$			$T_f = T_0 + a + bt$	
		a, K	$b, K/s$	$C, K \cdot s$	a, K	$b, K/s$
5	FEM	137.4	0.0832	528.6	140.1	0.0826
	Analytical	136.18	0.0834	484.8	139.28	0.0824
10	FEM	80.79	0.0833	261.72	82.39	0.0830
	Analytical	79.93	0.0834	239.33	82.35	0.0821
20	FEM	52.54	0.0834	128.45	53.54	0.0832
	Analytical	51.81	0.0834	117.83	53.32	0.0826
30	FEM	43.13	0.0835	84.22	43.85	0.0833
	Analytical	42.43	0.0834	77.60	43.80	0.0823
80	FEM	31.21	0.0836	29.76	31.41	0.0835
	Analytical	30.68	0.0834	28.13	31.21	0.0830

circumferential stress at the inner surface was calculated using discrete form of the Duhamel integral. The unknown parameters a , b , and c in function (29) or parameters a and b in function (30) were determined by the least squares method applying the Levenberg- Marquardt method. In the second approach thermal stresses were computed using the Ansys software [22] without introducing the influence function. Thermal stresses at the cylinder inner surface were calculated for given time points using the FEM method at every iteration step of the Levenberg-Marquardt algorithm. The optimum fluid temperature changes are shown in Figs. 8 and 9. In addition, the optimum parameter values appearing in the functions (29) and (30) are given in Table 4. The analysis of the results shown in Figs. 8 and 9 indicates

Fig. 8 Comparison of optimum fluid temperature changes for boiler drum approximated by function (29) using the Duhamel integral and FEM

Fig. 9 Comparison of optimum fluid temperature changes for boiler drum approximated by function (30) using the Duhamel integral and FEM

Fig. 10 Comparison of the calculated and allowable circumferential stress during optimum heating of the boiler drum—fluid temperature given by three parameter function (29)

Fig. 11 Comparison of the calculated and allowable circumferential stress during optimum heating of the boiler drum—fluid temperature given by two parameter function (30)

that optimum temperature changes determined by the both described methods differ slightly. During optimum heating the circumferential thermal stresses at the inner surface are equal approximately the allowable value (Figs. 10 and 11). If the optimum temperature is approximated by the function (29) containing two unknown parameters then the agreement between the calculated and the allowable stress

is better (Fig. 10) in comparison with two parameter curve given by the function (30). The discrepancy between the calculated and allowable stress is larger at the beginning of the heating process (Fig. 11). The form of the optimum temperature change given by (30) is too simple to make calculated circumferential stress equal to the allowable value. However, the fluid time temperature changes resulting

Fig. 12 Influence of inner pressure on optimum heating of the boiler drum

from (30) can be easily conducted in the practice. Fig. 12 illustrates the influence of the inner pressure on optimum fluid temperature changes. With increasing pressure the initial stepwise temperature increase, as well as, the heating rate v_T goes up.

Conclusions

Two methods for determining optimum fluid temperature changes during heating of the hollow cylinder were presented. During optimum heating of the cylinder the circumferential thermal stress at the inner surface of the cylinder is equal to the allowable value. Optimum fluid temperature was approximated by simple functions. Unknown parameters appearing in these functions were estimated using the Levenberg—Marquardt method. The circumferential thermal stress was evaluated using the Duhamel integral or the Finite Element Method. Both methods give almost identical results. Since the influence function is calculated from an exact analytical solution, the first method based on the Duhamel integral method can be used to assess the accuracy of numerical solutions, for example solutions obtained by the FEM. When the FEM is used to compute thermal stresses then time optimum fluid temperature changes may be determined for temperature dependent physical material properties and complex form of the construction element can easily be accounted for.

Compliance with Ethical Standards

Conflict of interest The authors declare that there are no actual or potential conflicts of interest in relation to this article.

Open Access This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

References

1. Rop P (2010) Drum Plus: a drum type HRSG with Benson benefits. *Mod Pow Syst* 30:35–40
2. Staff report (2007) Dealing with cycling: TRD 301 and Euro Norm compared. *Mod Pow Syst* 27:33–38
3. Taler J (1997) Überwachung von instationären Wärmespannungen in dickwandigen Bauteilen. *Forsch Ingenieurwes* 63:127–135
4. Krüger K, Franke R, Rode M (2004) Optimization of boiler start-up using a nonlinear boiler model and hard constraints. *Energy* 29:2239–2251
5. Kim TS, Lee DK, Ro ST (2000) Analysis of thermal stress evolution in the steam drum during start-up of a heat recovery steam generator. *Appl Therm Eng* 20:977–992
6. Kehlhofer R, Hannemann F, Stimmann F, Rukes B (2009) Combined-cycle gas and steam turbine power plants, 3rd edn. PennWell Corporation, Tulsa
7. EN 12952-3 Water-tube boilers and auxiliary installations—Part 3: design and calculation for pressure parts. European Committee for Standardization, Brussels
8. Dzierwa P (2013) Quasi-steady-state approach for solving transient heat conduction problems. In: Hetnarski R (ed) *Encyclopedia of thermal stresses*. Springer, Berlin (in print)
9. Haneke R, Speitkamp L (1987) Optimierung dickwandiger Kesselbauteile im Hinblick auf kurze An- und Abfahrzeiten. *VGB Kraftwerkstechnik* 67(9):858–862
10. Taler J, Dzierwa P, Taler D (2009) Optimum heating of pressure components of large steam boilers. *Forsch Ingenieurwes* 73(3):183–192
11. Taler J, Dzierwa P (2011) A new method for optimum heating of steam boiler pressure components. *Int J Energy Res* 35:897–908
12. Dzierwa P (2013) Optimum heating of pressure components of complex shape. In: Hetnarski R (ed) *Encyclopedia of thermal stresses*. Springer, Berlin (in print)
13. Taler D, Taler J (2009) Optimum heating of thick plate. *Int J Heat Mass Transf* 52(9–10):2335–2342
14. Timoshenko SP, Goodier JN (1970) *Theory of elasticity*. McGraw-Hill, New York
15. Parkus H (1976) *Thermoelasticity*. Springer, New York
16. Carslaw HS, Jaeger JC (2000) *Conduction of heat in solids*, 2nd edn. Oxford University Press, New York
17. Taler J, Duda P (2006) *Solving direct and inverse heat conduction problems*. Springer, Berlin
18. Schweigerer S (1978) *Festigkeitsberechnung im Dampfkessel-, Behälter- und Rohrleitungsbau*. Springer, Berlin
19. Elizarov DP (1971) Thermal shock in steam lines of thermal power stations. *Tieploenergetika (Thermal Engineering)* 18(2):78–82
20. The IMSL Fortran 90 MP mathematics and statistics libraries (F90 MP). Visual Numerics Inc., Houston
21. Press WH, Teukolsky SA, Vetterling WT, Flannery BP (2008) *Fortran numerical recipes*, 2nd edn. Cambridge University Press, Cambridge
22. Ansys, Release 13.0, ANSYS, Inc., Southpointe, 275 Technology Drive, Canonsburg